

Spin injection into strongly interacting matter as a probe to the topological sector in gauge systems

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ABSTRACT

A rapidly rotating system of strongly interacting matter at high temperature and density has been created and observed in relativistic heavy-ion collision experiments. However, the theoretical framework for describing the dynamical evolution of spin degrees of freedom still requires further development. In this research project, we have formulated a thermodynamic system in which spin is introduced as an external environmental parameter, and we have investigated the thermal properties induced by spin. We have discovered that the spin carried by gauge particles is closely related to the topological current in the gauge sector, opening a novel direction for exploring the nonperturbative nature of hot and dense matter.

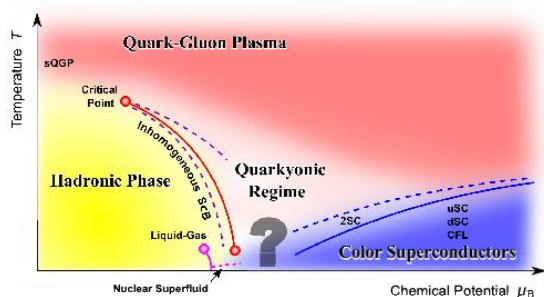
1- Introduction

Quarks and gluons are elementary particles that constitute nucleons as their bound states. The dynamics of quarks and gluons is formulated in a form of the gauge field theory that is called Quantum Chromodynamics (QCD). This fundamental theory of the Strong Interaction belongs to a family of gauge theories; one well-known member of this family is the dynamics between electrons and photons, i.e., Quantum Electrodynamics (QED). In QED, electrons and positrons, antiparticles of electrons, are charged and they feel attractive or repulsive Electromagnetic force as a result of photon exchange. In nature, there are two more family members representing the fundamental interactions, namely, the Weak Interaction, and the Gravity. Among these four interactions in the Standard Model, the Strong Interaction plays a unique role in our understanding of the Universe. The Electromagnetic Interaction, the

Weak Interaction, and the Gravity have their validity ranges, and it is believed that a more complete theory beyond the Standard Model should exist to take them over at some energy scales. However, the Strong Interaction has no breakdown in theory and so far, it has been successful in explaining all phenomena from microscopic scale $\sim 10^{-16}$ m inside the nucleon to macroscopic scale $\sim 10^4$ m inside the neutron star. It is only QCD that possesses such a powerful predictive power for phenomena of different energy scales over 20 orders of magnitude.

There are many interesting applications of QCD; see a review [1]. One example is the existence of the QCD phase transition in extreme environments such as high temperature, high density, strong external fields, and so on. In a quantum field theory like QCD, the ground state called the QCD vacuum is not an empty state, but it has rich structures.

It is a widely accepted idea that the QCD vacuum has special properties to confine any charges in the Strong Interaction and generate the mass of materials we observe in the Universe. The former is called “Color Confinement” because the QCD charge, a counterpart of the electric charge in QED, has a name, color, associated with the nature that there are three independent directions and their superposition makes white color or color neutral. The latter, mass generation, is closely related to Color Confinement. The theory of QCD tells us that these special properties are lost at high enough temperatures, and (nearly) massless quarks and gluons are deconfined if matter is heated up to the extreme temperature scale of $\sim 10^{12}$ K. This is the QCD phase transition, which is expected to occur in the heavy-ion collision experiments. Above the phase transition temperature, a new state of matter, a quark-gluon plasma (QGP), has been



experimentally confirmed. This is a state analogous to the plasma in which atoms are deconfined into electrons and ions.

Fig. 1: An example of a conjectured QCD phase diagram in the plane of temperature and baryon chemical potential.

The phase diagram of QCD matter is one big unsolved problem in contemporary physics (see Fig. 1 for example). The phases along the temperature axis have only been understood both theoretically and experimentally, but the QGP is characterized by not only the temperature but also other mechanical variables. Among various possibilities, a particularly interesting one is the angular momentum or the angular velocity of a rotating QGP fluid. In realistic experimental situations, two heavy-ion beams collide, and more than a hundred million events are all recorded and

analyzed. Then, some are head-on collisions and others are peripheral collisions. The system of two peripherally colliding heavy ions has a large angular momentum, and the QGP created after the collision should be rapidly rotating. Actually, this theoretical expectation has been verified experimentally through the measurement of global polarization of charge-neutral particles [2]. Since the experimental report on the evidence for the rapidly rotating fluid, it has been serious research subject to explore the phase diagram in the plane of temperature and rotation velocity; see pioneering works [3,4]. The studies based on QCD-like models conclude that the critical temperature of the QCD phase transition becomes smaller as the rotation velocity goes up. This conclusion is also consistent with our intuition; if the fluid element moves fast due to rapid rotation, the volume of the fluid is Lorentz contracted by the relativistic effect, and the effective density and thus the effective temperature should become higher (this is called the Tolman-Ehrenfest effect). Therefore, the phase transition should be easier to happen, and the critical temperature is expected to be lowered by rotation.

In a general context, the rotation effect is driven by the nontrivial geometry of curved spacetime. This is because the metric is no longer the one in flat spacetime. The quantum field theory in curved spacetime has been well established and even the first-principles simulation is feasible by means of the Monte Carlo importance sampling method, as demonstrated in [5].

On the phenomenological side, the properties of spin have attracted a lot of interest. In terms of relativistic hydrodynamics [6,7], the orbital angular momentum is distributed over the fluid velocity at finite vorticity, while the spin should be introduced as an approximate hydrodynamic variable in the formulation. In this way, it is of increasing importance to clarify the equilibrium and the nonequilibrium features of spin. The hydrodynamic and kinetic theory formulations have been developed under the assumption that the equilibrium thermodynamic properties involving spin are already known. However,

surprisingly, the phase diagram research has focused on the global rotation that causes both the orbital and the spin angular momenta, and little has been known about thermal properties of QCD matter when only the spin is injected.

The distribution of angular momentum in a rotating quark-gluon medium may also have an intriguing relation to the proton spin crisis discovered with the help of deep-inelastic scattering experiments at the European Muon Collaboration (EMC) in the late 1980s. It states that the intrinsic spin of the proton cannot be fully explained by the spins of its constituent valence quarks. While the naïve quark model predicted quark spins to completely account for the spin of a proton, the experimental results indicated that quark spins contribute only about 20–30% [9]. This result calls for explorations of contributions arising from gluon polarization and orbital angular momenta of quarks and gluons, providing the ground for our joint work.

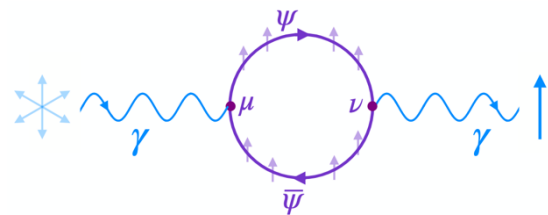
2- Theoretical setups

The angular momentum is a conserved quantity associated with rotation symmetry. The field-theoretical construction of the conserved quantity is well known as the Noether theorem. In electromagnetism, photons carry the angular momentum as well as the momentum. The photon momentum is expressed by the Poynting vector, and the angular momentum is then nothing but the vector product of the position vector and the Poynting vector. In the same way, we can identify the angular momentum carried by gluons in QCD. The angular momentum is further decomposed into the orbital part that contains spatial derivatives and the spin part. During the fellowship period, we had intensive discussions trying to prescribe the correct treatment of spin in QCD thermodynamics, starting with a simpler analogue of QED matter out of electrons and photons.

The decomposition of angular momentum to spin and orbital parts of quarks and gluons can also provide a key to the solution of the proton spin puzzle. The separation of the angular degrees of freedom has been extensively

studied through two principal approaches: the gauge-dependent Jaffe–Manohar construction, which explicitly separates spin and orbital contributions of quarks and gluons [10], and Ji’s gauge-invariant decomposition via generalized parton distributions [11]. According to our approach, the solution of the proton spin puzzle is rooted in the spin-injection mechanism in which the spin of valence quarks is “injected” into the gluon medium and polarizes it.

As mentioned above, given the complexity of the vortical quark-gluon plasma and its rich phenomenology, we adopt a stepwise approach to the problem of spin injection. As an initial step, we apply our framework to a simpler and well-characterized reference system such as QED. This step allows us to validate the methodology and clarify the underlying mechanisms of the spin injection. Once the construction is established and the subtleties of



its implementation understood, we extend, in the second step, the analysis to the more interesting case of the quark-gluon plasma subjected to rotation. The properties of the spin injection in small-volume systems will also allow us to tackle the spin problem of the proton in the MIT-bag approach.

Fig. 2: Feynman diagram that describes the effect of polarization of an initially unpolarized photon propagating through the spin-polarized fermionic environment.

Coming back to QED, we consider the induction of photon spin polarization resulting from the interaction with polarized electrons—that is, if the electrons are polarized, initially unpolarized photons acquire a net polarization. This effect exemplifies the spin injection mechanism via quantum interaction.

Our approach also suggests the existence of an equilibrium state, where spins of photons and electrons achieve thermal equilibrium with a

nonzero total spin polarization. The corresponding calculation can be carried out within a perturbative. The pertinent Feynman diagram is shown in Fig. 2.

The thermal equilibrium is achieved by the inverse process, in which the spin-polarized background of fermions makes the photons propagating through this polarized medium acquire the polarization as well (Fig. 3).

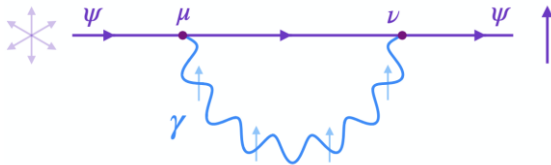


Fig. 3: Feynman diagram for polarization of a fermion in the spin-polarized photon bath.

3- Results and discussion

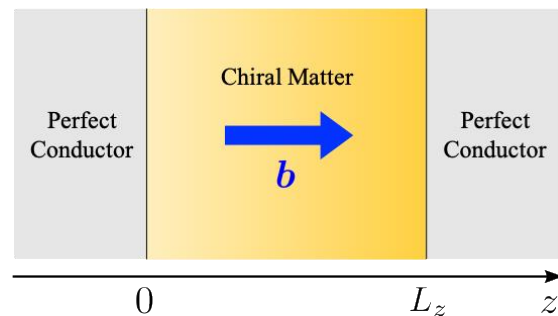
The spin part seems to be gauge dependent, but its volume integral is gauge invariant. Then, a source term coupled to the spin can be implemented in finite-temperature field theory. In this way, we reached a clear picture that the spin-injected gauge system reduces to the simpler gauge system with a spacetime-varying coefficient in the topological Chern-Simons term. This is a beautiful relation that nobody had considered before we discovered it.

The topological Chern-Simons term is a necessary ingredient in the Strong Interaction. Theoretically speaking, symmetry at the fundamental level is an experimental fact, and there is no requirement from theory. The Strong Interaction has parity symmetry, that is, symmetry under spatial conversion, and this means that the coefficient of the topological Chern-Simons term that violates P (parity) and CP (charge parity multiplied by parity) symmetries must be vanishing. In convention, this coefficient is denoted as θ , and the lack of theoretical explanation of unnaturally small θ is called the Strong CP Problem.

Though the Strong CP Problem is an unsolved mystery, the effective θ term can be induced and even controlled in the condensed matter

system in the tabletop experiments using the Weyl semimetals and the Dirac semimetals.

We proceeded to field-theoretical calculations of the photon dispersion relation in the presence of spin injection through finite spin chemical potential. This gauge system is equivalent to the extended QED system with a spatially modulating coefficient of the topological Chern-Simons term. Interestingly, the energy dispersion relation of the photon is distorted by the spin effect and this distortion turns out to be the same as considered previously in the context



of the anomalous Casimir effect [8] (see Fig. 4).

Fig. 4: A concrete realization of the photon system with the spin potential. The vector b represents the direction of the introduced spin potential or, equivalently, the direction of the spatial derivative of the topological θ coefficient.

Our new discovery has interdisciplinary impacts. As explained above, the Strong CP Problem is one of the most puzzling questions, and to resolve this, undiscovered elementary particles called axions were invented in elementary particle physics. The precise dynamics of axions is not known, but our discovery makes it clear that the spin chemical potential is nothing but the space-dependent axion background field or Spin-Axion shortened for simplicity. Another aspect of our discovery is the anomalous transport induced by the topological current. It is a grand challenge of physics to access phenomena arising from the quantum anomaly with a probe of macroscopic observables.

The chiral magnetic effect is one realization of such correspondence between the quantum anomaly and the macroscopic observable. In the

case of the chiral magnetic effect, the topological θ angle is assumed to be an increasing function of time. However, the experimental setup to manipulate time-dependent θ is very difficult. In contrast, in our case of the spin potential, the system is fully under theoretical and experimental control, and we can naturally expect the spatial counterpart of the chiral magnetic effect, which is seen in the form of the topological current.

4- Conclusion

We revisited the old and yet profound problem of the angular momentum in electromagnetism at the quantum level. We showed that the spin of a photon is induced by the spin chemical potential and this coupling reduces to the topological Chern-Simons term. We can utilize the spin degrees of freedom in gauge systems to emulate the axion dynamics in elementary particle physics, which can be named the Spin-Axion effective theory. Also, using the condensed-matter physics system, such as the Weyl semimetals and the Dirac semimetals, we can probe the topological current induced by the coupling with the spin injection, which paves a novel passage toward further investigations of topological matter.

5- Perspectives of future collaborations with the host laboratory

The fellow, Kenji Fukushima, keeps close contact with the host scientist, Maxim Chernodub, to continue the collaborations. Since our discovery is so insightful, it is worth extending the applications of the Spin-Axion corresponding in a wider range of perspectives. Furthermore, we are further discussing the future organization of workshops and symposiums together to promote the collaborative works.

6- Articles prepared in the framework of the fellowship

We, Kenji Fukushima and Maxim Chernodub, are completing a first publication to report our discovery with full details of physical backgrounds and mathematical procedures. The Fellow, Kenji Fukushima, has finalized a new

work about the strong-coupling expansion of rotating QCD matter with Yusuke Shimada while staying at the host laboratory, which benefits from discussions with Maxim Chernodub.

7- Acknowledgments

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8- References

- [1] F. Gross, E. Klempt, K. Fukushima et al., “50 Years of Quantum Chromodynamics,” *Eur. Phys. J. C* **83** (2023) 1125.
- [2] L. Adamczyk et al. (Star Collaboration), “Global Λ hyperon polarization in nuclear collision: evidence for the most vortical fluid,” *Nature* **548** (2017) 62.
- [3] H-L. Chen, K. Fukushima, X-G. Huang, K. Mameda, “Analogy between rotation and density for Dirac fermions in a magnetic field,” *Phys. Rev. D* **93** (2016) 104052.
- [4] Y. Jiang, J. Liao, “Pairing phase transitions of matter under rotation,” *Phys. Rev. Lett.* **117** (2016) 192302.
- [5] M. Chernodub, V. Goy, A. Molochkov, “Inhomogeneity of rotating gluon plasma and Tolman-Ehrenfest law in imaginary time: lattice results for fast imaginary rotation,” *Phys. Rev. D* **107** (2023) 114502.
- [6] K. Hattori, M. Hongo, X-G. Huang, M. Matsuo, H. Taya, “Fate of spin polarization in a relativistic fluid: An entropy-current analysis,” *Phys. Lett. B* **795** (2019) 100-106.
- [7] K. Fukushima, S. Pu, “Spin hydrodynamics and symmetric energy-momentum tensors – A current induced by the spin vorticity –,” *Phys. Lett. B* **817** (2021) 136346.
- [8] K. Fukushima, S. Imaki, Z. Qiu, “Anomalous Casimir effect in axion electrodynamics,” *Phys. Rev. D* **100** (2019) 045013.

[9] Ashman et al. (EMC Collaboration), “An investigation of the spin structure of the proton in deep inelastic scattering of polarised muons on polarised protons,” Nucl. Phys. B328 (1989) 1.

[10] R. L. Jaffe and A. Manohar, “The g_1 problem: Deep inelastic electron scattering and the spin of the proton,” Nucl. Phys. B337 (1990) 509.

[11] X. Ji, “Gauge-Invariant Decomposition of Nucleon Spin,” Phys. Rev. Lett. 78 (1997) 610.