Superconducting non-Abelian vortices in Weinberg-Salam theory – electroweak thunderbolts

Mikhail S. Volkov

LMPT, University of Tours, FRANCE

Condensed matter physics meets relativistic quantum field theory,
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Contents

- Superconducting vortices in the Electroweak Theory

  M.S.V. Phys.Lett. B 648, 249 (2007)
  J.Garaud and M.S.V. Nucl.Phys. B 826, 174 (2010)

- Their stability analysis

  J.Garaud and M.S.V. Nucl.Phys. B 799, 430 (2008)
  Nucl.Phys. B839, 310 (2010)

- Spinning vortex loops

  E.Radu and M.S.V. Physics Reports, 468, 101-151 (2008)
  J.Garaud, E.Radu, M.V.S. Phys.Rev.Lett. 111, 171602 (2013)

- Spinning sphalerons

  E.Radu and M.S.V. Phys.Rev. D79, 065021 (2009)

- Superconducting vortices in Weinberg-Salam
- Spinning vortex loops
- Spinning sphalerons in Weinberg-Salam

Superconducting vortices in electroweak theory

U(1) Abrikosov-Nielsen-Olesen vortex

Abelian Higgs model

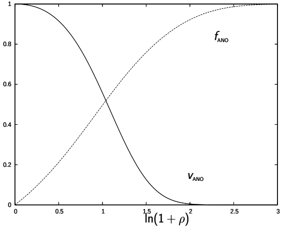
$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2$$

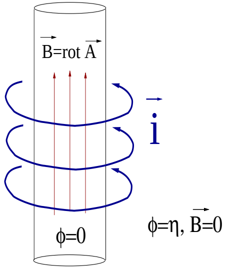
Cylindrically symmetric fields

$$\phi^{\text{ANO}} = \eta f_{\text{ANO}}(\rho) e^{in\varphi}, \quad A^{\text{ANO}} = (n - v_{\text{ANO}}(\rho))d\varphi,$$

$$n \in \mathbb{Z}, \qquad \Psi = 2\pi n \;\Rightarrow\; \text{magnetic flux quantization}$$

$n = 1$ vortex is topologically stable



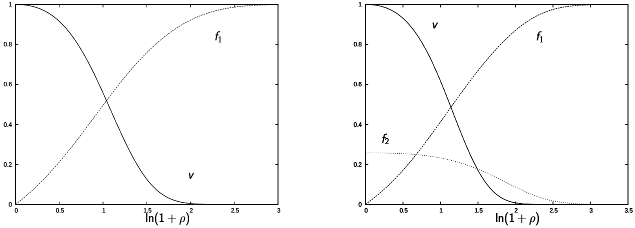


Witten's U(1)×U(1) model '85

$$\mathcal{L}_W = - \frac{1}{4}(F^{(1)}_{\mu\nu})^2 + |D_\mu\phi_1|^2 - \frac{\lambda_1}{4}\left(|\phi_1|^2 - \eta_1^2\right)^2$$
$$- \frac{1}{4}(F^{(2)}_{\mu\nu})^2 + |D_\mu\phi_2|^2 - \frac{\lambda_2}{4}\left(|\phi_2|^2 - \eta_2^2\right)^2 - \gamma|\phi_1|^2|\phi_2|^2$$

'bare vortex': $A^{(1)}_\mu = A^{\mathrm{ANO}}_\mu,\ \phi_1 = \phi^{\mathrm{ANO}}, \qquad A^{(2)}_\mu = \phi_2 = 0.$

'dressed vortex': $A^{(1)}_\mu \sim A^{\mathrm{ANO}}_\mu,\ \phi_1 \sim \phi^{\mathrm{ANO}}, \qquad \phi_2 = f_2(\rho) \neq 0.$

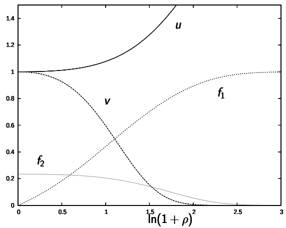


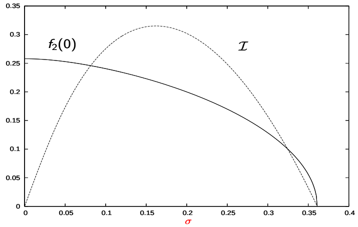
Witten's superconducting strings

Solutions with $A_\mu^{(2)} \neq 0$ interpolating between the 'bare' and 'dressed', with current $J_\mu = \partial^\nu F_{\nu\mu}^{(2)}$ along the string.

$$A^{(2)} = (\sigma_0 dt + \sigma_3 dz)\,(1 - u(\rho)), \quad \phi_2 = f_2(\rho) e^{i\sigma_0 t + i\sigma_3 z},$$

Twist vector $\sigma_\alpha = (\sigma_0, \sigma_3)$ with norm $\sigma^2 = \sigma_3^2 - \sigma_0^2$





$\exists$ critical current. GUT $\Rightarrow$ cosmological applications

What about Standard Model ?

- Weinberg-Salam theory also contains two complex scalar fields and two vector fields (U(1) and SU(2)).
- It has embedded ANO vortices = Z strings

$$\mathcal{W}_Z = 2(g'^2 + g^2\tau^3)A^{\text{ANO}}, \qquad \Phi_Z = \begin{pmatrix} \phi^{\text{ANO}} \\ 0 \end{pmatrix}.$$

/Nambu '77; Vachaspaty '93/

- Perhaps there exist also dressed Z strings that could be generalized for non-zero currents ? /Perkins '93; Olesen '93/
- Z strings are unstable and can be deformed to vacuum ⇒ no non-trivial lower bound for their energy /Klinkhamer, Olesen '94/
- Search for dressed Z strings gave no result /Achucarro, '94/.

Some known electroweak solutions

- Spinning dumbbells. /Nambu '77; Urrestilla et al. '02/
- Sphalerons = energy saddle points.
  /Klinkhamer & Manton '84/
- Vortex lattices /Ambjorn and Olsen '88/
- Oscillons /Graham '07/
- Twisted superconducting strings in the $g \to 0$ limit
  /Forgacs, Reuillon, M.S.V '06/

Perhaps one can find electroweak analogs of Witten's strings ?

SU(2)×U(1) Weinberg-Salam theory

$$\mathcal{L} = -\frac{1}{4g^2}\,\mathrm{W}^a_{\mu\nu}\mathrm{W}^{a\mu\nu} - \frac{1}{4g'^2}\,B_{\mu\nu}B^{\mu\nu} + (D_\mu\Phi)^\dagger D^\mu\Phi - \frac{\beta}{8}\left(\Phi^\dagger\Phi - 1\right)^2 ,$$

$$\begin{aligned}
\mathrm{W}^a_{\mu\nu} &= \partial_\mu \mathrm{W}^a_\nu - \partial_\nu \mathrm{W}^a_\mu + \epsilon_{abc}\mathrm{W}^b_\mu \mathrm{W}^c_\nu\,, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,, \\
\Phi &= \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}, \qquad D_\mu\Phi = \left(\partial_\mu - \frac{i}{2}\,A_\mu - \frac{i}{2}\,\tau^a \mathrm{W}^a_\mu\right)\Phi.
\end{aligned}$$

$$g = \cos\theta_{\mathrm{w}}, \qquad g' = \sin\theta_{\mathrm{w}}$$

$$m_{\mathrm{z}} = \frac{1}{\sqrt{2}}, \qquad m_{\mathrm{w}} = m_{\mathrm{z}}\cos\theta_{\mathrm{w}}, \quad \beta = \left(\frac{m_{\mathrm{h}}}{m_{\mathrm{z}}}\right)^2 = 1.88$$

Field equations

$$\partial_\mu B^{\mu\nu} = g'^2\,\Re(i\Phi^\dagger D^\nu\Phi)\,,$$
$$\partial_\mu \mathrm{W}_a^{\mu\nu} + \epsilon_{abc}\mathrm{W}_\sigma^b\mathrm{W}^{c\sigma\nu} = g^2\,\Re(i\Phi^\dagger\tau^a D^\nu\Phi)\,,$$
$$D_\mu D^\mu\Phi = \frac{\beta}{4}(\Phi^\dagger\Phi - 1)\Phi.$$

$n^a = \Phi^\dagger\tau^a\Phi/(\Phi^\dagger\Phi) \Rightarrow$ electromagnetic, Z fields /Nambu '77/

$$F_{\mu\nu} = \frac{g}{g'}B_{\mu\nu} - \frac{g'}{g}n^a\mathrm{W}^a_{\mu\nu}\,, \qquad Z_{\mu\nu} = B_{\mu\nu} + n^a\mathrm{W}^a_{\mu\nu}\,,$$

$\Rightarrow$ electromagnetic current density

$$J_\mu = \partial^\nu F_{\nu\mu}$$

Vortex symmetries

$$K_{(t)} = \frac{\partial}{\partial t}, \qquad K_{(z)} = \frac{\partial}{\partial z}, \qquad K_{(\varphi)} = \frac{\partial}{\partial \varphi}$$

$\Rightarrow$ energy, momentum, angular momentum

$$\int T^0_\mu K^\mu_{(t)} d^2x, \qquad \int T^0_\mu K^\mu_{(z)} d^2x, \qquad \int T^0_\mu K^\mu_{(\varphi)} d^2x,$$

electric charge and current $(\alpha = 0, 3)$

$$\mathcal{I}^\alpha = \int J^\alpha d^2x$$

Field ansatz

Symmetries commute $\Rightarrow \exists$ a gauge where the fields depend only on ρ. Let $\sigma_\alpha = (\sigma_0, \sigma_3)$ be a twist vector then

$$\mathcal{W} = u(\rho)\,\sigma_\alpha dx^\alpha - v(\rho)\,d\varphi + \tau^1\,[u_1(\rho)\,\sigma_\alpha dx^\alpha - v_1(\rho)\,d\varphi]$$

$$+\,\tau^3\,[u_3(\rho)\,\sigma_\alpha dx^\alpha - v_3(\rho)\,d\varphi], \qquad \Phi = \begin{pmatrix} f_1(\rho) \\ f_2(\rho) \end{pmatrix}$$

- $\mathcal{W}_\rho = 0$ gauge condition – remain 8 out of 16 real functions
- $\mathcal{W} = \mathcal{W}^*, \qquad \Phi = \Phi^*$
- Boosts along $z = x^3$ axis
- Residual global symmetry $(f_1 + if_2) \to e^{\frac{i}{2}\Gamma}(f_1 + if_2)$, $(u_1 + iu_3) \to e^{-i\Gamma}(u_1 + iu_3)$, $(v_1 + iv_3) \to e^{-i\Gamma}(v_1 + iv_3)$
- Only the "twist" (norm) $\sigma^2 = \sigma_3^2 - \sigma_0^2$ appears in the equations
- Maxwell and Z fluxes are no longer quantized.

8 coupled equations

$$\frac{1}{\rho}(\rho u')' = \frac{g'^{\,2}}{2}\left\{(u+u_3)f_1^2 + 2\,u_1 f_1 f_2 + (u-u_3)f_2^2\right\},$$

$$\rho\left(\frac{v'}{\rho}\right)' = \frac{g'^{\,2}}{2}\left\{(v+v_3)f_1^2 + 2\,v_1 f_1 f_2 + (v-v_3)f_2^2\right\},$$

$$\begin{aligned}
\frac{1}{\rho}(\rho f_1')' &= \left\{\frac{\sigma^2}{4}\left[(u+u_3)^2 + u_1^2\right] + \frac{1}{4\rho^2}\left[(v+v_3)^2 + v_1^2\right] + \frac{\beta}{4}(f_1^2+f_2^2-1)\right\} f_1 \\
&\quad + \left(\frac{\sigma^2}{2}\,uu_1 + \frac{1}{2\rho^2}\,vv_1\right) f_2, \\
\frac{1}{\rho}(\rho f_2')' &= \left\{\frac{\sigma^2}{4}\left[(u-u_3)^2 + u_1^2\right] + \frac{1}{4\rho^2}\left[(v-v_3)^2 + v_1^2\right] + \frac{\beta}{4}(f_1^2+f_2^2-1)\right\} f_2 \\
&\quad + \left(\frac{\sigma^2}{2}\,uu_1 + \frac{1}{2\rho^2}\,vv_1\right) f_1, \\
\frac{1}{\rho}(\rho u_1')' &= -\frac{1}{\rho^2}\,(v_1 u_3 - v_3 u_1)\,v_3 + \frac{g^2}{2}\left[u_1(f_1^2+f_2^2) + 2uf_1 f_2\right], \\
\frac{1}{\rho}(\rho u_3')' &= +\frac{1}{\rho^2}\,(v_1 u_3 - v_3 u_1)\,v_1 + \frac{g^2}{2}\left[(u_3+u)f_1^2 + (u_3-u)f_2^2\right], \\
\rho\left(\frac{v_1'}{\rho}\right)' &= +\sigma^2\,(v_1 u_3 - v_3 u_1)\,u_3 + \frac{g^2}{2}\left[v_1(f_1^2+f_2^2) + 2vf_1 f_2\right], \\
\rho\left(\frac{v_3'}{\rho}\right)' &= -\sigma^2\,(v_1 u_3 - v_3 u_1)\,u_1 + \frac{g^2}{2}\left[(v_3+v)f_1^2 + (v_3-v)f_2^2\right].
\end{aligned}$$

Boundary conditions

- At the symmetry axis $\rho = 0$ the fields are regular, energy density is finite.
- At infinity $\rho \to \infty$ the fields approach the Biot-Savart field of an infinitely long electric wire:

$$A_\mu = \frac{Q}{gg'}\,\sigma_\alpha dx^\alpha \ln\frac{\rho}{\rho_0} + c\,d\varphi$$

$$Z_\mu = 0, \quad \mathbf{W}^\pm_\mu = 0, \quad \Phi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

The current of the wire

$$\mathcal{I}_\alpha = \int J_\alpha\, d^2x = -\frac{2\pi Q}{gg'}\,\sigma_\alpha$$

Local solutions at the origin

$$u = a_1 + \ldots, \quad u_1 = a_2\rho^{\nu} + \ldots, \quad u_3 = 1 + \ldots,$$
$$v_1 = O(\rho^{\nu+2}), \quad v_3 = \nu + a_3\,\rho^2 + \ldots, \quad v = 2n - \nu + a_4\,\rho^2 + \ldots,$$
$$f_1 = a_5\,\rho^{n} + \ldots, \quad f_2 = q\rho^{|n-\nu|} + \ldots$$

If $n, \nu \in \mathbb{Z}$ then one can pass to a regular gauge where $\mathcal{W}_\varphi = 0$ at $\rho = 0$, but fields depend also on t, z, φ.

Infinity = Biot-Savart+corrections

$$u = Q \ln\rho + c_1 + \frac{c_3 g'^2}{\sqrt{\rho}}\, e^{-m_z\rho} + \dots$$

$$v = c_2 + c_4 g'^2 \sqrt{\rho}\, e^{-m_z\rho} + \dots$$

$$u_1 + iu_3 = e^{-i\gamma}\left\{\frac{c_7}{\sqrt{\rho}}\, e^{-\int m_\sigma d\rho} + i\left[-Q\ln\rho - c_1 + \frac{c_3 g^2}{\sqrt{\rho}}\, e^{-m_z\rho}\right]\right\} + \dots$$

$$v_1 + iv_3 = e^{-i\gamma}\left\{c_8\sqrt{\rho}\, e^{-\int m_\sigma d\rho} + i\left[-c_2 + c_4 g^2\sqrt{\rho}\, e^{-m_z\rho}\right]\right\} + \dots$$

$$f_1 + if_2 = e^{\frac{i}{2}\gamma}\left\{1 + \frac{c_5}{\sqrt{\rho}}\, e^{-m_h\rho} + i\,\frac{c_6}{\sqrt{\rho}}\, e^{-\int m_\sigma d\rho}\right\} + \dots$$

depend on

$$m_z, \quad m_h, \quad m_\sigma = \sqrt{m_w^2 + \sigma^2(Q\ln\rho + c_1)^2} \sim \mathcal{I} = \sigma Q$$

$\Rightarrow$ fields are localized if only $\sigma^2 \geq 0$ (magnetic or chiral type).

Global solutions

- the local solutions at $\rho \ll 1$ and at $\rho \gg 1$ are numerically extended and matched at $\rho \sim 1$ within the multiple shooting method.
- there are 16 matching conditions and 17 parameters to resolve them: $a_1, \ldots, a_5$ and q at the origin, also $c_1, \ldots, c_8$, C, γ at infinity and also σ^2.
- there is one parameter left to label the global solutions: condensate parameter $\boxed{q = f_2(0)}$.

$$q = 0 \Rightarrow \textbf{Z strings}$$

$$\mathcal{W}_Z = 2(g'^2 + g^2\tau^3)A^{\rm NO}, \qquad \Phi_Z = \begin{pmatrix} f_{\rm NO}(\rho)e^{in\varphi} \\ 0 \end{pmatrix}.$$

$q = f_2(0) \ll 1$; perturbative solutions

small Z string deformations $(\mathcal{W}, \Phi) = (\mathcal{W}_Z, \Phi_Z) + (\delta\mathcal{W}, \delta\Phi)$,

$$(\delta\mathcal{W}, \delta\Phi) \sim e^{i\sigma_\alpha x^\alpha}\Psi(\rho)$$

$\Rightarrow$ eigenvalue problem for $\sigma^2 = \sigma_3^2 - \sigma_0^2$

$$\Psi'' = (\sigma^2 + V_Z[\beta, \theta_{\rm w}, n, \nu, \rho])\Psi,$$

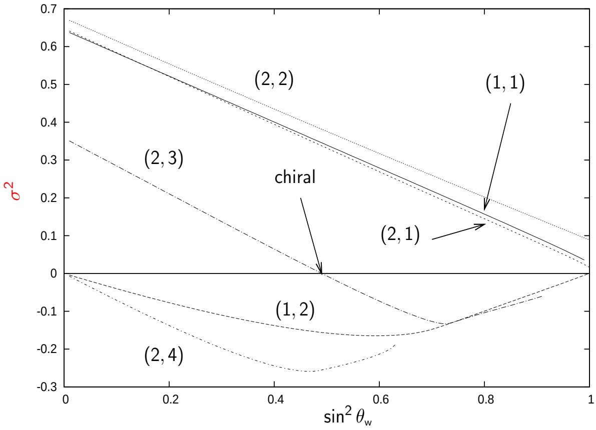
$\Rightarrow$ $2n$ bound states labeled by $\nu = 1, 2, \ldots 2n$

$$\Psi \sim \exp(-m_\sigma \rho), \qquad m_\sigma^2 = m_{\rm w}^2 + \sigma^2$$

describe Z string slightly perturbed by a current $\mathcal{I}_\alpha \sim \sigma_\alpha$.

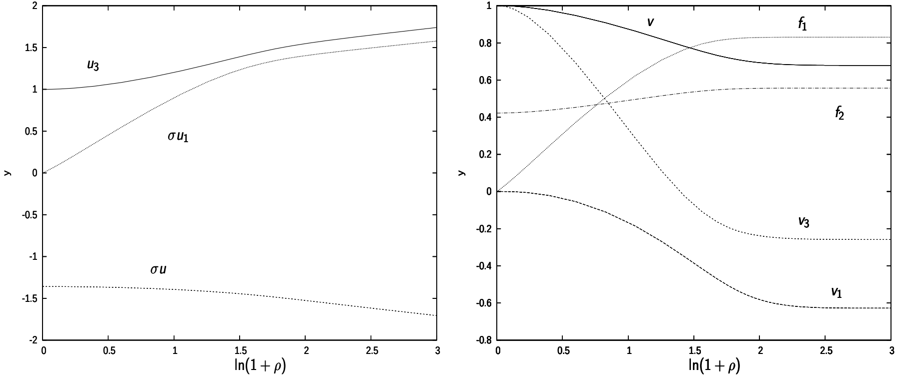
/One has $\sigma_0 = \sqrt{\sigma_3^2 - \sigma^2}$ $\Rightarrow$ vortices with $\sigma^2 > 0$ exist in the region where Z-strings are unstable: one can set momentum $\sigma_3 = 0 \Rightarrow \sigma_0 = \sqrt{-\sigma^2}$/

$\sigma^2(n,\nu)$-eigenvalue $(\beta = 2)$



$\sigma^2 = 0$ $\exists$ only for special values of β, θ_w, n, ν

Fully non-linear solutions, $q = f_2(0) \sim 1$

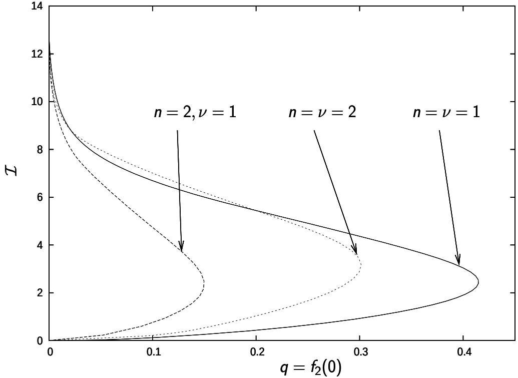


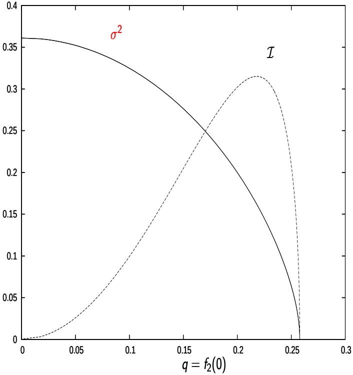
Generic superconducting vortices

are globally regular, with a regular vortex core containing a massive W-condensate that creates a current. The current produces a Biot-Savart field outside the core. They are field theory realisations of electric wires

- Exist for any value of the Higgs mass and for any $\theta_{\rm w}$
- Comprise a four parameter family labeled by current $\mathcal{I}$, electric charge Q and by two integers n, ν determining the values of the magnetic and Z fluxes.
- Vortices with different Q are related to each other by Lorentz boosts.
- For $\mathcal{I} \to 0$ reduce to Z strings.

Current $\mathbf{I}/\mathbf{I}_0 = \mathcal{I}$





$$\mathbf{I}_0 = \mathbf{c}\boldsymbol{\Phi}_0 = \mathbf{c} \times 54.26 \times 10^9 \text{ Volts} = 1.8 \times 10^9 \text{ Amperes}.$$

Large current limit

- In conventional superconductivity models $\mathcal{I}$ is bounded because it is carried by the scalar condensate, which is destroyed by the strong magnetic field.

- In the Weinberg-Salam theory the current is carried by the vector W-condensate, which is not quenched by the magnetic field, even though $\Phi \to 0$. As a result, $\mathcal{I}$ is unbounded (in classical theory).

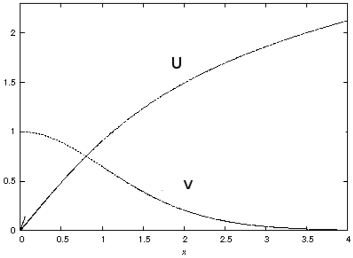
- For $\mathcal{I} \gg 1$ the system splits into the central W-condensate region and the external region.

Central W-condensate region, $\rho < 1/\mathcal{I}$

$B_\mu \approx$const., $\Phi \sim 1/\mathcal{I}^2 \approx 0 \Rightarrow \mathcal{L} = -\frac{1}{4g^2}\,\mathrm{W}^a_{\mu\nu}\mathrm{W}^{a\mu\nu}$

$\tau^a \mathrm{W}^a_\nu dx^\mu = \tau^1 \lambda \mathrm{U}(\xi) dz + \tau^3 \mathrm{V}(\xi) d\varphi \qquad \xi = \lambda\rho,\ \ \lambda = \text{scale parameter}$

$$\begin{aligned} \xi + \ldots \leftarrow \quad \mathrm{U} \quad &\rightarrow 0.85 + 0.91\ln(\xi) + \ldots \\ 1 \leftarrow \quad \mathrm{V} \quad &\rightarrow 0.32\sqrt{\xi}\, e^{0.06\xi}\xi^{-0.91\xi} + \ldots \end{aligned}$$



$\lambda = 0.17\frac{g}{g'}\,\mathcal{I}$

External region, $\rho > 1/\mathcal{I}$

U(1)×U(1) theory = Maxwell + Abelian Higgs

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(Z_{\mu\nu})^2 + |(\partial_\mu - \frac{i}{2} Z_\mu)\phi|^2 - \frac{\beta}{8}(|\phi|^2 - 1)^2$$

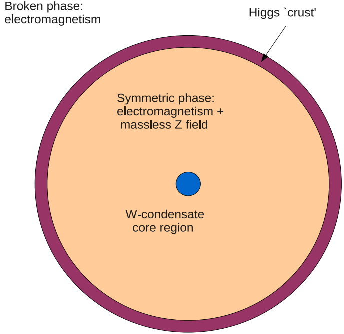
with

$$A_\mu = \frac{g}{g'} B_\mu - \frac{g'}{g} \mathrm{W}^1_\mu, \qquad Z_\mu = B_\mu + \mathrm{W}^1_\mu, \quad \phi_1 \approx \phi_2 \equiv \phi$$

Solution

$$\begin{aligned} 1/\mathcal{I} < \rho \leq \mathcal{I} \quad &: \qquad A_\mu \sim \mathcal{I} \ln \rho, \qquad Z_\mu \sim \mathcal{I} \ln \frac{\rho}{\mathcal{I}}, \qquad \phi \sim \left(\frac{\rho}{\mathcal{I}}\right)^{\mathcal{I}\rho} \\ \rho \geq \mathcal{I} \quad &: \qquad A_\mu \sim \mathcal{I} \ln \rho, \qquad Z = 0, \qquad \phi = 1 \end{aligned}$$

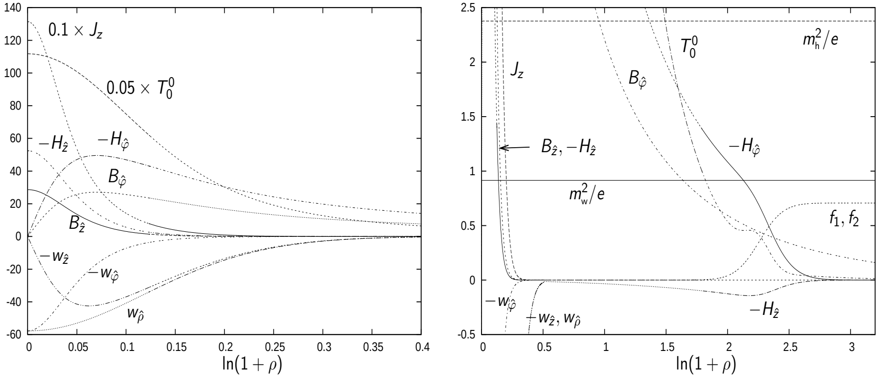
Large $\mathcal{I}$ vortex cross section



I. W-condensate core $\sim 1/\mathcal{I}$
II. symmetric phase $\sim \mathcal{I}$
III. Higgs 'crust' $\sim 1/m_h$

/Ambjorn & Olesen '88/

Inner structure of large $\mathcal{I}$ vortex



II. Stability analysis

J.Garaud and M.S.V. Nucl.Phys. B 799, 430 (2008)
Nucl.Phys. B839, 310 (2010)

Generic vortex perturbations

$$\Phi \to \Phi + \delta\Phi, \qquad B_\mu \to B_\mu + \delta B_\mu, \qquad W^a_\mu \to W^a_\mu + \delta W^a_\mu$$

$$\delta\Phi = \sum_\omega, k, m \{[\phi_\omega, k, m(\rho) + i\,\psi_\omega, k, m(\rho)] \cos(\omega t + m\varphi + \kappa z)$$
$$+[\pi_\omega, k, m(\rho) + i\,\chi_\omega, k, m(\rho)] \sin(\omega t + m\varphi + \kappa z)\} \ ,$$
$$\delta B_\mu = \sum_\omega, k, m \{\ldots\}$$
$$\delta W^a_\mu = \sum_\omega, k, m \{\ldots\}$$

Perturbation equations

Imposing the background gauge condition and separating the variables gives a Schroedinger system

$$-\Psi'' + \mathbf{U}_{m,\kappa}\Psi = \omega^2\Psi\,,$$

$\Psi(\rho)$ is a 16-component vector, $\mathbf{U}_{m,\kappa}(\rho)$ is a potential matrix determined by the background fields.

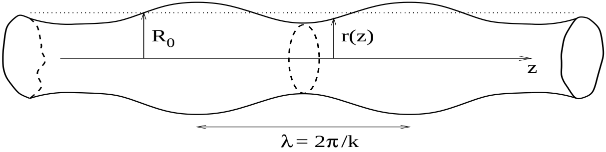
Bound states with $\omega^2 < 0 \Rightarrow$ unstable modes.

They exist only in the $m = 0$ channel.

String instabilities

Negative modes with $\omega^2(\kappa) < 0$ have the structure

$$e^{|\omega| t} \cos(\kappa t) \Psi(\rho) \quad \Rightarrow \quad \text{vortex fragmentation}$$



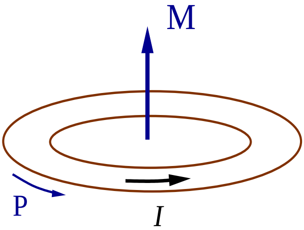
$$\kappa < \kappa_{max}(\mathcal{I}) \quad \Rightarrow \quad \boxed{\lambda > \lambda_{\min}(\mathcal{I}) = 2\pi/\kappa_{max}} \quad \Rightarrow$$

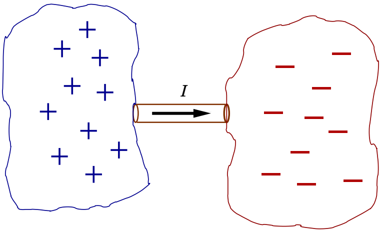
imposing periodicity with period $L < \lambda_{\min}(\mathcal{I})$ eliminates negative modes /Plateau-Rayleigh, Gregory-Laflamme/

Periodicity can be imposed by bending the vortex to a loop. $\Rightarrow$ small and thick vortex loops might be stable – because they are hard to pinch or bend

Stabilizing vortex segments

- Making loops (electroweak vortons ?)
- Attaching the ends to something (polarized clouds) ⇒ charge transfer





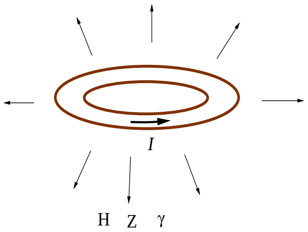
Electroweak thunderbolts

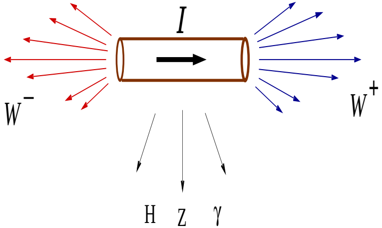
Finite vortex segments transferring charge between regions of space. Their current $\mathcal{I} \sim 10^9 - 10^{10}$ A

For atmospheric thunderbolts $\mathcal{I} \sim 3 \times 10^5$ A

Virtual vortex segments

- Closed segments – showers of neutral particles
- Open segments – charged jets





Perhaps they could be observed at the LHC ?

Summary of part I

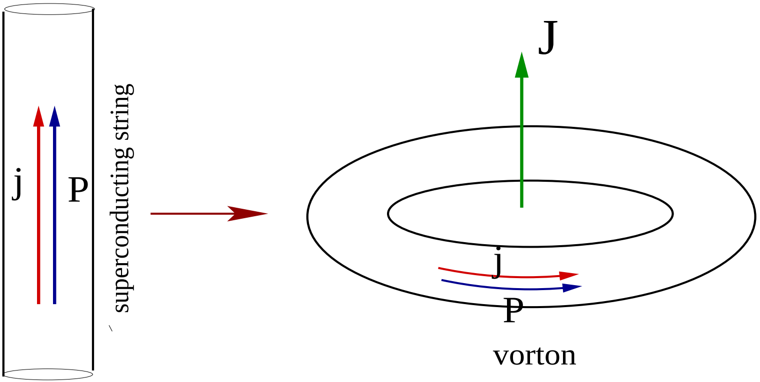
- There are superconducting vortices in the electroweak theory.
- Their current can typically attain billions of Amperes, and there seems to be no upper bound for it (in classical theory).
- For large currents the electroweak gauge symmetry is completely restored inside the vortex by the strong magnetic field.
- Vortices with $Q \neq 0$ could be stable upon imposing periodic boundary conditions.

Could vortex loops exist and be stable ?

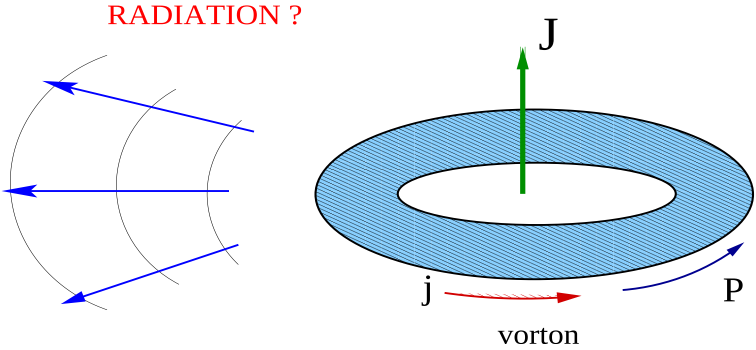
II. Spinning solitons and spinning vortex loops

E.Radu and M.S.V. Physics Reports, 468, 101-151 (2008)
J.Garaud, E.Radu, M.V.S. Phys.Rev.Lett. 111, 171602 (2013)

Effective macroscopic description: superconducting vortex = elastic rope /Davis, Shellard '88/



Perhaps vortons radiate ? Loop of current = accelerated motion of charges ⇒



Are they stationary ?

Constructing stationary vortons

Global limit of Witten's model

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + \partial_\mu \sigma^* \partial^\mu \sigma - U$$

$$U = \frac{1}{4}\lambda_\phi(|\phi|^2 - \eta_\phi^2)^2 + \frac{1}{4}\lambda_\sigma |\sigma|^2(|\sigma|^2 - 2\eta_\sigma^2) + \gamma|\phi|^2|\sigma|^2.$$

Axial symmetry

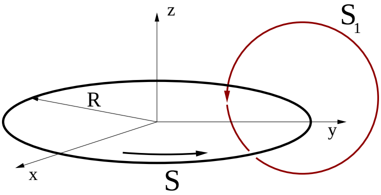
$$\phi = X(\rho, z) + iY(\rho, z), \qquad \sigma = Z(\rho, z) e^{i\omega t + im\varphi}$$

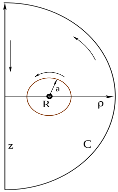
Noether charge Q and angular momentum J

$$Q = \omega \int |\sigma|^2 d^3x, \qquad J = \int T^0_\varphi d^3x = mQ$$

$$\text{spin:} \qquad m = J/Q$$

Equations + boundary conditions





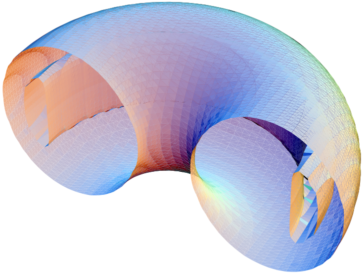
Phases of $\sigma = Ze^{i\omega t + im\varphi}$, $\phi = X + iY$ increase along S, S_1.

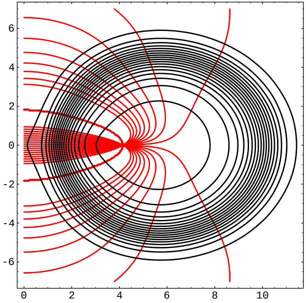
$$\Delta X = \left(\frac{\lambda_\phi}{2}(X^2 + Y^2 - 1) + \gamma Z^2 \right) X,$$

$$\Delta Y = \left(\frac{\lambda_\phi}{2}(X^2 + Y^2 - 1) + \gamma Z^2 \right) Y,$$

$$\Delta Z = \left(\frac{m^2}{\rho^2} - \omega^2 + \frac{\lambda_\sigma}{2}(Z^2 - \eta_\sigma^2) + \gamma(X^2 + Y^2) \right) Z.$$

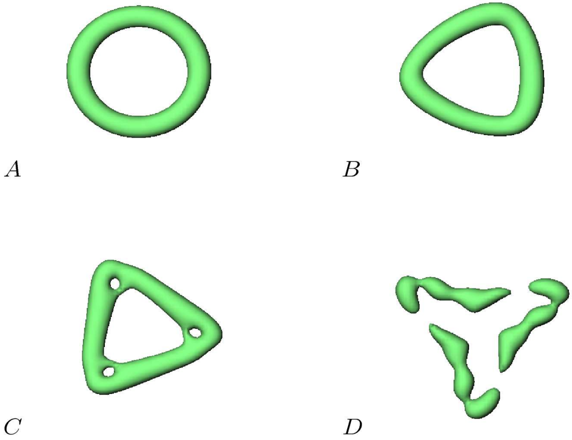
Thick vortons (small $m = J/Q$)





/Radu and M.S.V. '08/ – 20 years later.

Thin ring vortons – large $m = J/Q$



Pinching and bending instabilities /Battye and Sutcliffe '09/

Fully non-linear evolution in $3+1$

One solves the non-linear evolution problem (within a finite element numerical scheme)

$$\Box\phi + \frac{\partial U(\phi,\sigma)}{\partial|\phi|^2}\,\phi = 0, \qquad \Box\sigma + \frac{\partial U(\phi,\sigma)}{\partial|\sigma|^2}\,\sigma = 0$$

in a finite spatial box with reflecting boundary conditions. One starts from a stationary vorton configuration

$$\underline{t=0}: \quad \phi = \phi_{\rm vort}(\mathbf{x}),\ \ \sigma = \sigma_{\rm vort}(\mathbf{x}), \qquad \dot{\phi} = 0, \qquad \dot{\sigma} = -i\omega\sigma.$$

A non-trivial evolution is triggered by the discretization. The result depends on value of $m = J/Q$: **video**

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Large vorton

(m=6)

(m=3)

(m=1)

Stable – similar result for ”spinning light bullets”
/Michalache . . . Malomed . . . 2002/

Summary of part II

- The small and thick $n = m = 1$ global vortons with a sufficiently large charge are dynamically stable.

- Can global vortons be promoted to local solutions of the Weinberg-Salam theory = electroweak vortons ?

- If stable electroweak vortons exist, could they contribute to the dark matter ?

III. Electroweak vortons with $n = 0$, $m \neq 0$ winding
$\Rightarrow$ spinning sphalerons

E.Radu and M.S.V. Phys.Rev. D79, 065021 (2009)

Axial ansatz with 8 functions of ρ, z

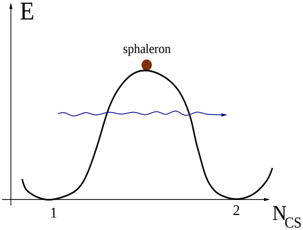
$$\mathcal{W} = (Y_a + \tau^1\psi^1_a + \tau^3\psi^3_a)dx^a + \tau^2 v_k dx^k, \quad \Phi = \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix}$$

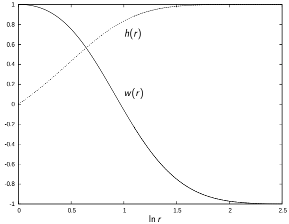
where $x^a = (t, \varphi)$ and $Y_a, \psi^1_a, \psi^3_a, v_k$ depend on $x^k = (\rho, z)$.

$$r \to \infty: \qquad \mathcal{W} = (\tau^3 - 1)\,(\omega dt + d\varphi) + \delta\mathcal{W}, \qquad \Phi = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \delta\Phi,$$

winds only about the azimuthal direction, $m \neq 0$, but $n = 0$

Sphaleron = saddle point



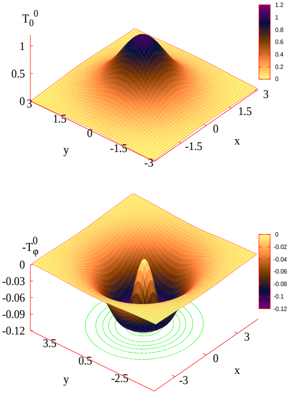


$\theta_{\mathrm{w}} = 0 \Rightarrow$ spherically symmetric /Klinkhamer & Manton '84/

$$W_{\mathrm{j}}^{a} = \epsilon_{\mathrm{ajk}} \frac{\mathrm{x}^{\mathrm{k}}}{r^2}(w(r)+1), \quad \Phi = \begin{bmatrix} h(r) \\ 0 \end{bmatrix}.$$

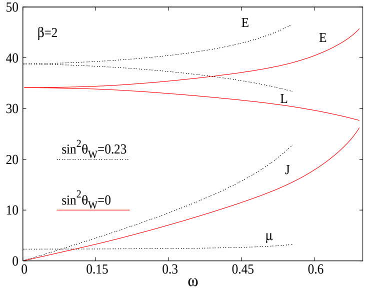
$\theta_{\mathrm{w}} > 0 \Rightarrow$ axially symmetric

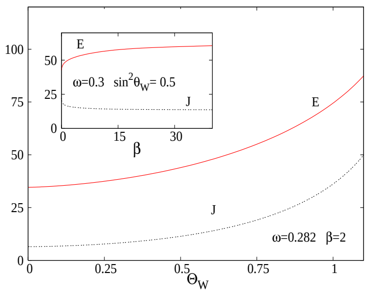
T_0^0 and $T_\varphi^0 \sim \omega$



Angular momentum

$$T_\varphi^0 = \frac{1}{gg'}\,\frac{1}{\rho}\,\partial_k(\rho F_{0k}) + \ldots \Rightarrow J = \int T_\varphi^0 d^3\mathbf{x} = \frac{1}{gg'}\oint \vec{\mathcal{E}}d\vec{S} \Rightarrow \boxed{\mathrm{Q} = \mathrm{eJ}}$$

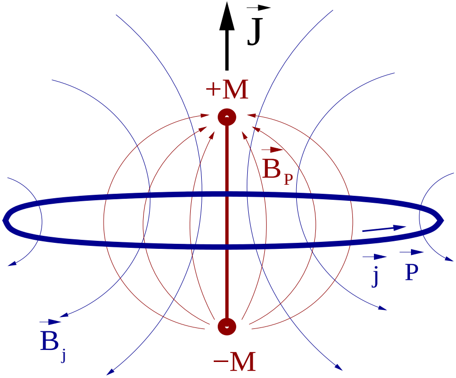




In quantum theory $J \in \mathbb{Z} \Rightarrow Q$ is quantized, sphalerons mediate transitions in sectors with $Q = 0$ (ZZ), $\pm e$ $(ZW^{\pm})$, $\ldots$

Inner structure

For large J shows the Regge behavior $J \sim E^2$ predicted by Nambu for dumbbells.



Contains a monopole-antimonopole pair and a spinning loop with zero winding number.

Summary of results

- Solutions describing superconducting vortices in the Weinberg-Salam theory.
- Stationary and stable vortex loops (vortons) within the U(1)$\times$U(1) global theory model.
- Spinning sphalerons in the Weinberg-Salam theory – $(n = 0, m > 0)$ vortons.

It is plausible that $n > 0, m > 0$ electoweak vortons also exist. The $n = 1, m = 1$ vorton should be stable – a possible dark matter candidate