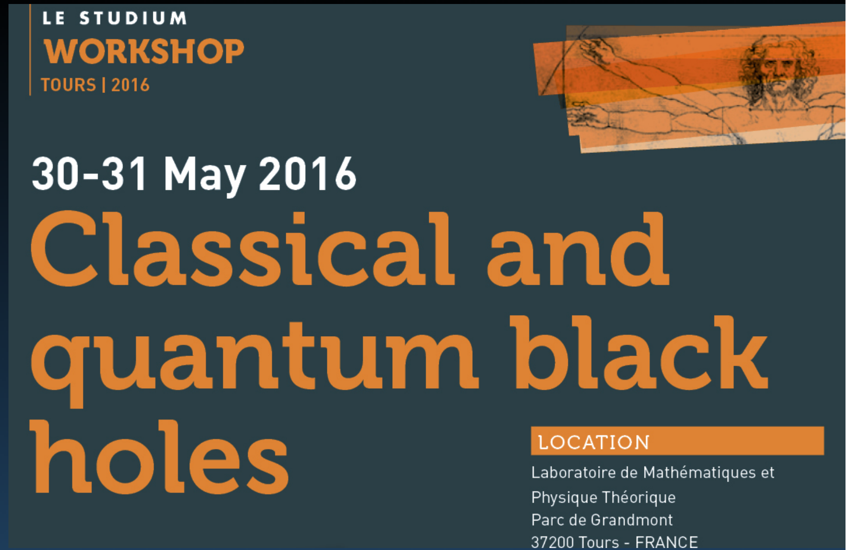
S-matrix approach to gravitational scattering, bremsstrahlung, and collapse

Gabriele Veneziano



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Introduction

For almost 30 years a small (and crazy?) group of theorists has taken Trans-Planckian-Energy (TPE) collisions of strings (and later of branes) as the thought experiment of choice for addressing some fundamental issues about the merging of General Relativity and Quantum Mechanics.

The aim was (and still is) to understand, within a consistent theory of quantum gravity, whether and how information is preserved in a process that leads, classically, to black hole formation and, semi-classically, to an apparent loss of information via Hawking's evaporation process (one of the main topics of this workshop)

The game started in 1987 with parallel work by:

Amati, Ciafaloni & GV (ACV)

and by

Gross, Mende & later Ooguri (GMO)

on TPE string-string collisions.

There was also parallel related work by 't Hooft and by Muzinich & Soldate in the QFT limit.

Much later (2010) the HE scattering of a closed string off a stack of N >> 1(coincident) D- branes was tackled:

D'Appollonio, Di Vecchia, R. Russo & GV (DDRV) +...

For lack of time only the first will be considered hereafter

Outline (a quick reminder for most)

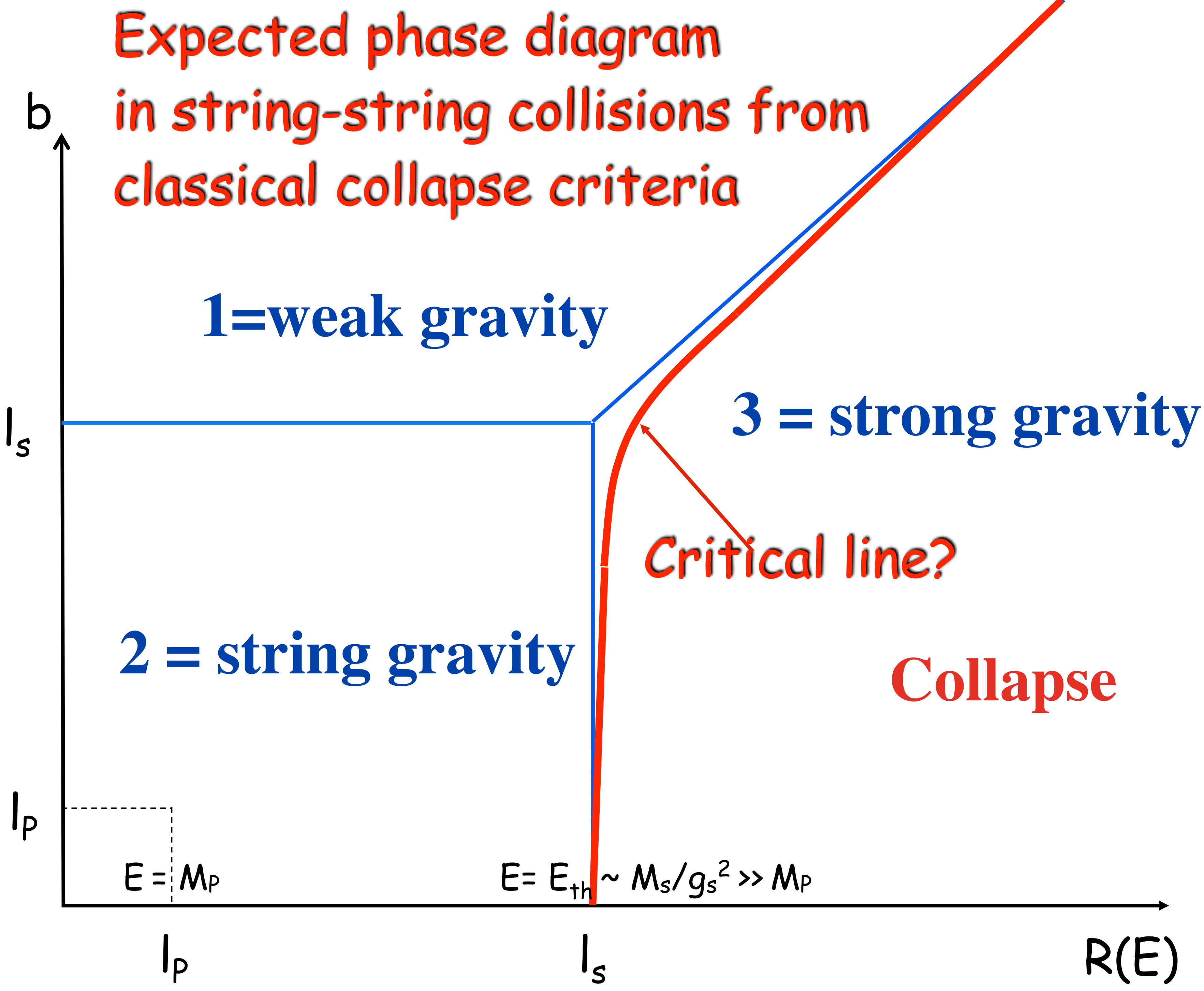
- Expected classical "phase diagram"
- Weak-gravity QFT regime (elastic unitarity):
  - Phase shift, gravitational deflection, time-delay.
- Weak-gravity QST regime (inelastic unitarity):
  - Tidal excitation of strings
- String-gravity regime (approximate unitarity):
  - (maximal deflection, minimal length, GUP)
  - (ST's resolution of a potential causality problem)
  - Precocious Black Hole Behavior (PBHB)
- Strong-gravity regime (D=4, no string corrections):
  - Reduction to a $CLFT_4$, $CLFT_2$, critical points/curves
  - Loss of unitarity below b_{crit} ?
  - **An energy crisis, its resolution, PBHB**
  - A comment on 1409.7405

Parameter-space for string-string collisions @ $s \gg M_P^2$

$$b \sim \frac{2J}{\sqrt{s}} \quad ; \quad R_D \sim \left(G\sqrt{s}\right)^{\frac{1}{D-3}} \quad ; \quad l_s \sim \sqrt{\alpha' \hbar} \quad ; \quad G\hbar = l_P^{D-2} \sim g_s^2 l_s^{D-2}$$

- 3 relevant length scales (neglecting l_P @ $g_s \ll 1$)
- Playing w/s and g_s we can make R_D/l_s arbitrary

Expected phase diagram in string-string collisions from classical collapse criteria



Actually there are subregions.
The semiclassical S-matrix takes the form:

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \right.$$

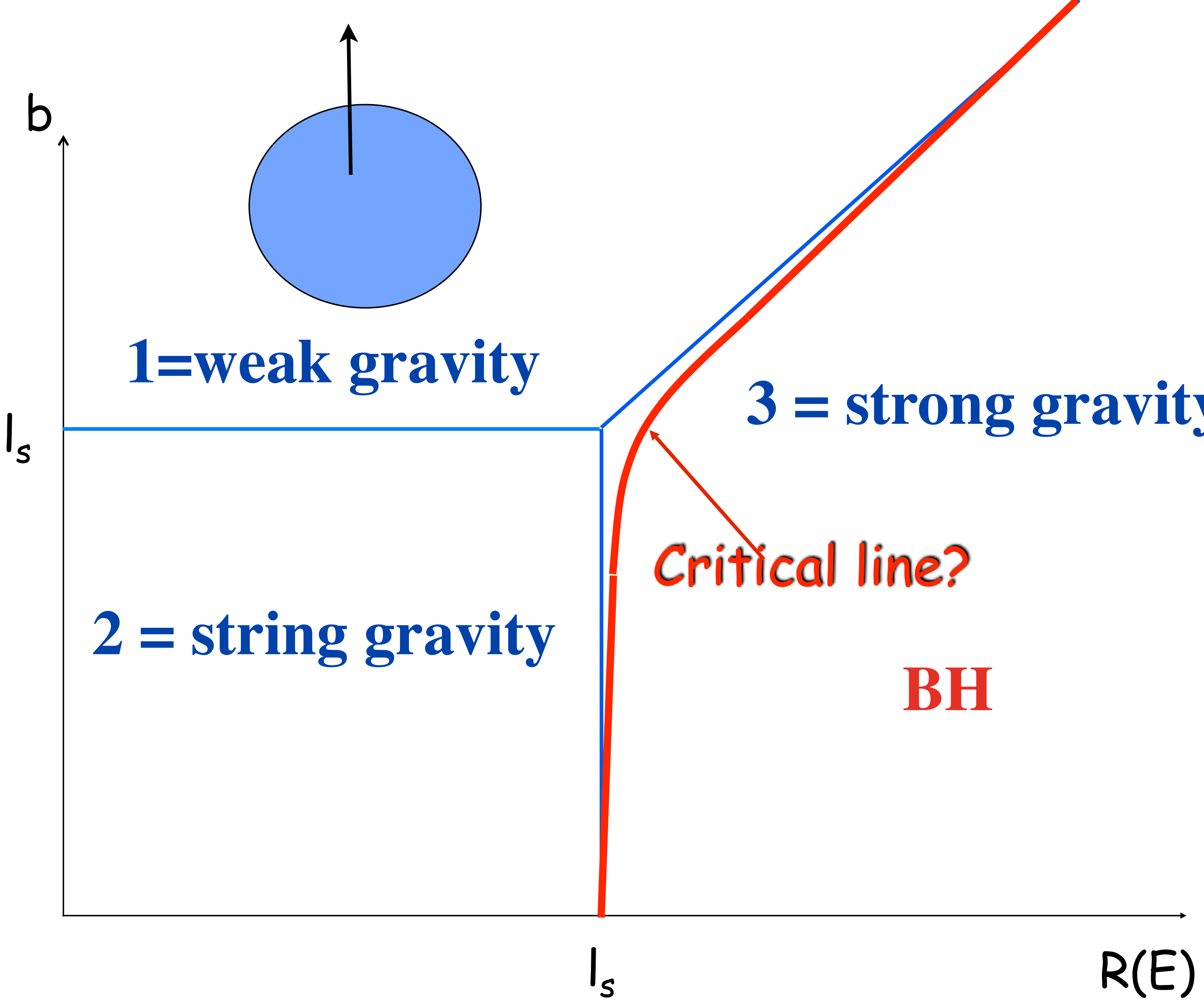
Since leading term is real, contributions to Im A_{cl} can be much more than corrections.

They give strong absorption ($|S_{el}| \ll 1$) if Im $A_{cl} \gg 1$.

This is what gives rise to the subregions.

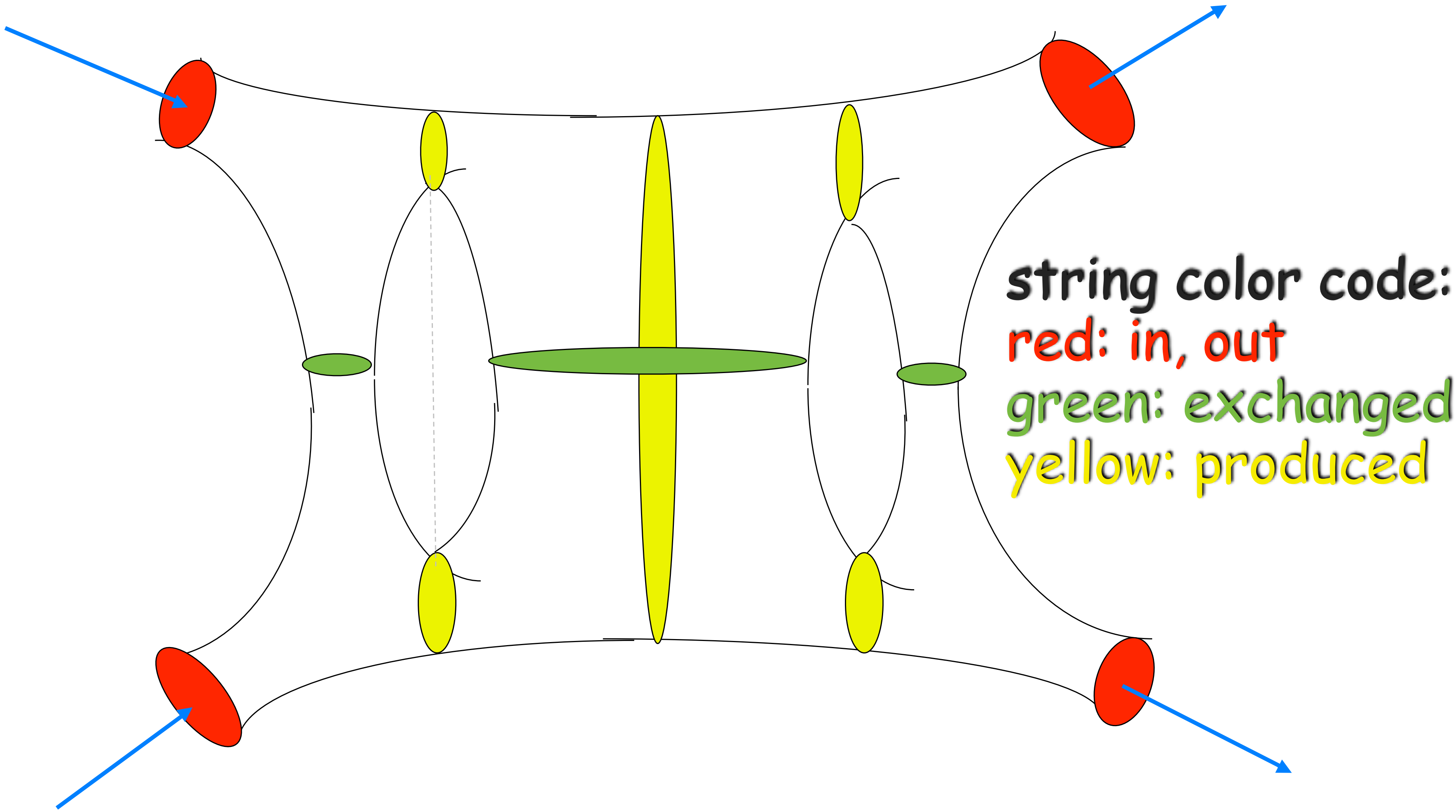
The weak-gravity QFT regime

(recovering elastic unitarity through loops)



b
1=weak gravity
l_s
3 = strong gravity
Critical line?
2 = string gravity
BH
l_s
R(E)

Typical contribution to TPE string-string collisions



After resummation @ fixed s,b…

A unitary elastic S-matrix

$$S(E,b) \sim exp\left(i\frac{Gs}{\hbar}c_D b^{4-D}\right) \;\; ; \;\; S(E,q) = \int d^{D-2}b \; e^{-iqb} S(E,b) \;\; ; \;\; s = 4E^2 \; , \; q \sim \theta E$$

The integral is dominated by a saddle point at:

$$b_s^{D-3} \sim \frac{G\sqrt{s}}{\theta} \; ; \; \theta \sim \left(\frac{R_S}{b}\right)^{D-3} \; ; \; R_S^{D-3} \sim G\sqrt{s}$$

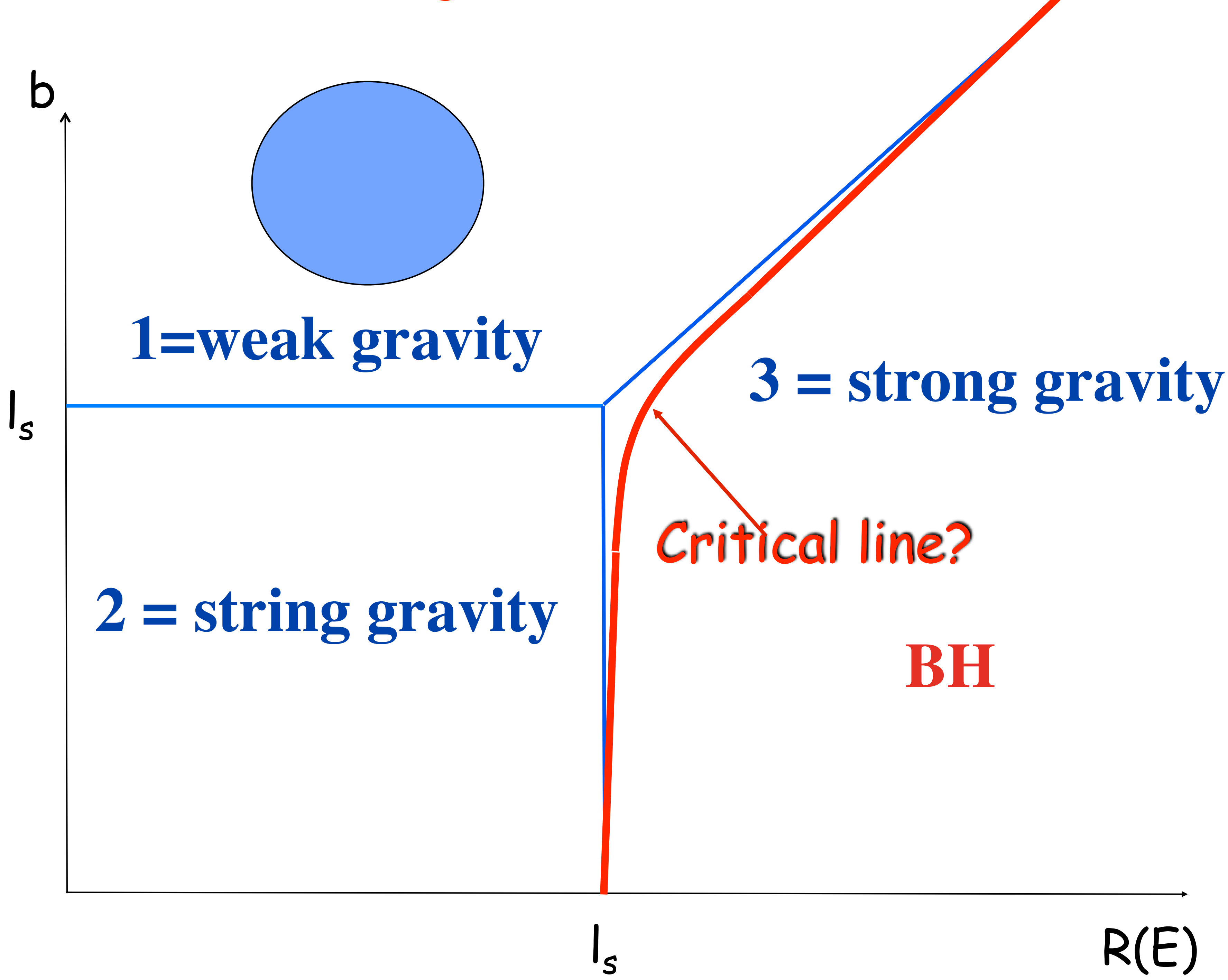
comments:

1. Deflection angle given by derivative of phase shift w.r.t. b. => correct generalization of Einstein's deflection formula to ultra-relativistic collisions & arbitrary D.
2. Derivative w.r.t. E gives correct Shapiro time delay.
3. Elastic unitarity is fullfilled.
4. Fixed small angle scattering probes large-distances!

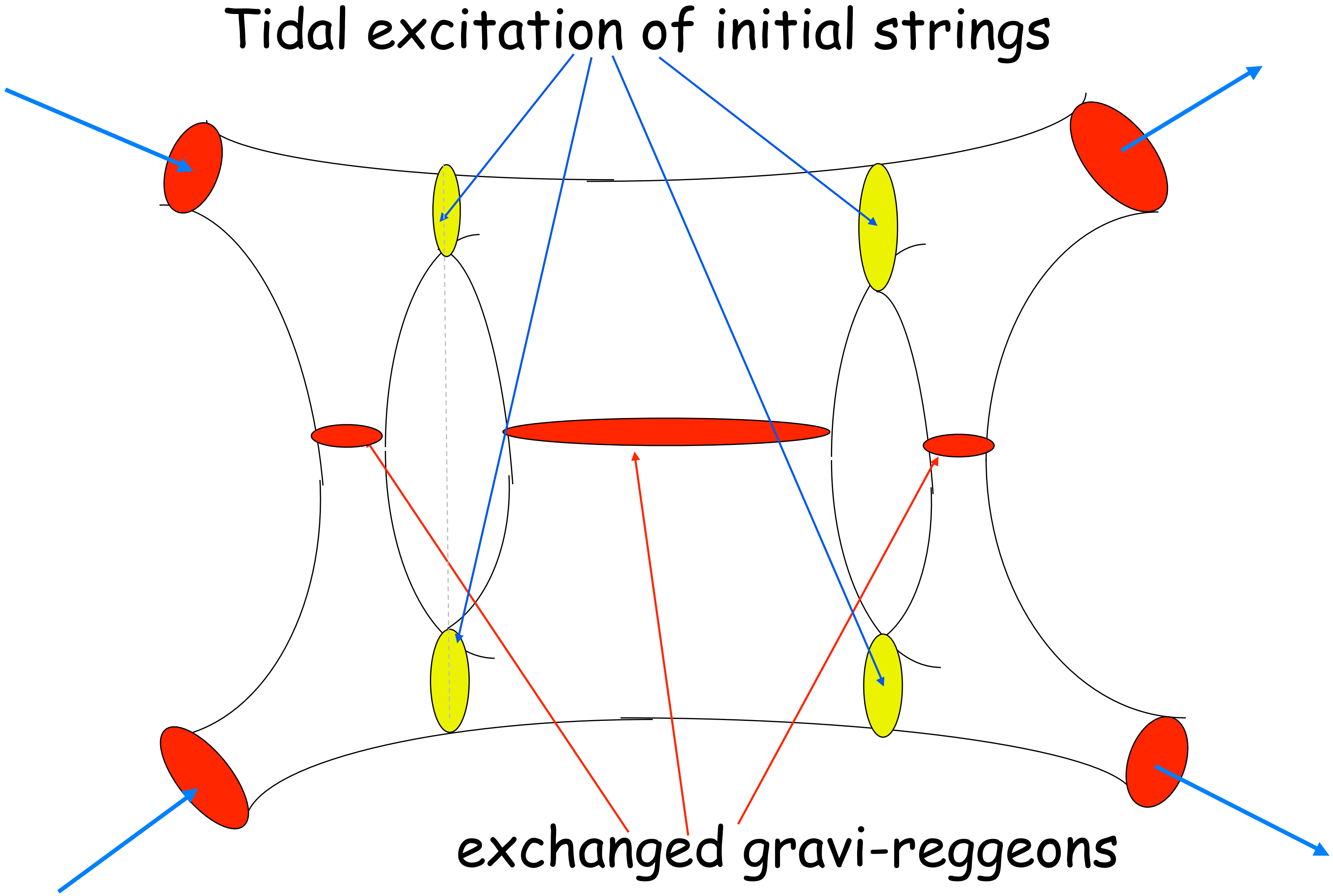
The weak-gravity QST regime

(w/ exact inelastic unitarity)

We are still in region 1 (but a little lower)!



Inelastic channels appear, dominate



String-string scattering @ large b

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D}\left(1 + O(\cancel{(R/b)}^{2(D-3)}) + O(l_s^2/b^2) + O(\cancel{(l_P/b)}^{D-2}) + \ldots\right)$$

Graviton exchange can excite one or both strings.
Reason (Giddings '06): a string moving in a non-trivial metric feels tidal forces as a result of its finite size.

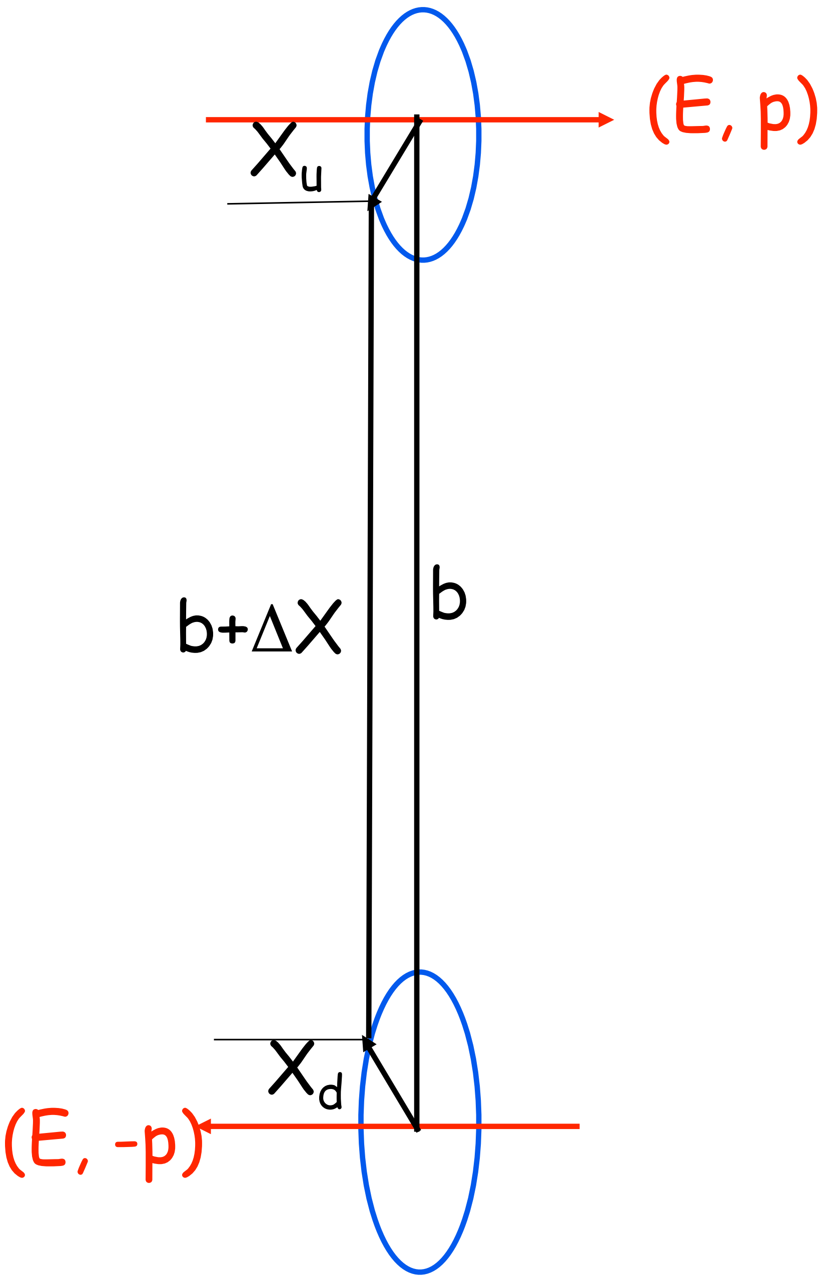
The critical impact parameter b_t below which the phenomenon kicks-in is parametrically larger than l_s:

$$b_t^{D-2} \sim \frac{Gs\ l_s^2}{\hbar}$$

It turns out (ACV '87,) that these effects are simply captured, at the leading eikonal level, by replacing the impact parameter **b** by a shifted impact parameter, displayed by each string's position operator evaluated at $\tau = 0$ (= coll. time) and averaged over σ (see figure).

This leads to a unitary operator eikonal (as long as the phase shift is real).

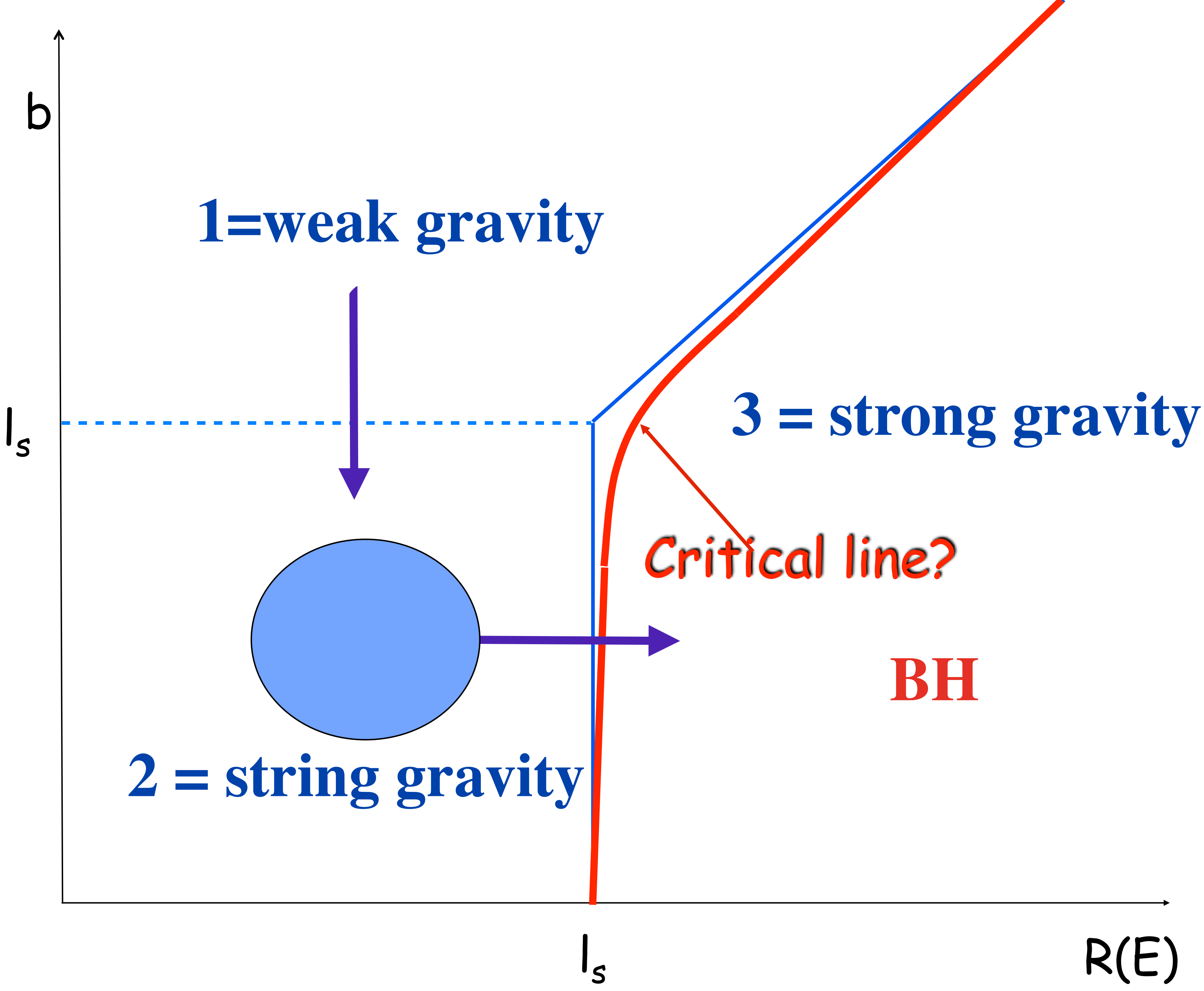
N.B. Elastic unitarity -> inelastic unitarity (analyzed in detail in DDRV '13)



(E, p)
X_u
b+ΔX
b
X_d
(E, -p)

The string-gravity regime

(rough unitarity, GV'04)



b
1=weak gravity
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Critical line?
BH
2 = string gravity
l_s
R(E)

String-string scattering @ b,R < l_s

$$S(E,b) \sim exp\left(i\frac{A}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + \cancel{O(R^2/b^2)} + O(l_s^2/b^2) + \cancel{O(l_P^2/b^2)} + \ldots)\right)$$

"Classical corrections" screened, string-corrected leading eikonal can be trusted even for b < R.

Phase shift is finite at b=0 and has a smooth expansion in $b^2/(l_s^2$ logs). Solves "causality problem", see DDRV/1502.

The maximal classical deflection angle is ~ $(R/l_s)^{D-3} \ll 1$, and is reached when the two strings graze each other. Agreement w/ GMO in classically forbidden region.

Scales shorter than l_s cannot be explored =>

Generalized Uncertainty Principle:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \Delta p \geq l_s$$

More important here: in string theory even single gravi-reggeon exchange gives a complex scattering amplitude.

For St-St scattering the imaginary part is due to formation of closed-strings in the s-channel (DHS duality, 1967).

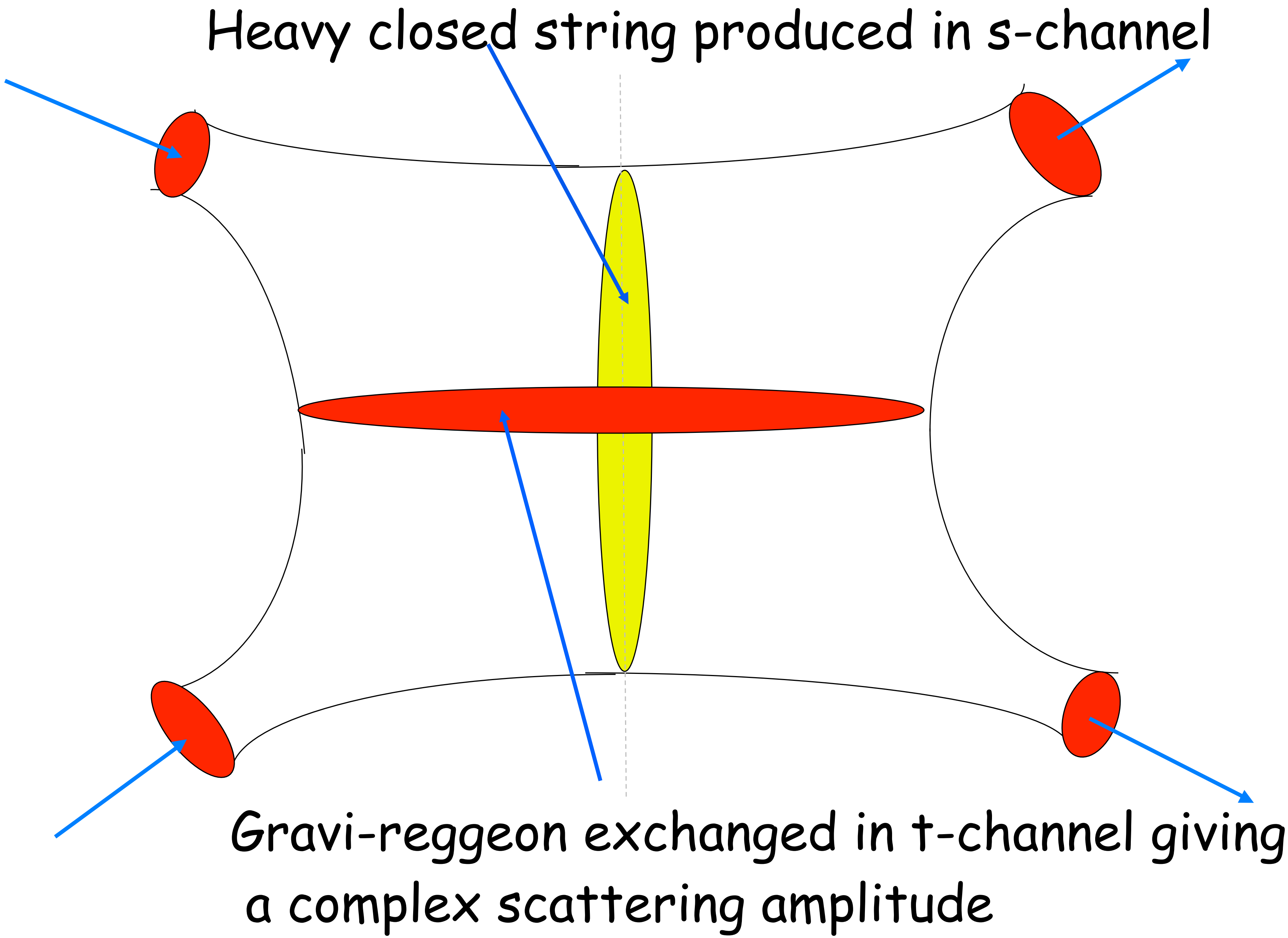
Exponentially small at $b \gg l_s$, important at $b < l_s . Y^{1/2}$

Like the real part it is smooth for $b \to 0$.

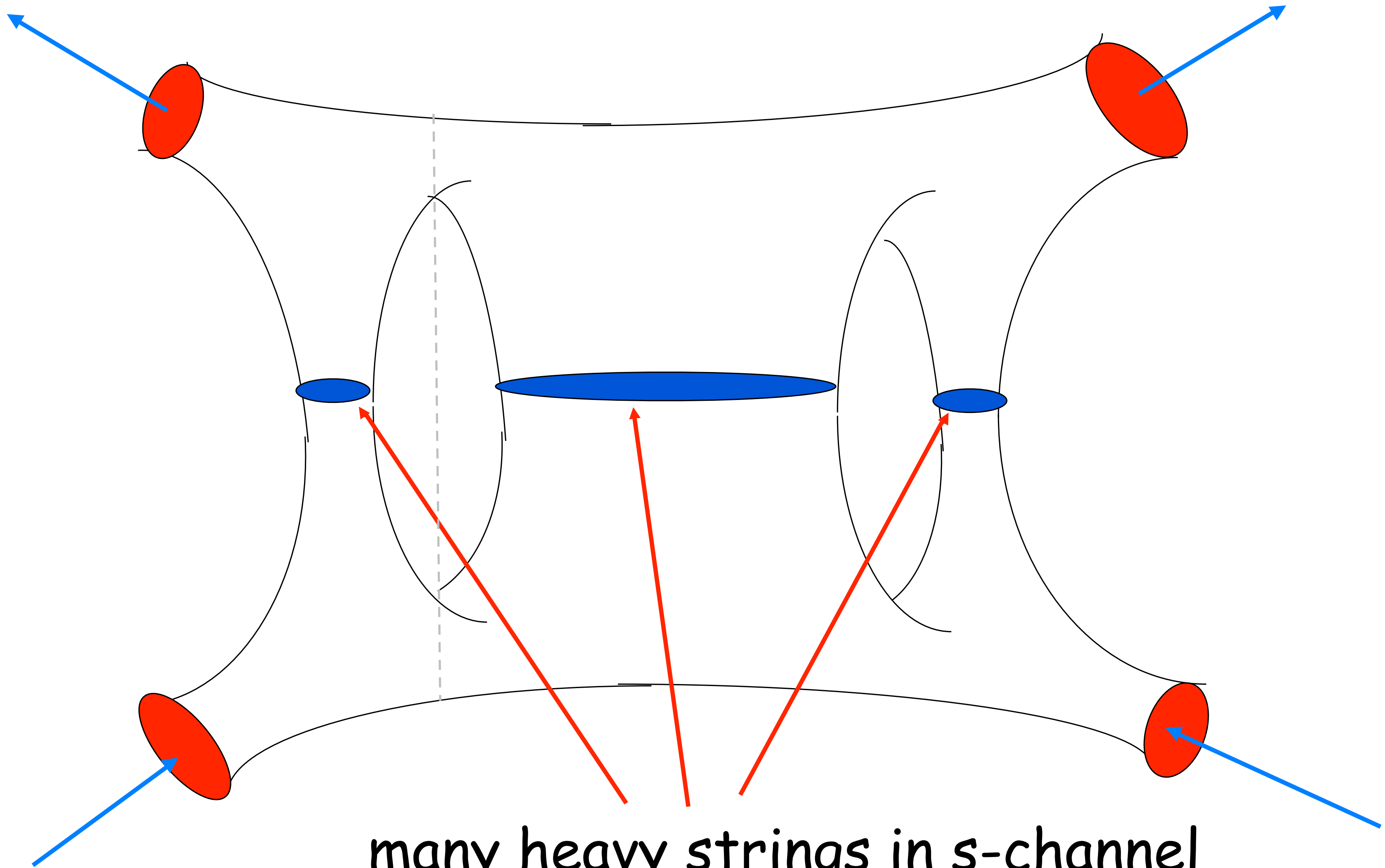
$$\mathrm{Im} A_{cl}(E,b) \sim \frac{G\ s}{\hbar}\ (l_s\sqrt{Y})^{4-D}\ \exp\left(-\frac{b^2}{l_s^2 Y}\right) \quad ; \quad Y = \log(\alpha' s)$$

$$\mathrm{Im} A_{cl}(E,b) \sim \langle n \rangle \to g^{-2} \sim S_{BH} \qquad \langle E_{final} \rangle \sim \frac{M_s^2}{g^2\sqrt{s}} \to M_s \ \text{at}\ \sqrt{s} = E_{th}$$

Fast growth of <n>, exp(-S) suppression of elastic scattering, softening of final state: all precursors of BH evaporation (see GV hep-th/0410166)?


Heavy closed string produced in s-channel
Gravi-reggeon exchanged in t-channel giving
a complex scattering amplitude

Turning the previous diagram by 90°

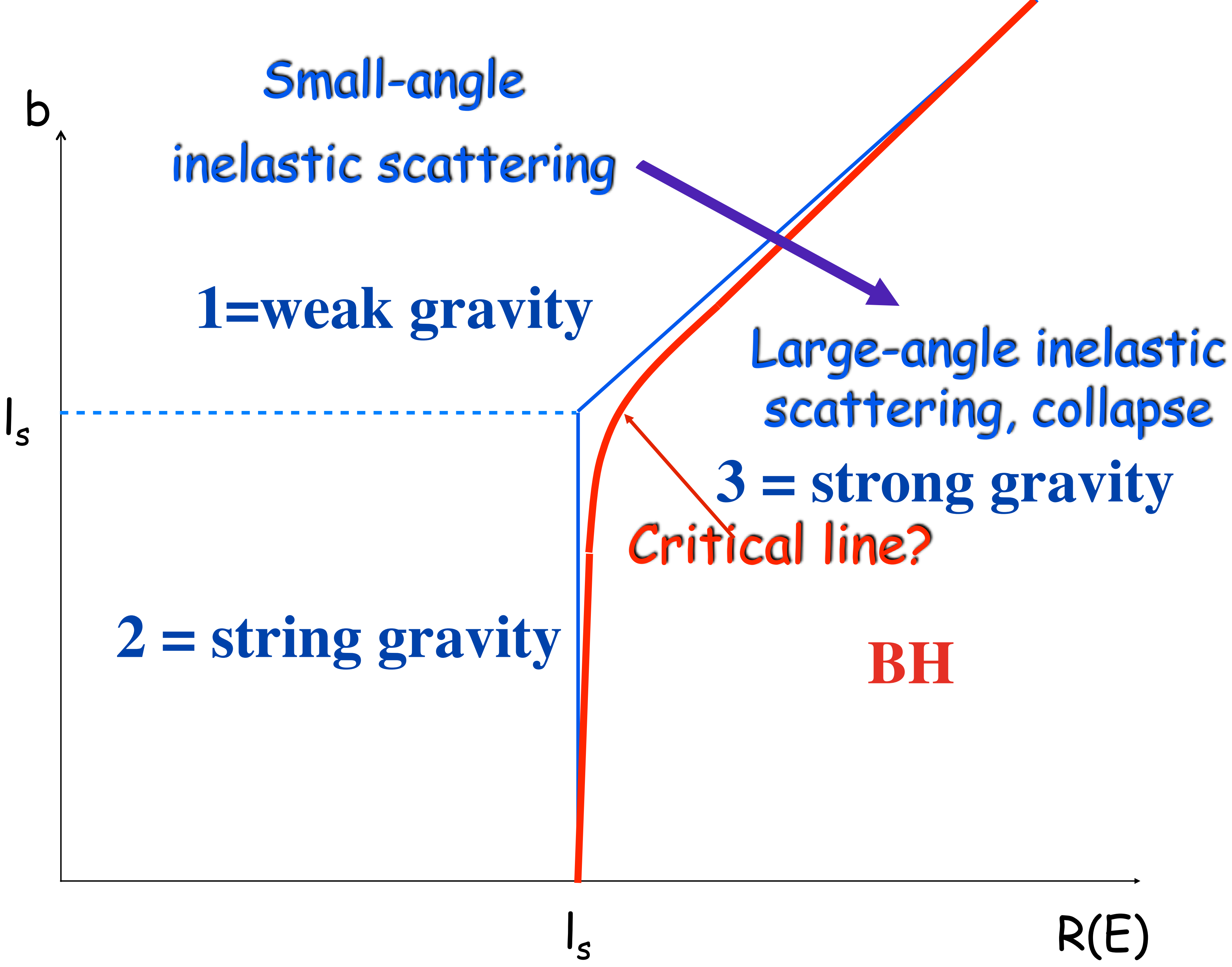


many heavy strings in s-channel

AGK cutting rules apply

The strong-gravity regime ($R > b, l_s$)

(D=4, no string corrections included!)


Small-angle
inelastic scattering
b
1=weak gravity
Large-angle inelastic
scattering, collapse
l_s
3 = strong gravity
Critical line?
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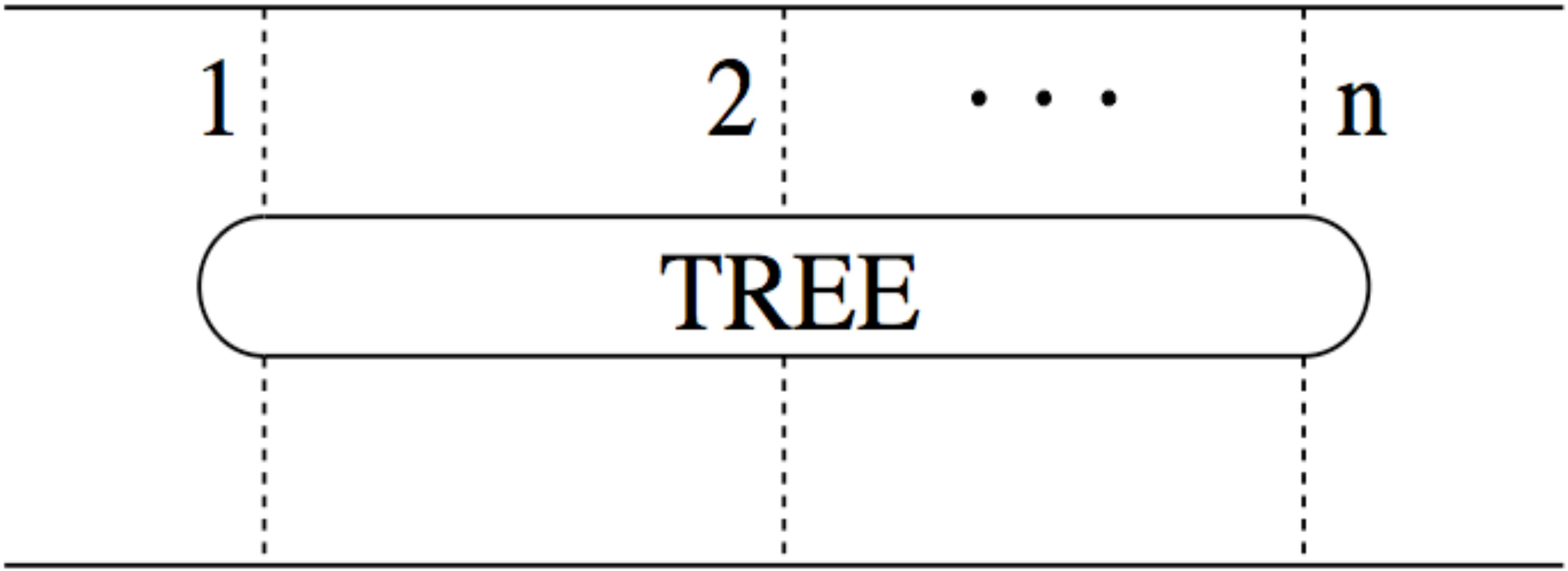
BH
l_s
R(E)

Classical corrections related to "tree diagrams"



Power counting for connected trees:

$$A_{cl}(E,b) \sim G^{2n-1} s^n \sim Gs\ R^{2(n-1)} \to Gs\ (R/b)^{2(n-1)}$$

Summing tree diagrams => solving a classical field theory.

Q: Which is the effective field theory for TP-scattering?

There is a D=4 effective action generating the leading diagrams (Lipatov, ACV '93). Too hard to solve!
After (approximately) factoring out the longitudinal dynamics: a D=2 effective action containing 4 fields. We also neglected "rescattering", see below.

ACV07 (+ Marchesini & Onofri, GV & Wosiek, Ciafaloni & Colferai) looked for real-regular solutions and found that they only exist for $b > b_c \sim R$ with a b_c consistent and in good quantitative agreement with b_c of classical (Closed Trapped Surface) collapse criteria.

For $b < b_c$ we had a choice between real-singular and complex-regular solutions and took the latter…

Loss of unitarity at $b < b_c$?

• A new elastic-unitarity deficit appears, for which we have found no physical interpretation

•Choice of complex-regular classical solution could be the reason (Ciafaloni & Colferai).

•Possibly string corrections are needed below b_c if real-singular option is taken (UV completion important?).

Or it could be our crude approximations…

We turned (momentarily?) to a simpler puzzle…

Graviton Spectra: an "energy crisis"?

Within ACV07's approximations the spectrum of produced gravitons implies the following GW energy-distribution:

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp\left(-|k||b| - \omega\frac{R^3}{b^2}\right)\ ;\ \frac{Gs}{\hbar}\frac{R^2}{b^2} >> 1$$

=> the fraction of energy emitted in GWs is O(1) already for $b = b^* >> R$ ($Gs/h\ (R/b^*)^2 = O(1)$). Looks puzzling from a GR perspective. Need answer to:

Q: What's the cutoff in ω for the GWs emitted in an ultra-relativistic small angle ($b >> R$) 2-body collision?

Possible answers: $1/b$, $1/R$ (my old guess), b/R^2, b^2/R^3 (ACV), γ/b (singular m=0 limit?), E/h (singular classical limit?)

PHYSICAL REVIEW D VOLUME 18, NUMBER 4 15 AUGUST 1978

High-speed black-hole encounters and gravitational radiation

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(Received 15 March 1977)

Encounters between black holes are considered in the limit that the approach velocity tends to the speed of light. At high speeds, the incoming gravitational fields are concentrated in two plane-fronted shock regions, which become distorted and deflected as they pass through each other. The structure of the resulting curved shocks is analyzed in some detail, using perturbation methods. This leads to calculations of the gravitational radiation emitted near the forward and backward directions. These methods can be applied when the impact parameter is comparable to $Gc^{-2}M\gamma^2$, where M is a typical black-hole mass and γ is a typical Lorentz factor (measured in a center-of-mass frame) of an incoming black hole. Then the radiation carries power/solid angle of the characteristic strong-field magnitude c^5G^{-1} within two beams occupying a solid angle of order γ^{-2}. But the methods are still valid when the black holes undergo a collision or close encounter, where the impact parameter is comparable to $Gc^{-2}M\gamma$. In this case the radiation is apparently not beamed, and the calculations describe detailed structure in the radiation pattern close to the forward and backward directions. The analytic expressions for strong-field gravitational radiation indicate that a significant fraction of the collision energy can be radiated as gravitational waves.

THE GENERATION OF GRAVITATIONAL WAVES. IV. BREMSSTRAHLUNG*†‡

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Received 1977 October 21; accepted 1978 February 28

ABSTRACT

This paper attempts a definitive treatment of "classical gravitational bremsstrahlung"—i.e., of the gravitational waves produced when two stars of arbitrary relative mass fly past each other with arbitrary relative velocity v, but with large enough impact parameter that

$$(\text{angle of gravitational deflection of stars' orbits}) \ll (1 - v^2/c^2)^{1/2} .$$

What's GR's answer for $\theta_s > 1/\gamma$?

A long standing problem, also hard numerically

Recent progress. Classical: Gruzinov & GV (1409.4555), Spirin &Tomaras (1503.02016); Quantum: Ciafaloni, Colferai & GV (1505.06619), CCColaldeschi & GV, (1512.00281).

Classical Calculation of Grav. Bremss.

(Andrei Gruzinov & GV, 1409.4555)

The calculation is done directly for massless particles at small θ_s. Frequency and the angular distribution of GW spectrum take the form:

$$\frac{dE^{GW}}{d\omega\ d^2\tilde{\theta}} = \frac{GE^2}{\pi^4}|c|^2 \ ; \ \tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}_s \ ; \ \boldsymbol{\theta}_s = 2R\frac{\boldsymbol{b}}{b^2}$$

$$c(\omega, \tilde{\boldsymbol{\theta}}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4}\ e^{-i\omega\mathbf{x}\cdot\tilde{\boldsymbol{\theta}}}\left[e^{-2iR\omega\Phi(\mathbf{x})} - 1\right]$$

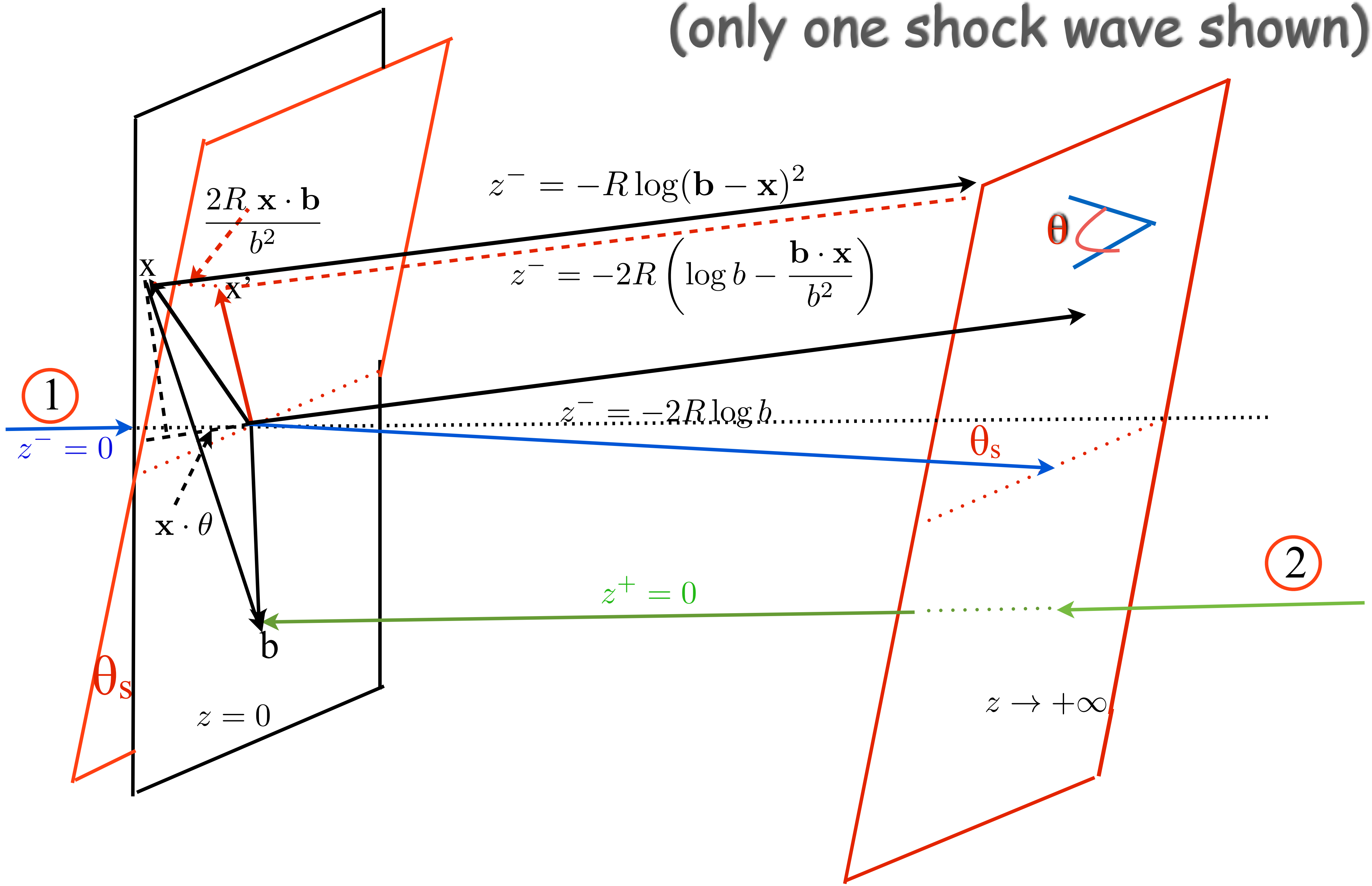
$$\zeta = x + iy \qquad \Phi(\mathbf{x}) = \frac{1}{2}\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2} + \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}$$

$$c(\omega, \boldsymbol{\theta}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4} e^{-i\omega\mathbf{x}\cdot\boldsymbol{\theta}}\left[e^{-iR\omega\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2}} - e^{+2iER\omega\frac{\mathbf{b}\cdot\mathbf{x}}{b^2}}\right]$$

where $(\theta-\theta_s)$ is the solid angle around (one of) the deflected trajectory(ies). Re ζ^2 and Im ζ^2 correspond to the two GW polarizations. Obtained via Huygens principle in Fraunhofer approx. Subtracting the deflected shock wave (cf. P. D'Eath) is crucial!

Schematic illustration of Huygens-Fraunhofer

(only one shock wave shown)



The frequency spectrum is almost flat ($dE/d\omega \sim \log \omega R$) for $b^{-1} < \omega < R^{-1}$. Below b^{-1} it freezes reproducing a known "zero-frequency-limit" (Smarr 1977) based on soft graviton theorems (F. Low, S. Weinberg, 1965):

$$\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi} \theta_s^2 E^2 \log(\theta_s^{-2})$$

Above $\omega = R^{-1}$ the spectrum becomes scale-inv., $dE/d\omega \sim \theta_s^2 E/\omega$, producing a log ω^* in the "efficiency". Using as cutoff $\omega^* \sim R^{-1} \theta_s^{-2}$ (where our approximations break down and the "Dyson bound" $dE/dt < 1/G$ is saturated) we find

$$\frac{E^{GW}}{\sqrt{s}} = \frac{1}{2\pi} \theta^2 \log(\theta^{-2})$$

For $\omega > \omega^*$ we argued for an ω^{-2} spectrum: it turns out to be that of a time-integrated BH evaporation!

Quantum calculation of grav. bremss.

(Ciafaloni, Colferai & GV (1505.06619),
CCColdeschi & GV, (1512.00281))

In CC(C)V (1505.06619 & 1512.00281) the same problem has been addressed at the quantum level improving earlier treatment

One observation is that the usual soft-graviton recipe (emission from external legs) has to be amended since the internal exchanged gravitons are almost on shell. Emission from such internal lines is important for not-so-soft gravitons (hence for the energy loss).

Another point is that, for gravitons with $\omega > R^{-1}$, there are decoherence effects. At fixed graviton helicity and momentum the production amplitudes depend in a precise way upon the incidence angle, which changes along the fast-particle trajectory.
This decoherence causes the break at $\omega \sim R^{-1}$.

If this effect is kept into account when summing over diagrams in which the graviton can be emitted by any rung in the ladder diagram, the result for $c(\omega, \theta)$ is:

$$\int \mathrm{d}^2\boldsymbol{z} \int_0^1 \mathrm{d}\xi \; h_s(\boldsymbol{z}) \mathrm{e}^{\mathrm{i}\omega b \boldsymbol{z}\cdot(\boldsymbol{\theta} - \xi \boldsymbol{\Theta}_s(b))}$$

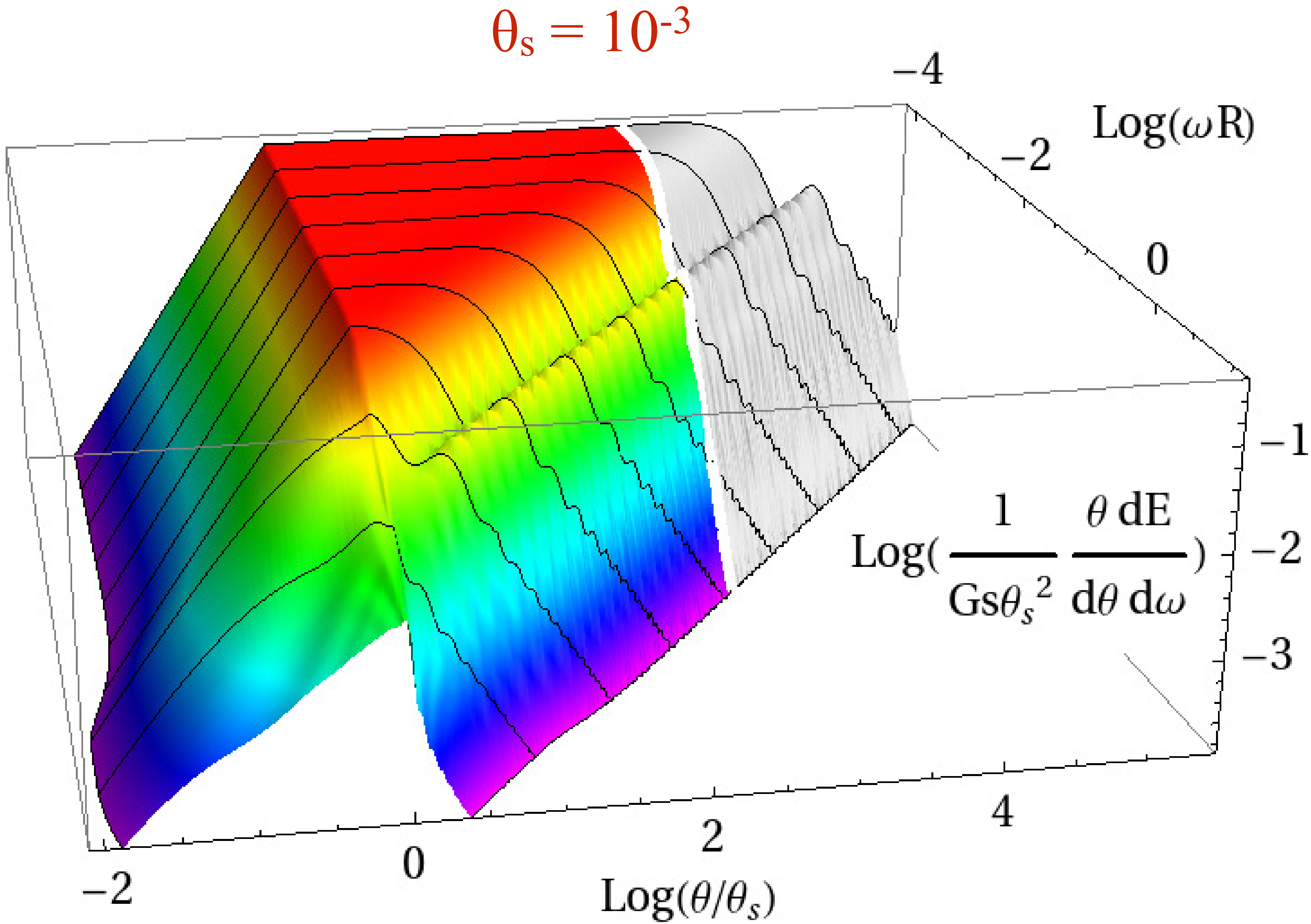
$$h_s(z) \equiv \frac{1}{\pi^2 z^{*2}} \left(\frac{E}{\omega} \log \left| \hat{\boldsymbol{b}} - \frac{\omega}{E}\boldsymbol{z} \right| - \log \left| \hat{\boldsymbol{b}} - \boldsymbol{z} \right| \right) \equiv -\frac{\Phi_R(\boldsymbol{z})}{\pi^2 z^{*2}}$$

$$\Phi_R(\boldsymbol{z}) \to \Phi(\boldsymbol{z}) \equiv \left(\hat{\boldsymbol{b}} \cdot \boldsymbol{z} + \log \left| \hat{\boldsymbol{b}} - \boldsymbol{z} \right| \right)$$

This is similar -but not identical- to the classical result of G+V. However, as argued in 1512.00281, one should also take into account the difference between the eikonal phase of the final 3-particle state and that of an elastic 2-particle state. When this is done, the classical result of 1409.4555 is exactly recovered in the limit $\hbar\omega/E \ll 1$.

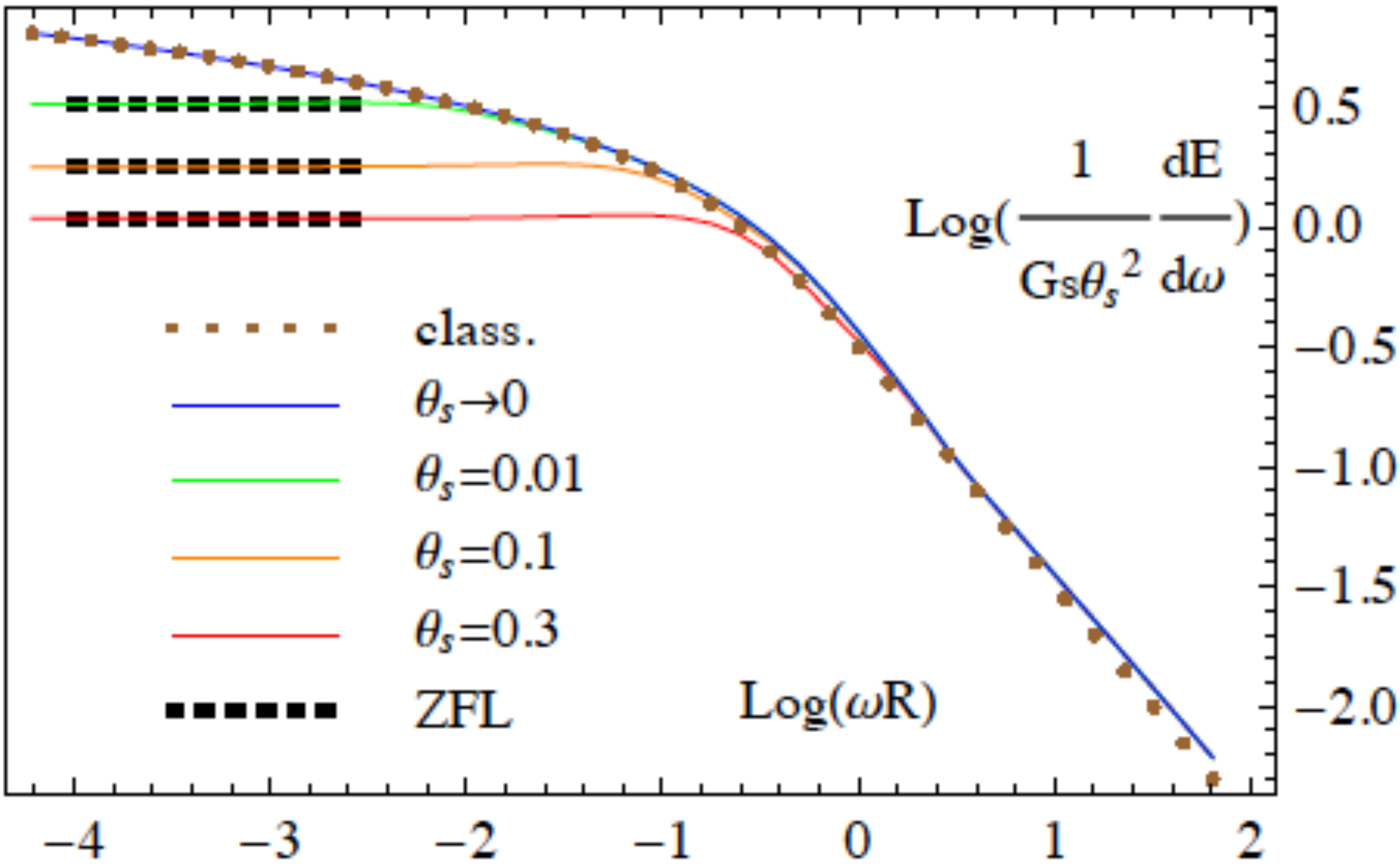
We have analyzed (mostly numerically) the properties of the spectrum with and without rescattering corrections and in the classical limit.

This is illustrated in a few pictures.



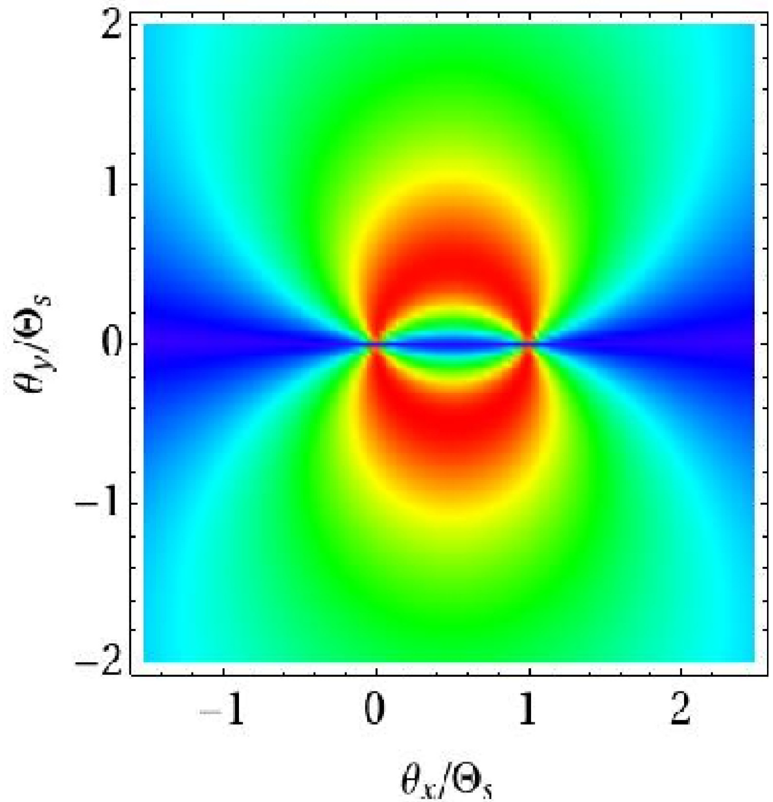
$\theta_s = 10^{-3}$
−4
−2
0
Log(ωR)
−1
−2
−3
$\mathrm{Log}\left(\frac{1}{Gs\theta_s^2}\frac{\theta\, dE}{d\theta\, d\omega}\right)$
−2
0
2
4
Log(θ/θs)

Frequency spectrum

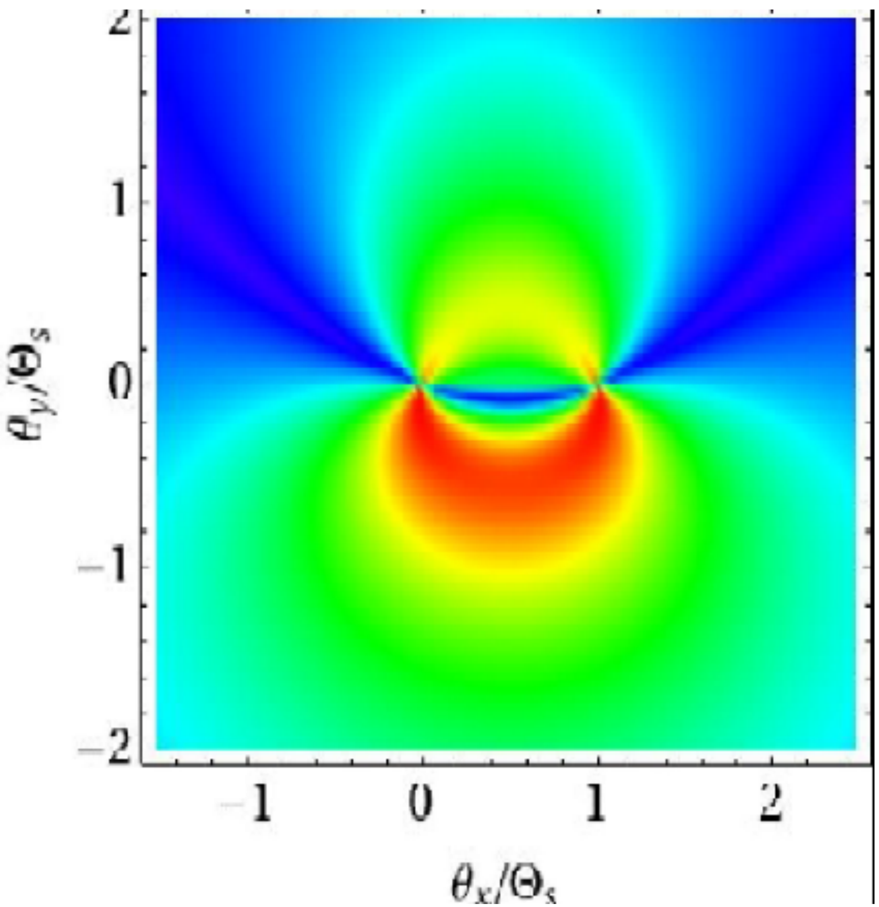


Angular (polar and azimuthal) distribution

$\omega R = 10^{-3}$



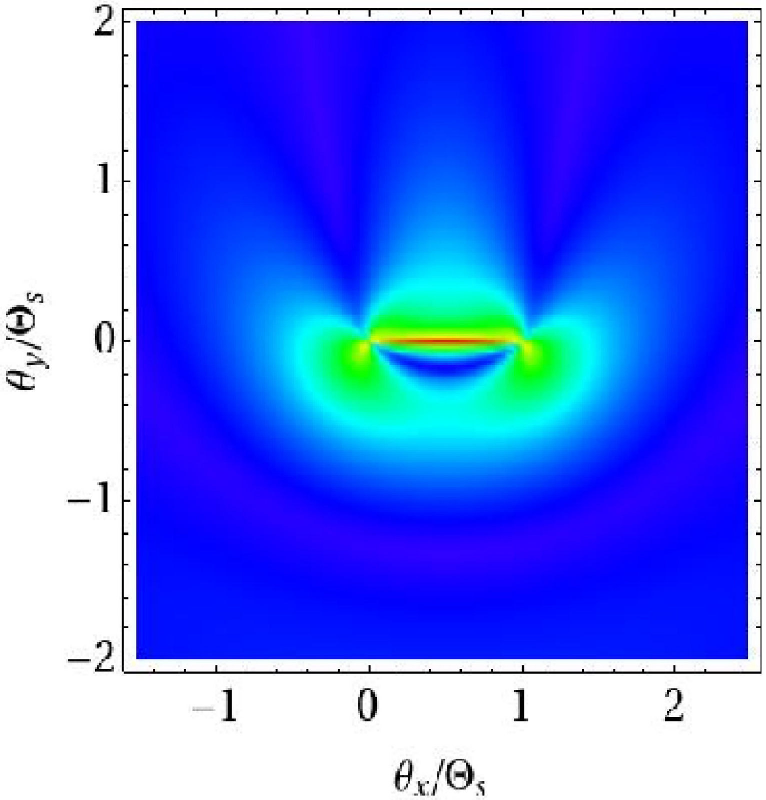
$\omega R = 0.125$



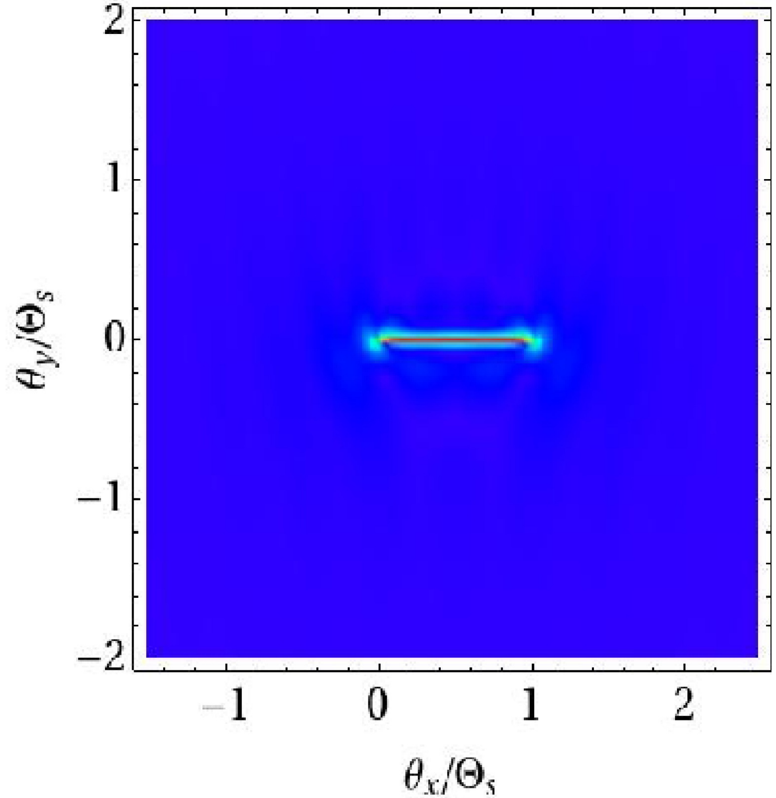
M. Ciafaloni, D. Colferai, F. Coraldeschi & GV, TH-2015-272

Angular (polar and azimuthal) distribution

$\omega R = 1.0$



$\omega R = 8.0$



M. Ciafaloni, D. Colferai, F. Coraldeschi & GV, TH-2015-272

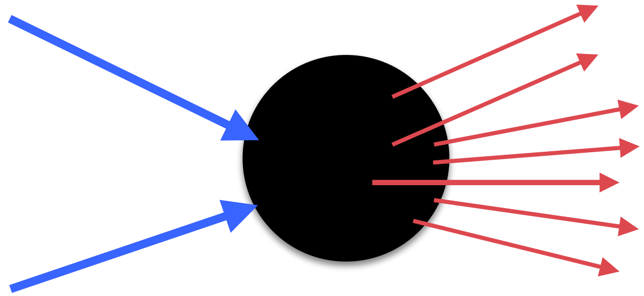
We now want to understand what, if any, provides a large-frequency cutoff and extend the reasoning towards the large-angle/collapse regime.

The emerging picture is quite appealing: transverse momenta are limited by $1/b$ while longitudinal ones (and energies) are controlled by the larger scale $1/R$ (with some leakage at higher frequencies)

If that behavior persists as $b \to b_c \sim R$, the GW/graviton distribution becomes more and more "isotropic" with $\langle n \rangle \sim Gs/\hbar$ and (again!) characteristic energy $O(\hbar/R \sim T_H)$.

A short digression (time permitting)

In 1409.7405 Dvali et al. have considered the process 2 UHE gravitons —> N ULE gravitons



Claim (in D=4 & both QFT & QST): most important contribution to unitarity comes from

$$N \sim \frac{E R_S}{\hbar} \sim S_{BH}(E)$$

and that tree-level saturates unitarity (after adding by hand BH entropy factor)!

Quite amazing if true.

however...

Exclusive x-sections have IR singularities:

1. At tree-level they blow up.
2. At fixed multiplicity w/ virtual corrections resummed they vanish;
3. Only suitably defined inclusive enough xsections are free from IR problems

=> Dvali et al's result needs to be reinterpreted

Work in progress (A. Addazi, M. Bianchi & GV)

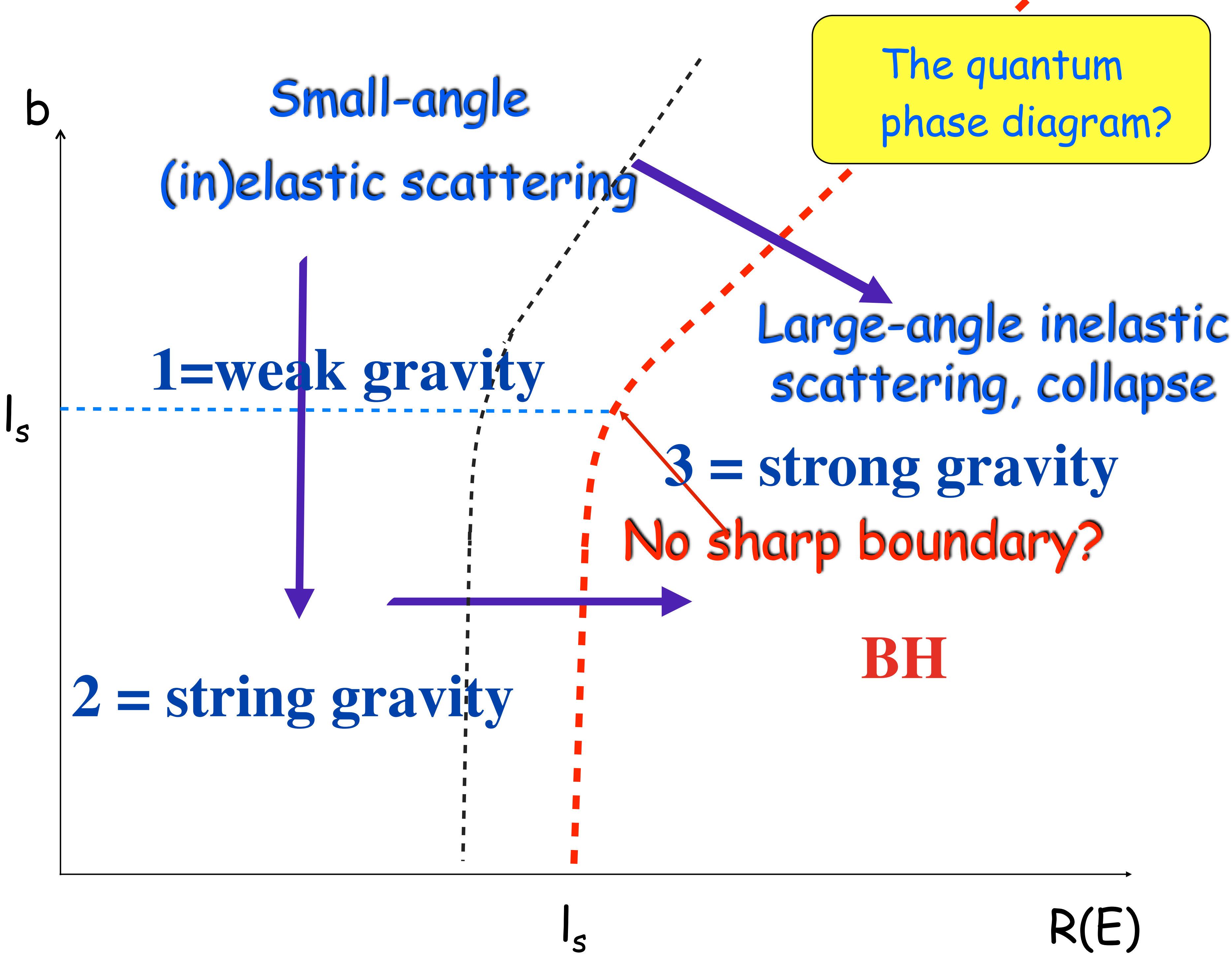
appears to justify qualitatively their basic picture!

One final remark

Is pre-collapse the gravitational analog of pre-confinement (Amati & GV,'79) in QCD?

- A general pattern seems to emerge where, at the quantum level, the transition between the dispersive and the collapse phase is smoothed out.

- As some critical value $b = b_c \sim R$ is approached, the nature of the final state appears to change smoothly from one characteristic of a dispersive state to one reminiscent of Hawking's radiation (high multiplicity & $\langle E_f \rangle = \sim h/R$).



The quantum phase diagram?
b
Small-angle (in)elastic scattering
1=weak gravity
Large-angle inelastic scattering, collapse
l_s
3 = strong gravity
No sharp boundary?
BH
2 = string gravity
l_s
R(E)

Thank you…