Catalysis of dynamical chiral symmetry breaking by chiral chemical potential

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15 June 2016

Outline:

- Introduction
- SU(2) QCD with nonzero chiral chemical potential
- SU(3) QCD with nonzero chiral chemical potential
- Theoretical explanation
- Conclusion

Introduction

Motivation:

- QCD action can be separated into right and left parts:
  $S_{QCD} = S_R + S_L$
- There is symmetry between right and left parts
- One can introduce asymmetry of the form $\mu_5 Q_5, \quad Q_5 = Q_R - Q_L$
- $S_{QCD}(\mu_5) = S_R(\mu = \mu_5) + S_L(\mu = -\mu_5)$
- $\mu_5 \neq 0$ can be created in
  - Heavy ion collisions
  - Neutron stars and supernovae
  - Early Universe
  - Dirac and Weyl semimetals ($\vec{E}||\vec{H}$)
  - Elastic deformation (talk of Maria Vozmediano)

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How nonzero chiral chemical potential influences the properties of QCD

Studies of the phase diagram of chiral QCD

- "Chiral magnetic effect in the PNJL model"
  Kenji Fukushima, Marco Ruggieri, Raoul Gatto, Phys.Rev. D81 (2010) 114031
- "Phase diagram of chirally imbalanced QCD matter"
  M.N. Chernodub, A.S. Nedelin, Phys.Rev. D83 (2011) 105008
- "Hot Quark Matter with an Axial Chemical Potential"
  Raoul Gatto, Marco Ruggieri, Phys.Rev. D85 (2012) 054013
- "Inverse magnetic catalysis induced by sphalerons"
  Jingyi Chao, Pengcheng Chu, Mei Huang, Phys.Rev. D88 (2013) 054009
- "Spontaneous generation of local CP violation and inverse magnetic catalysis
  Lang Yu, Hao Liu, Mei Huang, Phys.Rev. D90 (2014) 7, 074009
- "The effect of the chiral chemical potential on the chiral phase transition in the NJL model with different regularization schemes
  Lang Yu, Hao Liu, Mei Huang, arXiv:1511.03073
- ...

Results:

- Decrease of the critical temperature with chiral chemical potential
- Decrease of the chiral condensate with chiral chemical potential

Studies of the phase diagram of chiral QCD

- "Universality of phase diagrams in QCD and QCD-like theories
  M. Hanada, N. Yamamoto, PoS LATTICE2011, 221 (2011), arXiv:1111.3391
- "Chemical potentials and parity breaking: the Nambu-Jona-Lasinio model"Alexander A. Andrianov, Domenec Espriu, Xumeu Planells, Eur.Phys.J. C74 (2014) 2, 2776
- "Effect of the chiral chemical potential on the position of the critical endpoint"Bin Wang, Yong-Long Wang, Zhu-Fang Cui, Hong-Shi Zong, Phys.Rev. D91 (2015) 3, 034017
- "Chiral phase transition with a chiral chemical potential in the framework of Dyson-Schwinger equations"Shu-Sheng Xu, Zhu-Fang Cui, Bin Wang, Yuan-Mei Shi, You-Chang Yang, Hong-Shi Zong, Phys.Rev. D91 (2015) 5, 056003
- ...

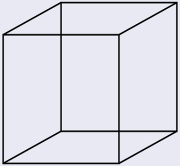
Results:

- Increase of the critical temperature with chiral chemical potential
- Increase of the chiral condensate with chiral chemical potential

SU(2) QCD
with
nonzero chiral chemical potential

Details of the calculation:

- Dynamical staggered fermions ($N_f = 4$) + Wilson action (SU(2))
- Link modification:
  $U \to U e^{\mu_5 \gamma_5}, \quad U^+ \to U^+ e^{-\mu_5 \gamma_5} \Rightarrow$ nonlocal action
- Chiral chemical potential:
  $\delta S_{\mu_5} = \frac{1}{2}\mu_5 a \sum_x (-1)^{x_2} (\bar{\psi}_{x+\delta} \bar{U}_{x+\delta,x} \psi_x - \bar{\psi}_x \bar{U}^{\dagger}_{x+\delta,x} \psi_{x+\delta})$

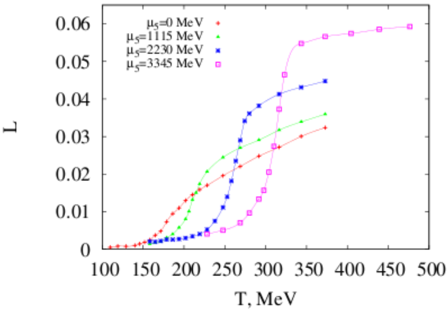


- Correct continuum limit: $\delta S_{\mu_5}|_{a\to 0} \to \mu_5 \int d^4x \; \bar{Q}(\gamma_4\gamma_5 \times 1) Q$
- $6 \times 20^3 (m_\pi \sim 300\text{MeV}), \; 10 \times 28^3 (m_\pi \sim 500\text{MeV})$

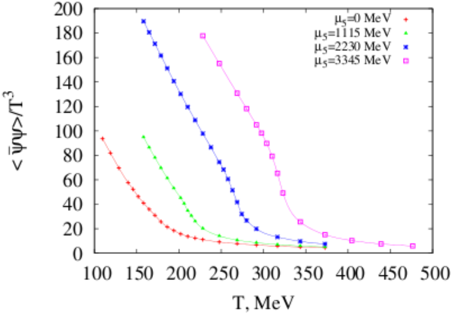
Observables:

- The Polyakov loop (confinement/deconfinement transition)
  $L = \frac{1}{N_\sigma^3} \sum_{n_1, n_2, n_3} \langle Tr \prod_{n_4=1}^{N_\tau} U_4(n_1, n_2, n_3, n_4) \rangle$
- The chiral condensate (chiral symmetry breaking/restotation transition)
  $a^3 \langle \bar{\psi}\psi \rangle = -\frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \frac{\partial}{\partial(ma)} \log Z = \frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \langle Tr \frac{1}{D+ma} \rangle$
- The Polyakov loop susceptibility
  (position of the transition)
  $\chi_L = N_\sigma^3 \left(\langle L^2 \rangle - \langle L \rangle^2 \right)$
- The disconnected part of the chiral susceptibility
  (position of the transition)
  $\chi_{disc} = \frac{1}{N_\tau N_\sigma^3} \frac{1}{16} (\langle (Tr \frac{1}{D+ma})^2 \rangle - \langle Tr \frac{1}{D+ma} \rangle^2)$

Polyakov loop and chiral condensate

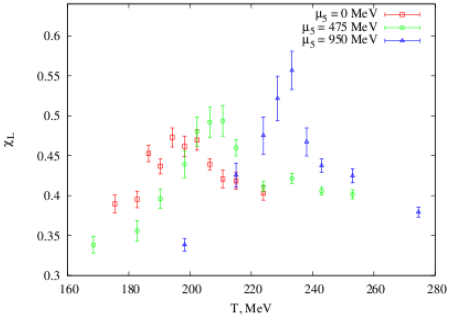


Polyakov loop

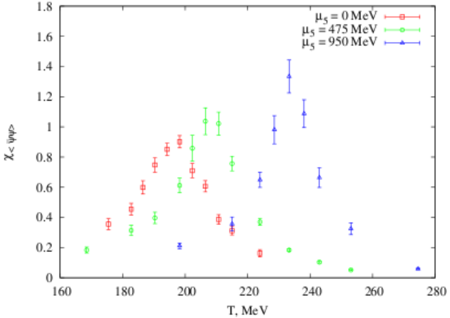


chiral condensate

Susceptibilities

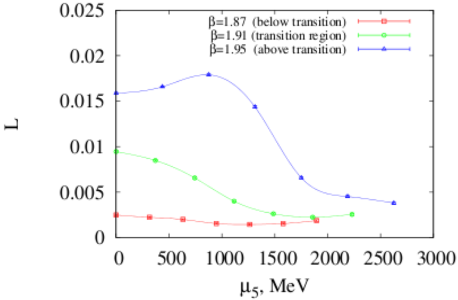


Polyakov loop susceptibility

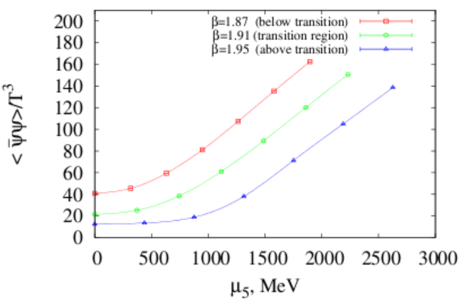


chiral susceptibility

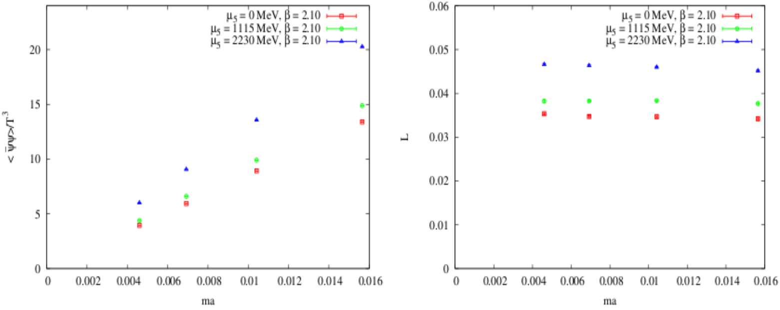
Fixed temperature scan



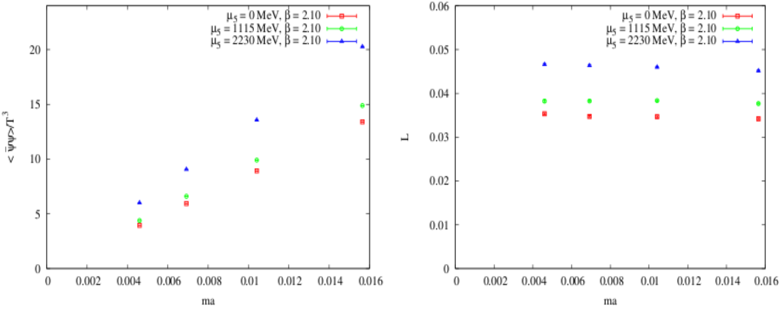
Polyakov loop



chiral condensate

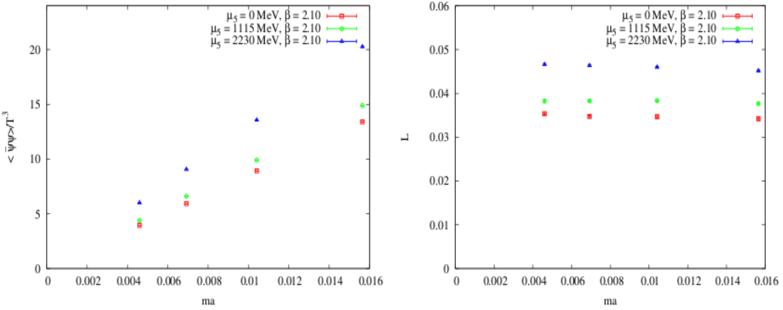


Uncertainties:



Uncertainties:

- Chiral limit



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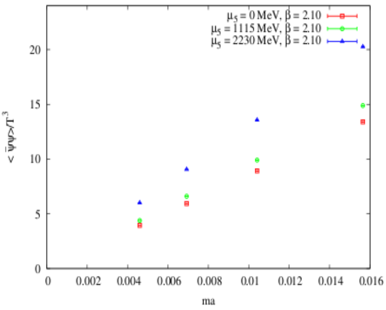
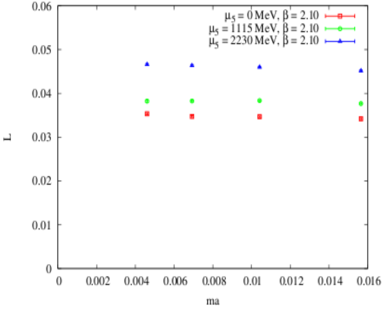
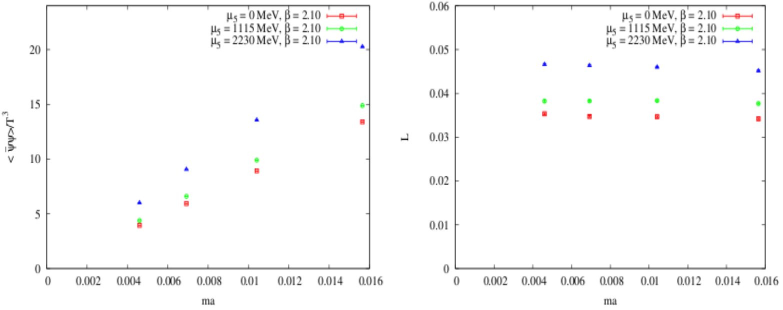




Uncertainties:

- Chiral limit ✓
- Final volume effects



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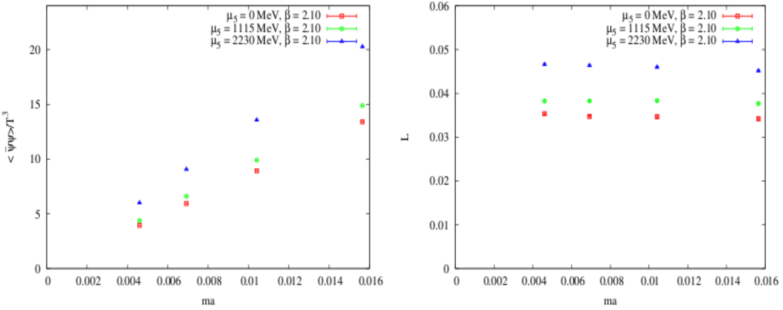

Uncertainties:

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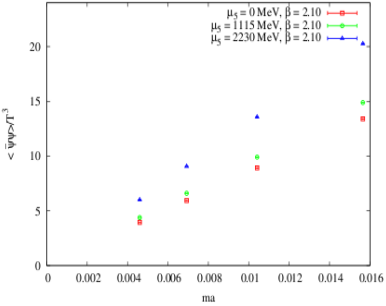
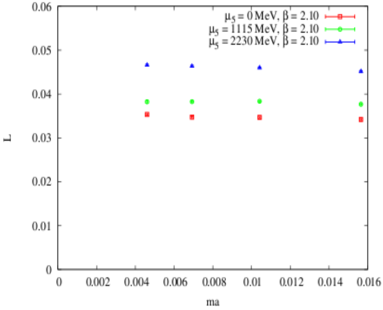
- Discretization effects





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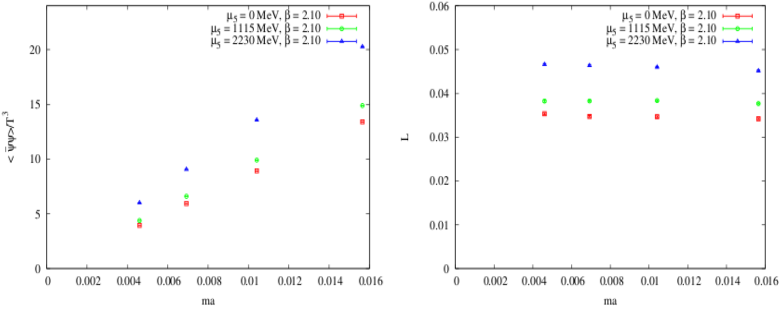

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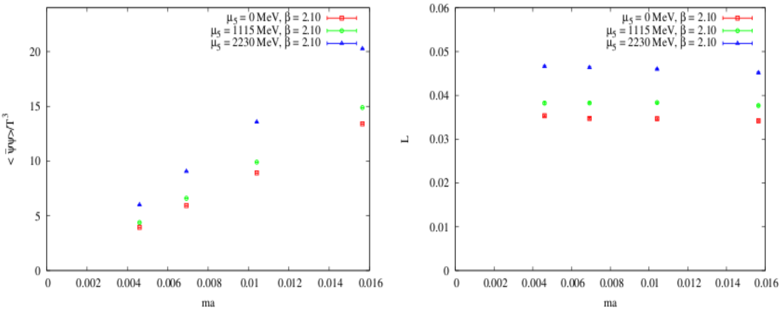
- Additional divergences



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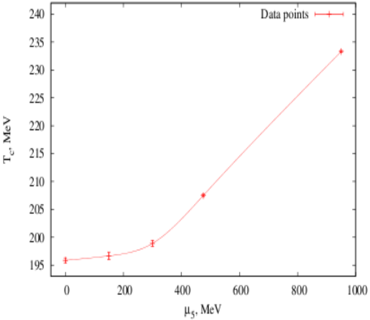
Results of the calculation:

- The critical temperatures increase
- The critical temperatures of the confinement/deconfinement phase transition and of the chiral symmetry breaking/restoration coincide

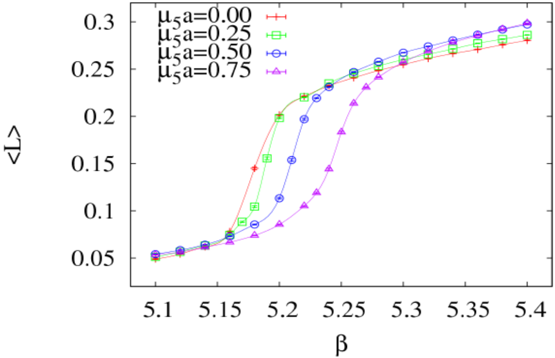


SU(3) QCD with nonzero chiral chemical potential

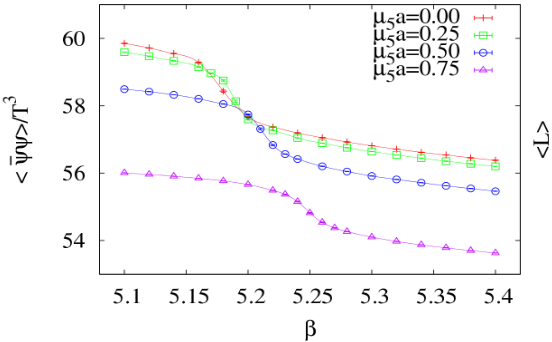
Details of the calculation:

- Dynamical Wilson fermions ($N_f = 2$) + Wilson action (SU(3))
- Link modification:
  $U \rightarrow U e^{\mu_5 \gamma_5}, \quad U^+ \rightarrow U^+ e^{-\mu_5 \gamma_5}$
- Continuum limit: $\delta S_{\mu_5}|_{a \rightarrow 0} \rightarrow \mu_5 \int d^4x \; \bar{Q}\big(\gamma_4 \gamma_5 \times 1\big) Q$
- $4 \times 16^3 (m_\pi \sim 400\text{MeV})$

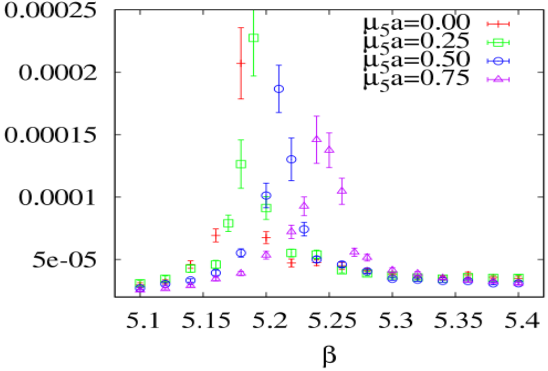
Polyakov loop



Chiral condensate



Polyakov loop susceptibility



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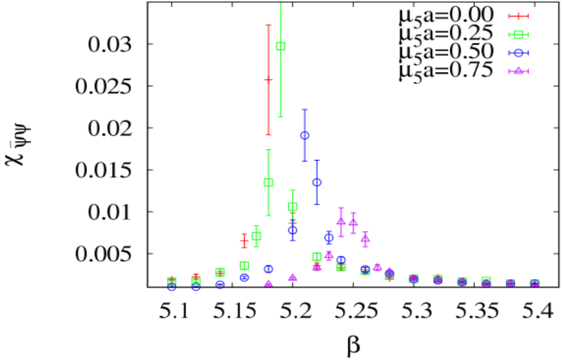

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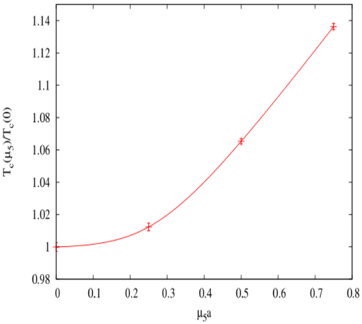

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- QCD-like theories:
  - SU(2) $N_f = 4$
  - SU(3) $N_f = 2$
- Different ways of introduction of chiral chemical potential
- Similar results (enhancement of chiral symmetry breaking)

Theoretical explanation

NJL model ($U_L(1) \times U_R(1)$, N_c colors)

- $S_E = \int d^4x \left(\bar\psi(\hat\partial + m - \mu_5\gamma_4\gamma_5)\psi - G\left[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\psi)^2\right] \right)$

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- $S_E = \int d^4x \Big(\bar\psi (\hat\partial + m - \mu_5 \gamma_4 \gamma_5 + \sigma + i\gamma_5 \pi) \psi + \frac{1}{4G} \left[\sigma^2 + \pi^2 \right] \Big)$

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- $S_{eff} = \int d^4x \Big(\frac{1}{4G}(\sigma^2 + \pi^2) - \mathrm{Tr}\log(\hat{\partial} + m - \mu_5 \gamma_4 \gamma_5 + \sigma + i\gamma_5 \pi) \Big)$

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Gap equation ($N_c \to \infty$)

- $\frac{\delta S_{eff}}{\delta\sigma} = \frac{\sigma}{2G} - N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{i\hat k + m - \mu_5\gamma_4\gamma_5 + \sigma + i\gamma_5\pi} \right] = 0$

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- $\frac{\pi^2}{GN_c} = \int_0^\Lambda k^2 dk \left[\frac{1}{\sqrt{(|\vec k| - \mu_5)^2 + M^2}} + \frac{1}{\sqrt{(|\vec k| + \mu_5)^2 + M^2}}\right]$

Gap equation

$$\frac{1}{\alpha_{NJL}} - 1 = \left(y^2 - \frac{x^2}{2}\right) \log \frac{1}{x^2}$$

$$\alpha_{NJL} = \frac{G N_c \Lambda^2}{\pi^2}, \quad x = \frac{M}{\Lambda}, \quad y = \frac{\mu_5}{\Lambda}$$

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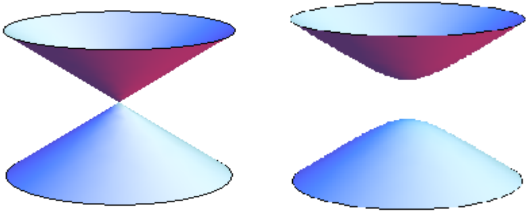
Properties ($\mu_5 = 0$):

- $\alpha_{NJL} < 1$ no solutions
- $\alpha_{NJL} > 1$ there is solution $M \neq 0$



Weakly coupled chiral plasma ($\alpha_{NJL} \ll 1$)

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μ_5 creates dynamical chiral symmetry breaking

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μ_5 creates dynamical chiral symmetry breaking

Strongly coupled chiral plasma ($\alpha_{NJL} > 1$)

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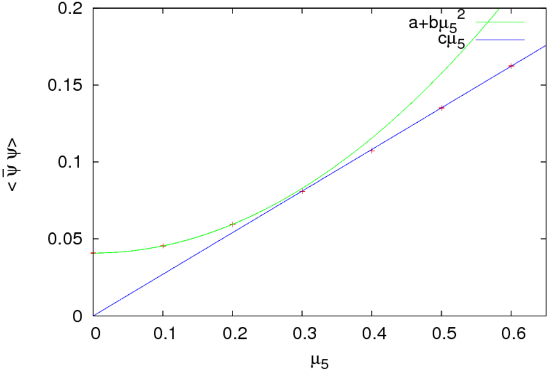
μ_5 enhances dynamical chiral symmetry breaking

Prediction:

In strong coupling region NJL model predicts that dynamical fermion mass is quadratically rising function at small μ_5 which switches to linear rising behaviour at large μ_5



$a+b\mu_5^2$
$c\mu_5$
$<\bar\psi\psi>$
μ_5
0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
0.6

Chiral symmetry breaking as condensation of Cooper pairs (BCS theory)

- Vacuum: $|vac\rangle = \hat{G}_1 \hat{G}_2 \hat{G}_3 |p_F\rangle$,

$$\hat{G}_1 = \prod_{\boldsymbol{p}} \Big(\cos(\theta_{\boldsymbol{L}}) - \sin(\theta_{\boldsymbol{L}}) \hat{a}^+_{\boldsymbol{L},\boldsymbol{p}} \hat{b}^+_{\boldsymbol{L},-\boldsymbol{p}} \Big)$$

$$\hat{G}_2 = \prod_{\boldsymbol{p}>\mu_5} \Big(\cos(\theta_{\boldsymbol{R}}) + \sin(\theta_{\boldsymbol{R}}) \hat{a}^+_{\boldsymbol{R},\boldsymbol{p}} \hat{b}^+_{\boldsymbol{R},-\boldsymbol{p}} \Big)$$

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- Energy:

$$E_{vac} = 2N_c\Big[\int_{p<\mu_5}\frac{d^3p}{(2\pi)^3}(p-\mu)\cos^2\tilde{\theta}_R + \int_{p>\mu_5}\frac{d^3p}{(2\pi)^3}(p-\mu)\sin^2\theta_R + \int\frac{d^3p}{(2\pi)^3}(p+\mu)\sin^2\theta_L\Big]$$

$$-GN_c^2\Big(\int_{p<\mu_5}\frac{d^3p}{(2\pi)^3}\sin 2\tilde{\theta}_R + \int_{p>\mu_5}\frac{d^3p}{(2\pi)^3}\sin 2\theta_R + \int\frac{d^3p}{(2\pi)^3}\sin 2\theta_L\Big)^2$$

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- Due the Fermi spheres additional fermion states participate in chiral symmetry breaking

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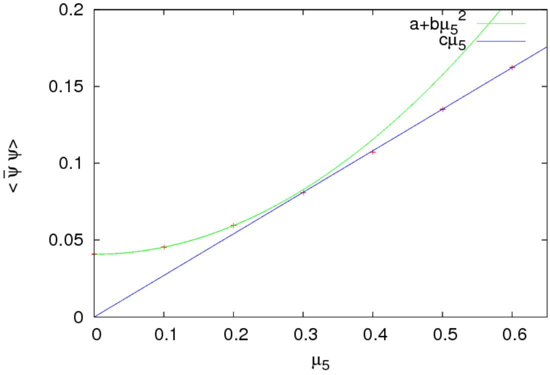
- Minimun energy is realized at the gap equation

- $\mu_5 > 0$ creates Fermi spheres of right particles and right antiparticles
- Due the Fermi spheres additional fermion states participate in chiral symmetry breaking
- μ_5 plays role of catalyst of chiral symmetry breaking due to additional fermion states (model independent result applicable not only in QCD)



0.2
0.15
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0.1
0.2
0.3
0.4
0.5
0.6
$a+b\mu_5^2$
$c\mu_5$
$<\bar{\psi}\psi>$
μ_5

Conclusion

Conclusion:

- CHIRAL CATALYSIS: Chiral chemical potential enhances chiral symmetry breaking and rises critical temperature of chiral symmetry breaking/restoration transition.
- Chiral plasma is unstable with respect to chiral symmetry breaking and condensation of Cooper pairs for $T < T_c$
- Gluonic sector of QCD with $\mu_5 \neq 0$?