UNIVERSITÀ DEGLI STUDI FIRENZE

Analytic bootstrap and Witten diagrams for (ABJM) Wilson line as defect CFT_1

Domenico Seminara

Department of Physics and Astronomy: University of Florence

Exploration of Duality, geometry and entanglement, 1st meeting 2nd-5th May 2020

LE STUDIUM CONSORTIUM

Based on: L. Bianchi, G. Bliard, L. Griguolo, V. Forini and D.S. arXiv: 2004.07849

L. Bianchi, L. Griguolo, M. Preti and D.S. arXiv: 1706.06590

May 4th, 2020

Outline

- Wilson line in ABJM as defects
- Operators insertions corresponding to broken symmetries

- Superspace formalism for correlators
- Independent four point functions

- Bootstrapping the four-point correlators (Displacement super-multiplet)
    - Constraints on the correlators and OPE
    - Solving the crossing equation
    - Extracting conformal Data

- String in $AdS_4 \times CP^3$
    - Minimal surface
    - Witten diagrams for our four point-correlators
    - Checking the boost-strap procedure

Motivations

- Why Wilsons line?
  - ➡ They are a simple and under-control laboratory to study defects
  - ➡ Provide simple examples of CFT_1 to test and use bootstrap techniques (in D=4 see e.g. Liendo, Meneghelli, Mitev 2018)
  - ➡ They provide examples non trivial RG-flows between different CFT_1 (in D=4 see e.g. Polchinski, Sully 2011, Beccaria, Giombi, Tseytlin 2017)

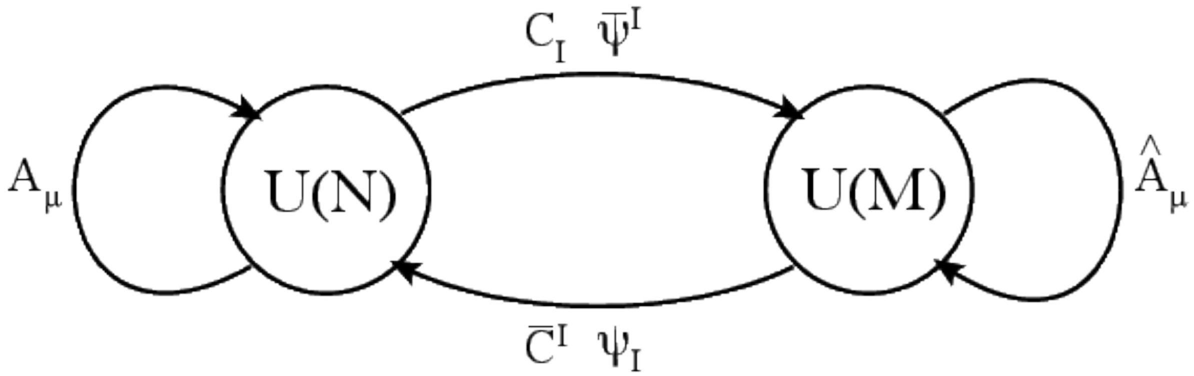
- Why 1/2 BPS?
  - ➡ The largest possible amount of symmetries
  - ➡ Despite of the large amount of symmetries, we have a non-trivial dynamics
  - ➡ A wide range of known exact results

- What does we learn from studying four-point correlators?
  - ➡ First large class of correlators which aren't fixed entirely through conformal symmetry
  - ➡ CFT data

ABJ(M) theories

[Aharony, Bergman, Jafferis, Maldacena, 2008]

N=6 ABJ(M) model for **U**



4 complex scalars $C_I, \bar{C}^I$

4 fermions $\Psi_I, \bar{\Psi}^I$ in the (anti)-bifundamental of the gauge group

A suitable choice of the superpotential makes the theory super-conformal. From now on N=M:

N=6 Super CS with matter D=3 SCFT

Integrability

(Localization)

Type IIA Superstrings on $AdS_4 \times CP^3$

0 $\quad \lambda = N/k \quad \infty$

In the large N-limit, the **theory is integrable**, but the integrability coupling **h** is a non trivial function of 't Hooft coupling λ

$$\lambda = \frac{\sinh 2\pi h(\lambda)}{2\pi}\, {}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};1,\frac{3}{2};-\sinh^2 2\pi h(\lambda)\right)$$

Conjecture
[Gromov-Sizov 2014]

Weak coupling test [Leoni, Mauri, Minahan, Ohlsson Sax, Santambrogio,Sieg 2010] Strong coupling test [L. Bianchi, M. Bianchi, Bres, Forini,Vescovi 2014]

Wilson line in ABJM theories

- The landscape of **supersymmetric/conformal Wilson loops** in ABJM and in general in D=3 is much richer than in D=4 (see rewiew: Drukker, Trancanelli, et al. 2020)

We have the merely bosonic 1/6 bosonic Wilson loops [Drukker, Plefka, Young; Chen, Wu; Rey, Suyama, Yamaguchi, 2008]

$$\langle W_{SCS}\rangle = \langle \mathrm{Tr}\ \mathrm{P} e^{-i\int_\Gamma d\tau(\dot{x}^\mu A_\mu(x) - \frac{2\pi i}{k}|\dot{x}| M_J{}^I C_I \bar{C}^J)}\rangle$$

with $M_I{}^J = \mathrm{diag}(1,1,-1,-1)$: line, circle (They exists from N=2 on, [Gaiotto, Yin 2007])

Locally 1/2 BPS Wilson loops in ABJM are realised as the holonomy of a superconnection $\mathscr{L}(t)$ of the supergroup U(N|N) [Drukker, Trancanelli 2009; Lee,Lee 2010; Cardinali, Griguolo, Martelloni, S. 2012]

$$W_F = \mathrm{Tr} P \exp\left[-i\int_\Gamma d\tau \mathcal{L}(\tau)\right]$$

$$\mathcal{L}(\tau) = \begin{pmatrix} A_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}| M_I{}^J C_J \bar{C}^I & -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\eta_I \bar{\psi}^I \\ -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\psi_I \bar{\eta}^I & \hat{A}_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}| M_I{}^J \bar{C}^I C_J \end{pmatrix}$$

The actual form of the couplings (the matrix M and the fermions $\eta, \bar{\eta}$) will be not relevant for our discussion. This operator is 1/2 BPS if we choose as contour the **line** or the **circle**.

There is also one-parameter family of 1/6 BPS/conformal fermionic Wilson lines: [Lietti, Mauri, Penati, Zhang 2017].

Many choices for studying W-line inspired superconformal defects in D=3 (non trivial conformal manifold with marginal directions: [Correa, Geraldo-Rivera, Silva 2019])

We choose the 1/2 BPS Wilson line

Operator insertions in the WLine

What do we want to compute?

[Similar investigation for the 1/2 BPS line in D=4: [Giombi, Roiban, Tseytlin, 2017; Giombi, Komatsu 2019]]

We want to evaluate **correlators** of operators inserted in the Wilson-line

$$\langle \mathcal{O}_1(t_1)\mathcal{O}_2(t_2)\ldots\mathcal{O}_n(t_n)\rangle_{\mathcal{W}} \equiv \frac{\langle \mathcal{W}[\mathcal{O}_1(t_1)\mathcal{O}_2(t_2)\ldots\mathcal{O}_n(t_n)]\rangle}{\langle \mathcal{W}\rangle}\,.$$

where $\mathcal{W}[\mathcal{O}_1(t_1)\mathcal{O}_2(t_2)\ldots\mathcal{O}_n(t_n)] \equiv \mathrm{Tr}\mathcal{P}\left[\mathcal{W}_{t_i,t_1}\mathcal{O}_1(t_1)\mathcal{W}_{t_1,t_2}\mathcal{O}_2(t_2)\ldots\mathcal{O}_n(t_n)\mathcal{W}_{t_n,t_f}\right]$

Since the line breaks the original super-conformal from

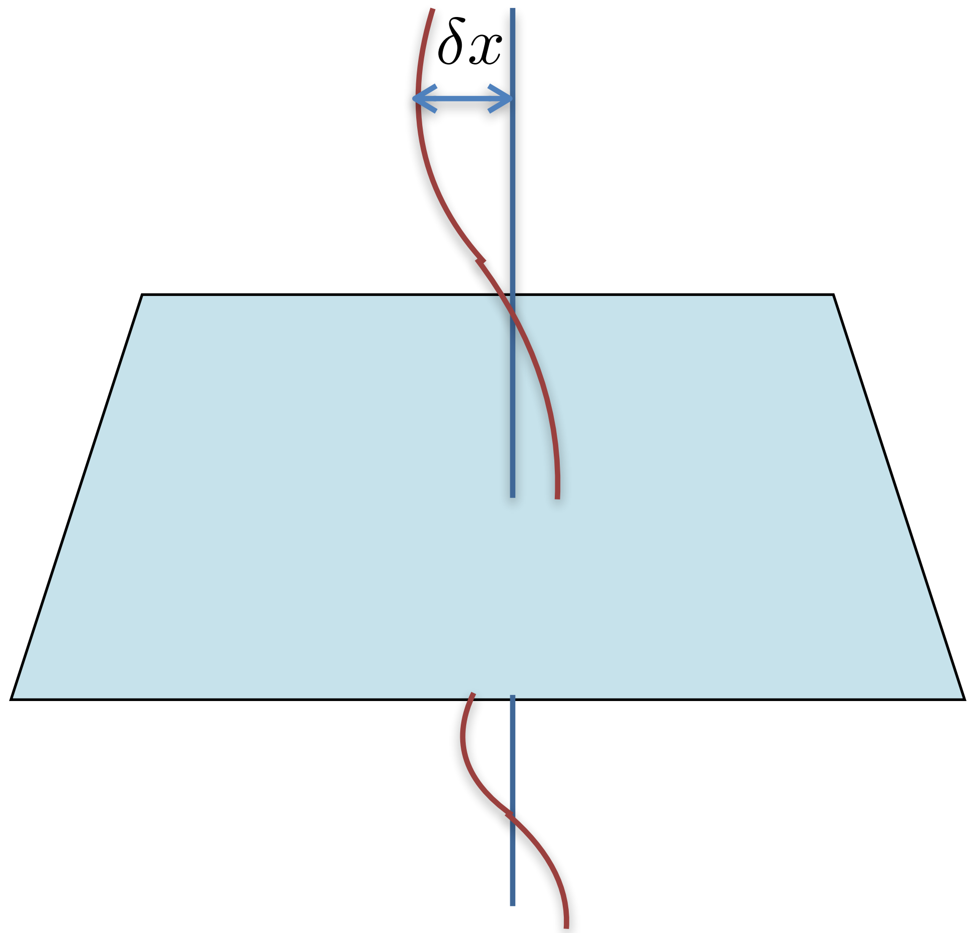
$$\mathrm{OSP}(6|4) \longrightarrow \mathrm{SU}(1,1|3),$$

which contains the conformal group in 1-dimension, we want look, abstractly at them as **Correlators in a Defect Conformal Field Theory in 1-dimension.**

In particular we look at the **operator insertions** generated by the action of **broken symmetries.**

The ubiquitous Displacement operator

[Present framework and conventions: Billo, Goncalves, Lauria, Meineri, 2016]



- Any defect breaks translations:

$$\partial_\mu T^{\mu i}(x) = \delta^p(x_\perp) D^i(x_\parallel)$$

$D^i(x_\parallel)$ is the **displacement operator**

- It implements small deformation of the contour

$$\delta\langle X\rangle_W = \int d^p x_\parallel \; \delta x_i(x_\parallel) \; \langle D^i(x_\parallel) X\rangle_W$$

- Its two point function is fixed by conformal symmetry

$$\langle D^i(x_\parallel) D^j(0)\rangle_W = C_D \frac{\delta_{ij}}{|x_\parallel|^{2(p+1)}}$$

- The normalization C_D is physical: in superconformal theories is proportional. to the one-point function of energy momentum tensor through a universal constant: [Lewkowycz, Maldacena 2013] [Bianchi, Lemos, Meineri, 2018], [Bianchi, Lemos 2019]

- In superconformal field theories, it gets together with other operators associated to other broken charges to form a short super-multiplet.

Displacement Supermultiplet

[L.Bianchi, L. Griguolo, M. Preti and D.S. 2017]

- The presence of the 1/2 BPS Wilson Line breaks the original symmetry

OSP(6|4) → SU(1,1|3)

ABJM → ABJM+1/2 BPS WL

Bosonic subgroup: $SO(1,4)\times SU_R(4) \rightarrow SO(1,2)\times SU_R(3)\times U(1)$

Broken symmetries:

Defect operators:

R-symmetry

$SU_R(4) \rightarrow SU_R(3)$ $\qquad \mathbb{O}_a \quad \bar{\mathbb{O}}_a, \quad \{a=1,2,3\}$

supersymmetries $\qquad \mathbb{\Lambda}_a \quad \bar{\mathbb{\Lambda}}_a, \quad \{a=1,2,3\}$

space-time symmetry (translations) $\qquad \mathbb{D} \quad \bar{\mathbb{D}}$

These operators do not exhaust the content of the super-multiplet. In fact none has the right quantum number to be highest weight of this multiplet:

Fermionic-superprimary $\qquad \mathbb{F} \quad \bar{\mathbb{F}}$

Displacement Supermultiplet

SU(1,1|3) representations:

$\bar{\mathcal{B}}^{\frac{1}{2}}_{\frac{3}{2},0,0}$: Chiral multiplet

$$\begin{array}{ll} \mathbb{F} & [\frac{1}{2},\frac{3}{2},0,0] \\ \downarrow \mathcal{Q}^a & \\ \mathbb{O}^a & [1,2,1,0] \\ \downarrow \mathcal{Q}^a & \\ \mathbb{\Lambda}_a & [\frac{3}{2},\frac{5}{2},0,1] \\ \downarrow \mathcal{Q}^a & \\ \mathbb{D} & [2,3,0,0] \end{array}$$

$\mathcal{B}^{\frac{1}{2}}_{\frac{3}{2},0,0}$: Anti-chiral multiplet

$$\begin{array}{ll} \bar{\mathbb{F}} & [\frac{1}{2},-\frac{3}{2},0,0] \\ \downarrow \bar{\mathcal{Q}}^a & \\ \bar{\mathbb{O}}_a & [1,-2,0,1] \\ \downarrow \bar{\mathcal{Q}}_a & \\ \bar{\mathbb{\Lambda}}^a & [\frac{3}{2},-\frac{5}{2},1,0] \\ \downarrow \bar{\mathcal{Q}}_a & \\ \bar{\mathbb{D}} & [2,-3,0,0] \end{array}$$

$[\Delta, j_0, j_1, j_2]$:

- Δ conformal dimension
- j_0 U(1) charge
- j_1, j_2 Dynkin label di $SU_R(3)$

Superspace Formalism

[Similar superspace for formulation in D=4: Liendo, Meneghelli, Mitev 2018]

- We can pack this multiplet into a chiral super-field

$$\Phi(y,\theta) = \mathbb{F}(y) + \theta_a \mathbb{O}^a(y) - \frac{1}{2}\theta_a\theta_b\,\epsilon^{abc}\,\Lambda_c(y) + \frac{1}{3}\theta_a\theta_b\theta_c\,\epsilon^{abc}\mathbb{D}(y)$$

where $y = t + \theta_a\bar{\theta}^a$ as usual.

Two point function in superspace:

$$\langle \Phi(y_1,\theta_1)\bar{\Phi}(\bar{y}_2,\bar{\theta}_2)\rangle = \frac{c_\Phi}{\langle 1\bar{2}\rangle} \qquad \langle i\bar{j}\rangle = y_i - y_j - 2\theta_{a\,i}\bar{\theta}^a{}_j$$

The symbol $\langle i\bar{j}\rangle$ stands for the so-called the **chiral distance**

$$\bar{\mathsf{D}}_i\,\langle i\bar{j}\rangle = \mathsf{D}_j\,\langle i\bar{j}\rangle = (Q_i + Q_j)\,\langle i\bar{j}\rangle = (\bar{Q}_i + \bar{Q}_j)\,\langle i\bar{j}\rangle = 0$$

The constant c_Φ is known exactly. In fact it is related to the Bremsstrahlung function:

$$C_\Phi(\lambda) = 2B_{1/2}(\lambda)$$

The three point function vanishes beacause of the U(1)-symmetry (charged superfield)

Four point Functions

- We have two possible four point functions in one dimension:

$$\langle \Phi(y_1,\theta_1)\bar{\Phi}(\bar{y}_2,\bar{\theta}_2)\Phi(y_3,\theta_3)\bar{\Phi}(\bar{y}_4,\bar{\theta}_4)\rangle = \frac{c_\Phi^2}{\langle 1\bar{2}\rangle \langle 3\bar{4}\rangle} f(\mathcal{Z}) \,,$$

$$\langle \Phi(y_1,\theta_1)\bar{\Phi}(\bar{y}_2,\bar{\theta}_2)\bar{\Phi}(\bar{y}_3,\bar{\theta}_3)\Phi(y_4,\theta_4)\rangle = -\frac{c_\Phi^2}{\langle 1\bar{2}\rangle \langle 4\bar{3}\rangle} h(\mathcal{X})$$

where we have introduced the following two harmonic ratios:

$$\mathcal{Z} = \frac{\langle 1\bar{2}\rangle \langle 3\bar{4}\rangle}{\langle 1\bar{4}\rangle \langle 3\bar{2}\rangle} \qquad \mathcal{X} = -\frac{\langle 1\bar{2}\rangle \langle 4\bar{3}\rangle}{\langle 1\bar{3}\rangle \langle 2\bar{4}\rangle}$$

Both correlates are ordered such that $t_1<t_2<t_3<t_4$.

In higher dimensions these two correlates are related by crossing, but not in one dimension

$$z = \frac{t_{12}t_{34}}{t_{14}t_{32}} \qquad \chi = \frac{t_{12}t_{34}}{t_{13}t_{24}} \quad \longrightarrow \quad z = \frac{\chi}{\chi - 1}$$

Even if we write f as function of χ , we cannot wiew h as its analytical continuation

A relation between f and h exists, but it will emerge from their s-channel block expansion

Four point Functions

In both cases all the information is carried by the superprimary. For instance, for the first type of 4-point function:

$$\langle \mathbb{F}(t_1)\bar{\mathbb{F}}(t_2)\mathbb{F}(t_3)\bar{\mathbb{F}}(t_4)\rangle = \frac{C_\Phi^2}{t_{12}t_{34}} f(z)$$

All other correlators are then obtained by expanding the superspace expressions and are built by f(z) and its derivatives.

$$\begin{aligned}\langle \mathbb{O}^{a_1}(t_1)\bar{\mathbb{O}}_{a_2}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = \frac{4C_\Phi^2}{t_{12}^2 t_{34}^2}\Big[&\delta^{a_1}_{a_2}\delta^{a_3}_{a_4}\left(f(z) - zf'(z) + z^2 f''(z)\right)\\ &- \delta^{a_1}_{a_4}\delta^{a_3}_{a_2}\left(z^2 f'(z) + z^3 f''(z)\right)\Big]\end{aligned}$$

$$\begin{aligned}\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{D}(t_3)\bar{\mathbb{D}}(t_4)\rangle = \frac{(12C_\Phi)^2}{t_{12}^4 t_{34}^4}\,\frac{1}{36}\Big[&36f(z) - 36(z^4+z)f'(z) + 18z^2(-14z^3+3z^2+1)f''(z)\\ &- 6z^3\left(55z^3 - 39z^2 + 3z + 1\right)f^{(3)}(z) - 3z^4\left(46z^3 - 63z^2 + 18z - 1\right)f^{(4)}(z)\\ &- 3(z-1)^2 z^5(7z-1)f^{(5)}(z) - (z-1)^3 z^6 f^{(6)}(z)\Big]\end{aligned}$$

$$\begin{aligned}\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = \frac{24C_\Phi^2}{t_{12}^4 t_{34}^2}\delta^{a_3}_{a_4}\frac{1}{6}\Big[&(1-z)\,z^4 f^{(4)}(z) - (3z+1)\,z^3 f^{(3)}(z)\\ &+ 3z^2 f''(z) - 6zf'(z) + 6f(z)\Big],\end{aligned}$$

Selection rules for the OPE

- If we want to use bootstrap to compute our four point correlators, we have two qualitatively different channels:

Chiral-Anti Chiral channel:

$$\Phi \times \bar{\Phi} \sim \mathbb{1} + \mathcal{A}^{\Delta}_{0,0,0}$$

Identity Operator

Any Long supermultiplet Δ>0

The same selection rule of SU(1,1|1) (see [Bianchi, Preti, Vescovi 2018])

Chiral-Chiral channel:

$$\Phi \times \Phi \sim \bar{\mathcal{B}}_3 + \bar{\mathcal{B}}_{7/2,1} + \bar{\mathcal{B}}_{4,0,1} + \mathcal{A}^{\Delta}_{9/2,0,0}$$

1/2 BPS Supermult.

1/3 BPS Supermult.

1/6 BPS Supermult.

Any Long supermultiplet Δ>3/2 (unitarity bound)

In the OPE of the superprimary operator **F**, every supermultiplet contributes with a single conformal family, whose conformal primary has quantum numbers [Δ, $2j_0$, 0, 0].

Bootstrapping 4 Point Functions

[Ferrara, Gatto, Grillo, Parisi 72,74; Polyakov 74; Rattazzi, Rychov, Tonni, Vichi,08………..]

[Liendo, Meneghelli, Mitev 2018]

Can we obtain the form of f(z) and h(χ) with help of symmetries and a minimal set of physical assumptions on their structure?

$$\langle \mathbb{F}(t_1)\bar{\mathbb{F}}(t_2)\mathbb{F}(t_3)\bar{\mathbb{F}}(t_4)\rangle = \frac{C_\Phi^2}{t_{12}t_{34}} f(z) \qquad \langle \mathbb{F}(t_1)\bar{\mathbb{F}}(t_2)\bar{\mathbb{F}}(t_3)\mathbb{F}(t_4)\rangle = \frac{C_\Phi^2}{t_{12}t_{34}} h(\chi)$$

Where z and χ are the two harmonic ratio and they are related by $z = \frac{\chi}{\chi - 1}$

Inserting a resolution of the identity in the s-channel, namely between t_2 and t_3, in both correlators, we find the following two expansions for the functions:

$$f(z) = 1 + \sum_\Delta c_\Delta G_\Delta(z) \qquad G_\Delta(z) = (-z)^\Delta {}_2F_1(\Delta, \Delta, 2\Delta + 3; z)$$

[Dolan, Osborn 2011]

$$h(\chi) = 1 + \sum_\Delta \tilde{c}_\Delta \tilde{G}_\Delta(\chi) \qquad \tilde{G}_\Delta(\chi) = \chi^\Delta {}_2F_1(\Delta, \Delta, 2\Delta + 3; \chi)$$

[Dolan, Osborn 2011]

Exploiting these two expansions we can show that h and f are related!

Parity restrictions

- The coefficient in the above expansions are related two the coefficient in the OPE expansions as follows:

$$c_\Delta = f_{\mathbb{F}\bar{\mathbb{F}}\mathcal{O}_\Delta} f_{\mathbb{F}\bar{\mathbb{F}}\mathcal{O}_\Delta} \qquad \tilde{c}_\Delta = f_{\mathbb{F}\bar{\mathbb{F}}\mathcal{O}_\Delta} f_{\bar{\mathbb{F}}\mathbb{F}\mathcal{O}_\Delta}$$

In 1D CFT, there is a Z_2 parity transformation $t \to -t$, which implies [Billò, Caselle, Gaiotto, Gliozzi, Meineri, Pellegrini 2013]

$$\langle \mathcal{O}_1(t_1)\mathcal{O}_2(t_2)\mathcal{O}_3(t_3)\rangle = (-1)^{T_1+T_2+T_3}\langle \mathcal{O}_3(-t_3)\mathcal{O}_2(-t_2)\mathcal{O}_1(-t_1)\rangle$$

T_i (i=1,2,3) are the charge of these operators under this symmetry. Then

$$f_{\mathbb{F}\bar{\mathbb{F}}\mathcal{O}_\Delta} = (-1)^{1+T_\mathcal{O}} f_{\bar{\mathbb{F}}\mathbb{F}\mathcal{O}_\Delta} \qquad (\text{for fermions})$$

Therefore

$$c_\Delta = (-1)^{T_\mathcal{O}+1}\tilde{c}_\Delta$$

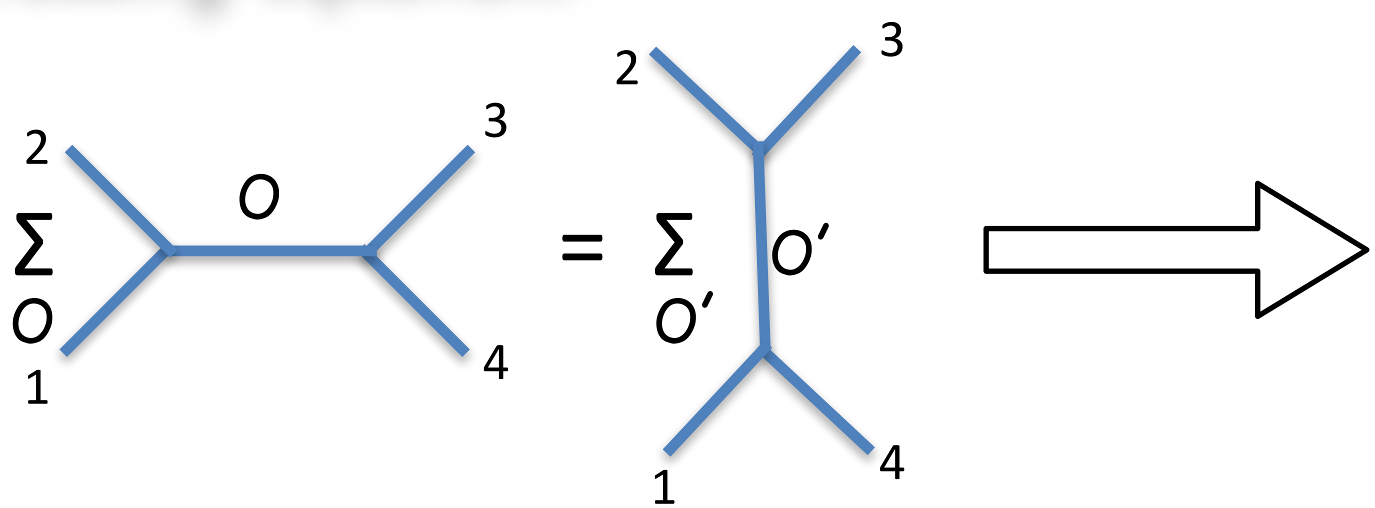
A similar relations holds for bosonic super-primary. You have simply to drop the one in the exponent.

Back to bootstrapping

- To apply analytic bootstrap it is convenient to redefine the function f

$$\hat{f}(\chi) = \frac{f(\frac{\chi}{\chi-1})}{\chi}$$

Crossing equation:



$$\hat{f}(\chi) = \hat{f}(1-\chi)$$

We assume that f can be expanded perturbatively in parameter ε

$$\hat{f}(\chi) = \hat{f}^{(0)}(\chi) + \epsilon\, \hat{f}^{(1)}(\chi) + O(\epsilon^2)$$

Back to bootstrapping

We can fix by performing the Wick contraction in free theory (at strong coupling):

$$f^{(0)}(z) = 1 - z \qquad \text{alternatively} \qquad \hat{f}^{(0)}(\chi) = \frac{1}{\chi(1-\chi)}$$

These form can be interpreted as the exchange of generalised free field schematically of the form,

$$[\mathbb{F}\bar{\mathbb{F}}]_n \sim \mathbb{F}\partial_t^n \bar{\mathbb{F}}$$

and of conformal dimension 1+n. By using the orthogonality relation,

$$\oint \frac{dz}{2\pi i} \frac{z}{(1-z)^3} G_{1+n}(z) G_{-3-n'}(z) = \delta_{n,n'}$$

for the conformal blocks. We can also extract the leading behaviour of the coefficients c_Δ

$$c_n^{(0)} = \frac{\sqrt{\pi}\, 2^{-2n-3}\Gamma(n+4)}{(n+1)\Gamma\left(n+\frac{5}{2}\right)}$$

Back to bootstrapping

Step1: To determine the first correction, we have to make an ansatz for the form of $f^{(1)}(\chi)$; we shall use using the minimal ansatz proposed by [Liendo, Meneghelli, Mitev 2018]

$$\hat{f}^{(1)}(\chi) = r(\chi)\log(1-\chi) + r(1-\chi)\log\chi + q(\chi)$$

where $r(\chi)$ and $q(\chi)$ are rational functions with $q(\chi) = q(1-\chi)$.

Step2: The above ansatz has infinite an infinite family of solutions. We have to impose additional **reasonable physical constraints** on the possibile solutions: [Liendo, Meneghelli, Mitev 2018]

- Mildest large n-behaviour for the anomalous dimensions γ_n : We assume $\gamma_n^{(1)} \sim n^2$. This picks out the solution where

$$q(\chi) = \frac{q_{-1}}{\chi(1-\chi)}$$

Back to bootstrapping

- Existence of poles only at physical position: namely when two operators collide
- Unitarity bounds in the $\mathbb{FF}$ channel: $\gamma_n^{(1)} = 0$ per $n < 3$.

These criteria allow us to fix completely the form of $f^{(1)}(z)$:

$$f^{(1)}(z) = -\frac{(1-z)^3}{z}\log(1-z) + z(3-z)\log(-z) + z - 1$$

The same kind of analysis allow us to construct the leading correction to $h(\chi)$: $h^{(0)}(\chi) = 1 - \chi$

$$h^{(1)}(\chi) = \frac{(1-\chi)^3}{\chi}\log(1-\chi) - \chi(3-\chi)\log(\chi) + 1 - \chi$$

Extracting conformal data

A natural question if wether the knowledge of this analytical solution of the crossing equation allow us to extract the anomalous dimensions:

$$\Delta_n = 1 + n + \epsilon\gamma_n^{(1)} + O(\epsilon^2)$$

and the OPE the first correction to the OPE coefficient.

The **naive** inversion procedure gives for the chiral-antichiral channel

$$\gamma_n^{(1)} = -n^2 - 4n - 3$$

and

$$c_n^{(1)} = c_n^{(0)}\left[-2 - 4n + \gamma_n^{(1)}\left(\psi(n+4) - \psi(n+\tfrac{5}{2}) - 2\log(2) - \frac{1}{n+1}\right)\right]$$

Mixing Problem: Mixing between 2 particle operators $\mathbb{F}\partial_t^n\bar{\mathbb{F}}$ with multi particle opearators. For instance, n=2 $\mathbb{F}\partial_t^2\bar{\mathbb{F}}$ mixes with the four particle operator $\mathbb{F}\bar{\mathbb{F}}\mathbb{F}\bar{\mathbb{F}}$.

Thus $\gamma_n^{(1)}$ is only a linear combination of the anomalous dimensions of the operator appearing in the mixing.

$AdS_4 \times CP^3$ String

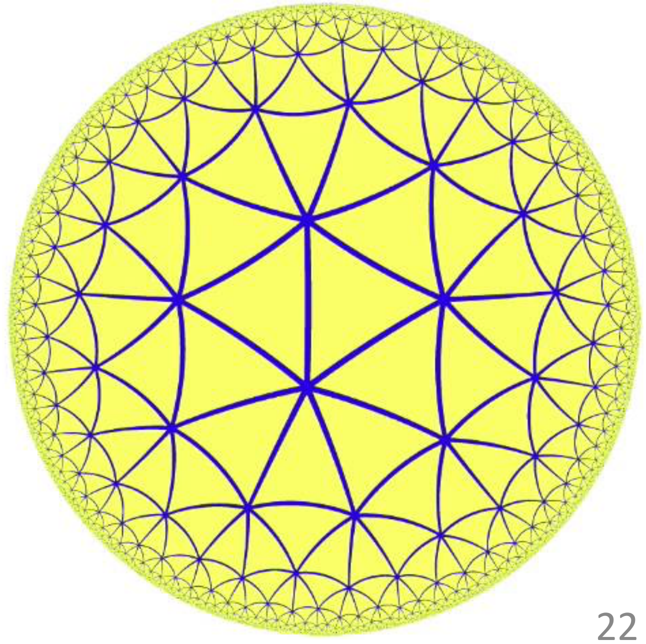
Our 1/2 BPS Wilson line is dual to the fundamental string ending on this defect at the boundary AdS_4

The minimal area solution is embedded in AdS_4 [Forini, Puletti, Sax 2012]:

$$ds^2_{\mathrm{AdS}_4} = \frac{dz^2 + dx^r dx^r}{z^2} \qquad z = s\,, \quad x^0 = t\,, \qquad x^i = 0\,, \qquad i = 1{,}2\,,$$

(Minimal surface)

$$ds_{\mathrm{AdS}_2} = \frac{dt^2 + ds^2}{s^2}$$



Fluctuation modes of the world-sheet are naturally associated to contour deformations.

String fluctuations are the AdS dual of the displacement supermultiplet

Grading	Fluctuation	Operator	Δ	m^2
Fermion	ψ	$\mathbb{F}(t)$	$\frac{1}{2}$	0
Boson	w^a	$\mathbb{O}^a(t)$	1	0
Fermion	ψ_a	$\Lambda_a(t)$	$\frac{3}{2}$	1
Boson	X	$\mathbb{D}(t)$	2	2

$$m^2 = \Delta(\Delta - 1)$$

(Bosons)

$$m^2 = (\Delta - \frac{1}{2})^2$$

(Fermions)

Witten diagrams

Thus our bootstrap analysis can be checked by computing Witten diagrams in AdS_2

It is sufficient (at this order) to consider only the bosonic part of the string-sigma model in $AdS_4 \times CP^3$ developed up to the quartic interactions:

$$
\begin{aligned}
S_B \equiv & T \int d^2\sigma \sqrt{g}\, L_B\,, \qquad L_B = \; L_2 + L_{4X} + L_{2X,2w} + L_{4w} + \ldots\,, \\
L_2 = & g^{\mu\nu}\partial_\mu X \partial_\nu \bar{X} + 2|X|^2 + g^{\mu\nu}\partial_\mu w^a \partial_\nu \bar{w}_a\;, \\
L_{4X} = & 2|X|^4 + |X|^2\left(g^{\mu\nu}\partial_\mu X \partial_\nu \bar{X}\right) - \frac{1}{2}(g^{\mu\nu}\partial_\mu X \partial_\nu X)(g^{\rho\kappa}\partial_\rho \bar{X}\partial_\kappa \bar{X})\,, \\
L_{2X,2w} = & \left(g^{\mu\nu}\partial_\mu X \partial_\nu \bar{X}\right)\left(g^{\rho\kappa}\partial_\rho w^a \partial_\kappa \bar{w}_a\right) - \left(g^{\mu\nu}\partial_\mu X \partial_\nu w^a\right)\left(g^{\rho\kappa}\partial_\rho \bar{X}\partial_\kappa \bar{w}_a\right) \\
& - \left(g^{\mu\nu}\partial_\mu \bar{X}\partial_\nu w^a\right)\left(g^{\rho\kappa}\partial_\rho X \partial_\kappa \bar{w}_a\right)\;, \\
L_{4w} = & \; -\tfrac{1}{2}(w^a\bar{w}_a)(g^{\mu\nu}\partial_\mu w^b \partial_\nu \bar{w}_b) - \tfrac{1}{2}(w^a\bar{w}_b)(g^{\mu\nu}\partial_\mu w^b \partial_\nu \bar{w}_a) + \tfrac{1}{2}\left(g^{\mu\nu}\partial_\mu w^a \partial_\nu \bar{w}_a\right)^2 \\
& - \tfrac{1}{2}(g^{\mu\nu}\partial_\mu w^a \partial_\nu \bar{w}_b)\left(g^{\rho\kappa}\partial_\rho \bar{w}_a \partial_\kappa w^b\right) - \tfrac{1}{2}(g^{\mu\nu}\partial_\mu w^a \partial_\nu w^b)\left(g^{\rho\kappa}\partial_\rho \bar{w}_a \partial_\kappa \bar{w}_b\right)
\end{aligned}
$$

$X \to \mathbb{D}$ $\qquad\qquad$ $w^a \to \mathbb{O}^a$

Witten diagrams

The simplest correlator to compute is

$$\langle w^{a_1}(t_1)\,\bar{w}_{a_2}(t_2)\,w^{a_3}(t_3)\,\bar{w}_{a_4}(t_4)\rangle = \frac{\left[C_w(\lambda)\right]^2}{t_{12}^2 t_{34}^2} G^{a_1\,a_3}_{a_2\,a_4}(\chi)$$

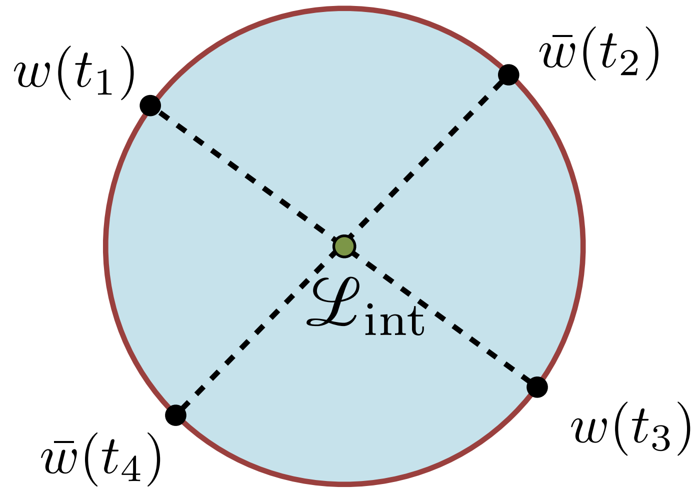
dual to $\langle \mathbb{O}^{a_1}(t_1)\bar{\mathbb{O}}_{a_2}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle$.

Leading Order:

$$\frac{\left[C_w(\lambda)\right]^2}{t_{12}^2 t_{34}^2}\left[\delta^{a_1}_{a_2}\delta^{a_3}_{a_4} + \frac{\chi^2}{(1-\chi)^2}\,\delta^{a_1}_{a_4}\delta^{a_3}_{a_2}\right]$$

Witten diagrams

Subleading Order: Connected contribution [only in terms of D-function [Dolan-Osborn 01,03]



$$\langle w^{a_1}(t_1)\, \bar{w}_{a_2}(t_2)\, w^{a_3}(t_3)\, \bar{w}_{a_4}(t_4) \rangle_{\text{conn.}} = \epsilon\, \frac{\left[\mathscr{C}_{\Delta=1}\right]^2}{t_{12}^2 t_{34}^2} \left[\delta^{a_1}_{a_2}\delta^{a_3}_{a_4}\, G_1(\chi) + \delta^{a_1}_{a_4}\delta^{a_3}_{a_2}\, G_2(\chi)\right]$$

with

$$G_1(\chi) = -3 + \frac{1}{(\chi-1)} - \frac{\chi^2}{(1-\chi)^2}\log\chi + \left(1 - \frac{4}{\chi}\right)\log(1-\chi)$$

$$G_2(\chi) = -\frac{\chi(3\chi+1)}{(1-\chi)^2} + \frac{\chi^2(\chi+3)}{(\chi-1)^3}\log\chi - \log(1-\chi)$$

We can now reconstruct f(z) by solving the system of differential equation connecting this correlator to that of $\mathbb{F}\bar{\mathbb{F}}\mathbb{F}\bar{\mathbb{F}}$

$$f(z) = 1 - z + \epsilon\left[z - 1 + z(3-z)\log(-z) - \frac{(1-z)^3}{z}\log(1-z)\right]$$

Perfect agreement!!!

Conclusions

- We have identified the Displacement supermultiplet for 1/2 BPS in ABJM theories
- We have identified the dual of these operators in the correspondence
- We have constructed a superspace formalism to study the four point correlation functions.
- We were able to determine the four point functions of this supermultiplet at the lowest non trivial order
  - by means of booststrap techniques
  - via computation of Witten diagrams in the dual picture
- We were able to extract a certain amount of conformal data of the underlying CFT_1

Outlooks

- Go beyond the tree-level approximation in the string analysis: Witten diagrams with loops.
- Bootstrapping the four point function at higher order
- Interplay with integrability
- Investigating the existence of topological sector: computable through localisation techniques
- Exploring the RG flow between different Wilson line in three dimensions: much richer situation than in four
- Apply these techniques to different Wilson lines and defects