Origin of dissipative Fermi arc transport in Weyl semimetals

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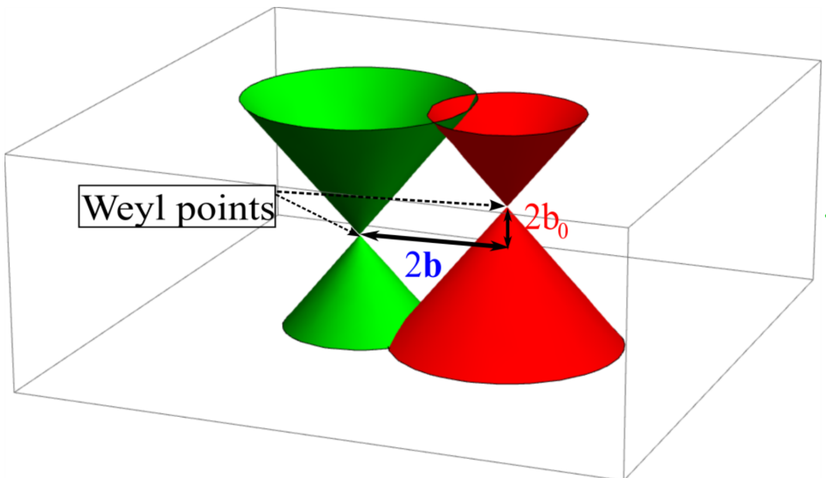
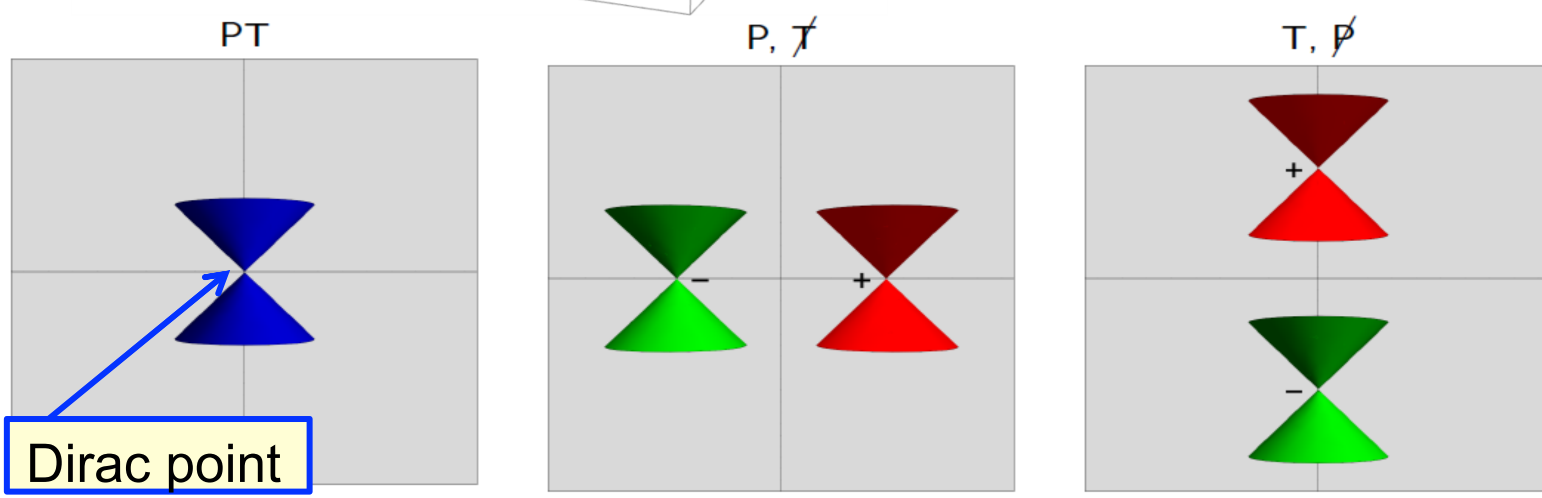
Low energy Dirac and Weyl fermions

$$H(\boldsymbol{k}) = \begin{pmatrix} v_F \boldsymbol{\sigma}\cdot(\boldsymbol{k}-\boldsymbol{b}) + b_0 & 0 \\ 0 & -v_F \boldsymbol{\sigma}\cdot(\boldsymbol{k}-\boldsymbol{b}) - b_0 \end{pmatrix}$$

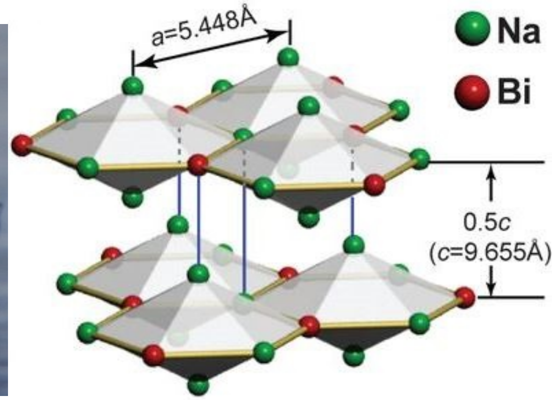


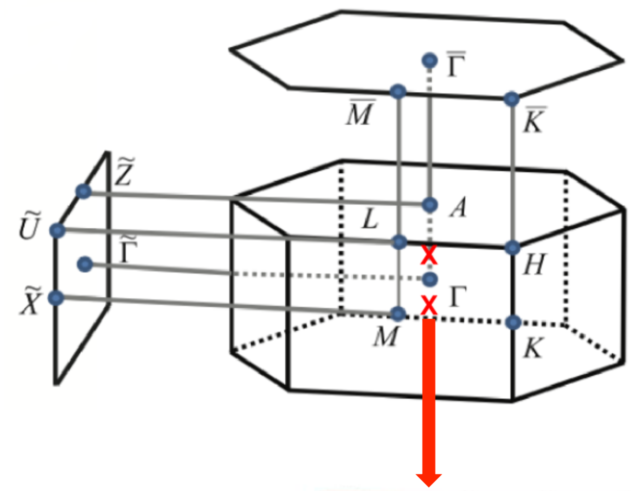
Energy spectrum of the Weyl semimetal

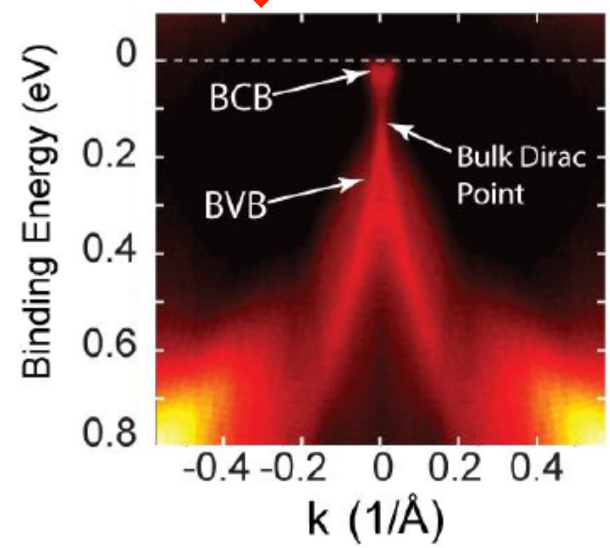


Energy spectrum for the different symmetry breakings:
T - time reversal,
P - inversion

Dirac semimetals





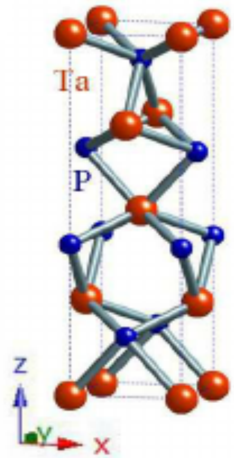


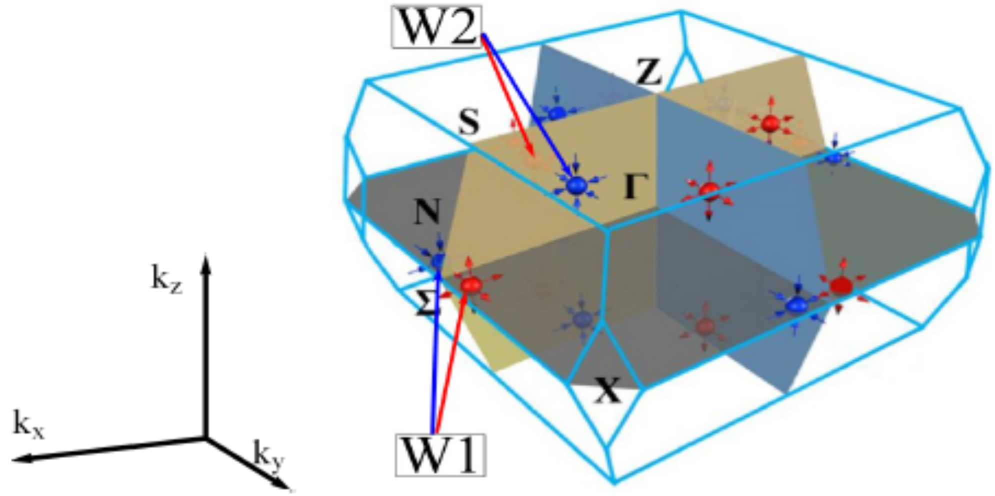
The image, the crystal structure, the Brillouin zone, and the ARPES spectrum of the Dirac semimetal Na_3Bi.[Z. K. Liu et al., Science **343**, 864, (2014)]

❖ Other materials:

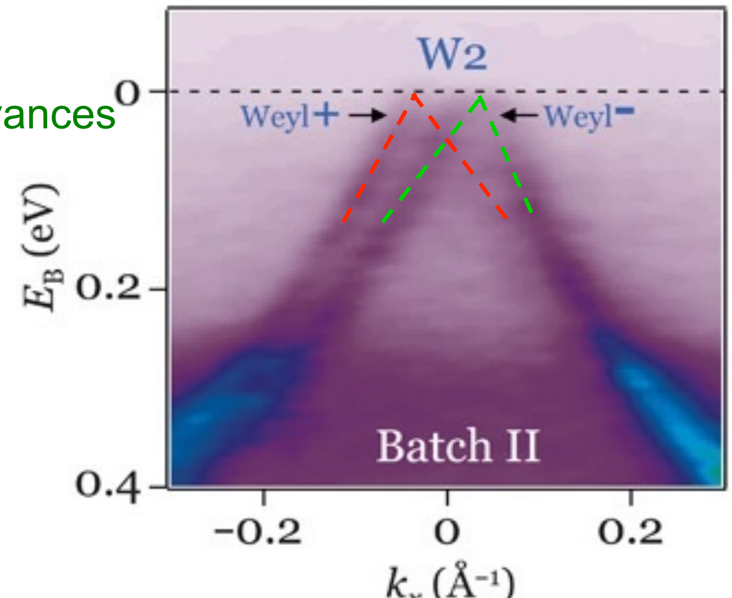
- Alloy $Bi_{0,96}Sb_{0,04}$ [D. Hsieh, et al. Nature **452**, 970, (2008)]
- Cd_3As_2 [M. Neupane et al., Nature Com. **5**, 3786, (2014)]
- $ZrTe_5$ [X. Li et al., Nature Physics **12**, 550 (2016)]

Weyl semimetals





The image, the crystal structure, the Brillouin zone, and the ARPES spectrum of the Weyl semimetal TaP. [S.-Y. Xu et al., Science Advances **1**, 1501092 (2015)]

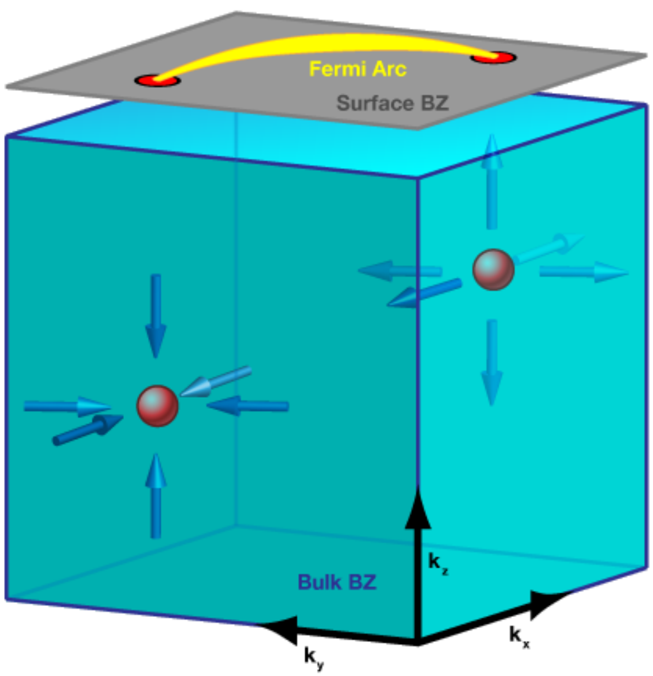


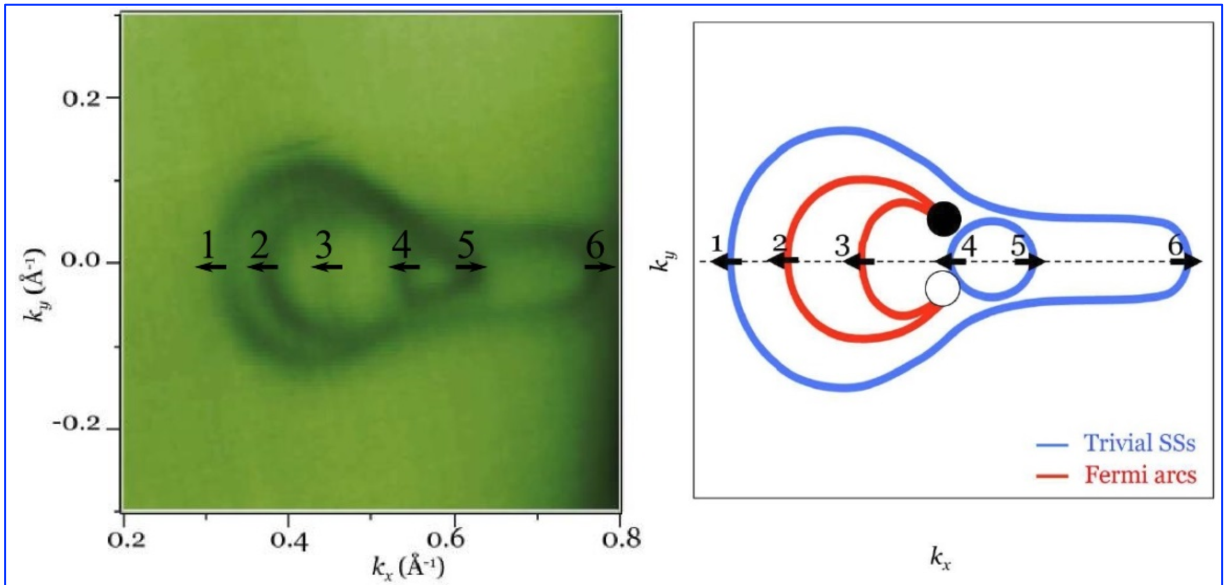
❖ Other materials:

- TaAs [M. Z. Hasan et al., Science **349**, 613 (2015)]
- NbAs [S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
- NbP [I. Belopolski et al. arXiv:1509.07465], [D. F. Xu et al., arXiv:1509.03847]
- WTe_2 [F. Y. Bruno et al., arXiv: 1604.02411]

Fermi arcs

❖ Fermi arcs are open segments of the Fermi surface connecting projections of the bulk cones onto the surface Brillouin zone (BZ). [X. Wan et al., Phys. Rev. B **83**, 205101 (2011)]





ARPES-measured Fermi surface and schematic showing the Fermi arcs and the trivial surface states. [S.-Y. Xu et al., Science Advances **1**, 1501092 (2015)]

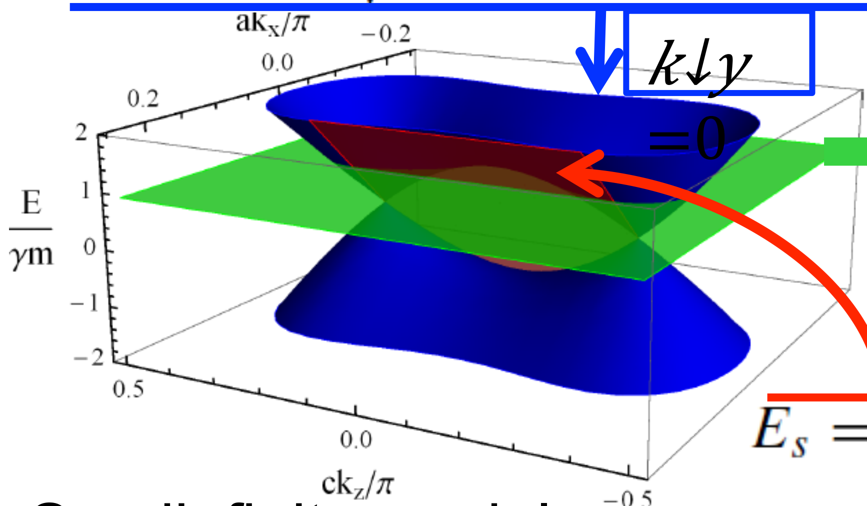
Effective model of the Fermi arcs

$$H(\mathbf{k}) = \gamma(k_z^2 - m)\sigma_z + v_F(k_x\sigma_x + k_y\sigma_y),$$

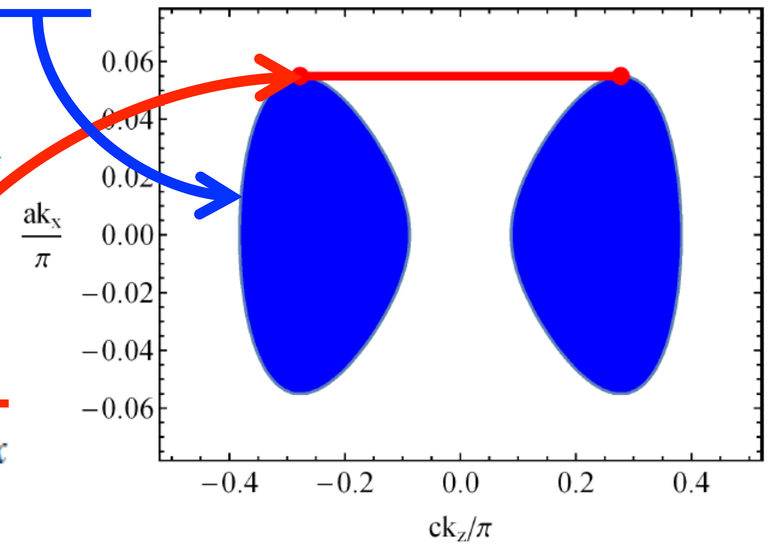
[R. Okugawa and S. Murakami, Phys. Rev. B **89**, 235315 (2014)].

$$E(k) = \pm\sqrt{\gamma^2(k_z^2 - m)^2 + v_F^2(k_x^2 + k_y^2)}.$$ ← The bulk energy spectrum

$k_y = 0$



$E_s = v_F k_x$



Semiinfinite model:

$m(y)=\{$■$m,\ y>0, -|m|,\ y<0,$

$$\psi_{y>0} = \sqrt{p(k_z)}\, e^{ik_x x + ik_z z - p(k_z)y}\begin{pmatrix}1\\1\end{pmatrix},$$

where $m \to \infty$, $p(k_z) = \frac{\gamma}{v_F}(m - k_z^2)$, $\psi_{y<0} = \sqrt{p(k_z)}\, e^{ik_x x + ik_z z + \gamma \tilde{m} y}\begin{pmatrix}1\\1\end{pmatrix}.$

Disorder and Kubo theory

Quenched disorder model:

$$H_{\text{dis}} = \int d^3\mathbf{r}\, \psi^\dagger(\mathbf{r})U(\mathbf{r})\psi(\mathbf{r}), \quad U(\mathbf{r}) = \sum_j u(\mathbf{r}-\mathbf{r}_j) = \sum_j u_0\delta(\mathbf{r}-\mathbf{r}_j).$$

Kubo formula for the dc conductivity: $\sigma_{xx}(\mathbf{r}) = -\lim_{\Omega\to 0}\frac{i}{\Omega}\Pi_{xx}(\mathbf{r}).$

Approximate model (common for the 3D):

$$\Pi_{xx}(i\Omega_n,\mathbf{q}) \simeq e^2 v_F^2 T \sum_l \int \frac{d^2\mathbf{k}}{(2\pi)^2} \boxed{\langle G(i\omega_l,\mathbf{k})\rangle_{\text{dis}} \langle G(i\omega_l - i\Omega_n, \mathbf{k}-\mathbf{q})\rangle_{\text{dis}},}$$

No crossing impurities lines

Exact model:

$$\Pi_{xx}(i\Omega_n;\mathbf{r},\mathbf{r}') = e^2 v_F^2 T \sum_l \langle G(i\omega_l;\mathbf{r},\mathbf{r}')G(i\omega_l - i\Omega_n;\mathbf{r}',\mathbf{r})\rangle_{\text{dis}}.$$

Conductivity in the 1D model

Effective 1D Hamiltonian for the surface states: $\boxed{H_{\rm surf} = -i\, v_F \partial_x .}$

The 1D effective model is valid only for $-\sqrt{m} < k\downarrow z < \sqrt{m}$.

Green's function is: $G_{\rm arc}(\omega, \mathbf{k}) = i \dfrac{\boxed{\theta(m - k_z^2)}}{\omega + \mu - v_F k_x + i0} .$

Approximate solution:

$\Sigma_{\rm arc}(\Omega, \mathbf{q}) \equiv$  $= -i n_{\rm imp} u_0^2 \dfrac{\sqrt{m}}{2\pi\, v_F} , \rightarrow -\Im\, m\perp \quad \Gamma = n_{\rm imp} u_0^2 \dfrac{\sqrt{m}}{2\pi v_F} .$

$\Re\mathfrak{e}\, \sigma_{xx}(0,0) = \dfrac{e^2 v_F \sqrt{m}}{4\Gamma \pi^2} .$ ← well-known, Drude-like, result

1D chiral surface states are dissipative!?

Correct conductivity in the 1D model

1D problem: $[-iv_F\,\partial_x+\sum_j u_0\,\delta(x-x_j)]\psi(x)=E\psi(x)$,

has exact solution:

$$G(\omega;x,x')=i\int\frac{dE}{2\pi}\frac{1}{\omega+\mu-E}e^{\frac{i}{v_F}\left[E(x-x')-\boxed{\sum_j u_0\left(\theta(x-x_j)-\theta(x'-x_j)\right)}\right]}.$$

odd under exchange $x\leftrightarrow x'$

$$\Pi_{xx}(i\Omega_n;\mathbf{r},\mathbf{r}')=e^2v_F^2T\sum_{i\omega_l}\langle G(i\omega_l;\underline{\mathbf{r},\mathbf{r}'})G(i\omega_l-i\Omega_n;\underline{\mathbf{r}',\mathbf{r}})\rangle_{\text{dis}},$$

$$\Re\mathrm{e}\,\sigma_{xx}(0,0)=\infty$$

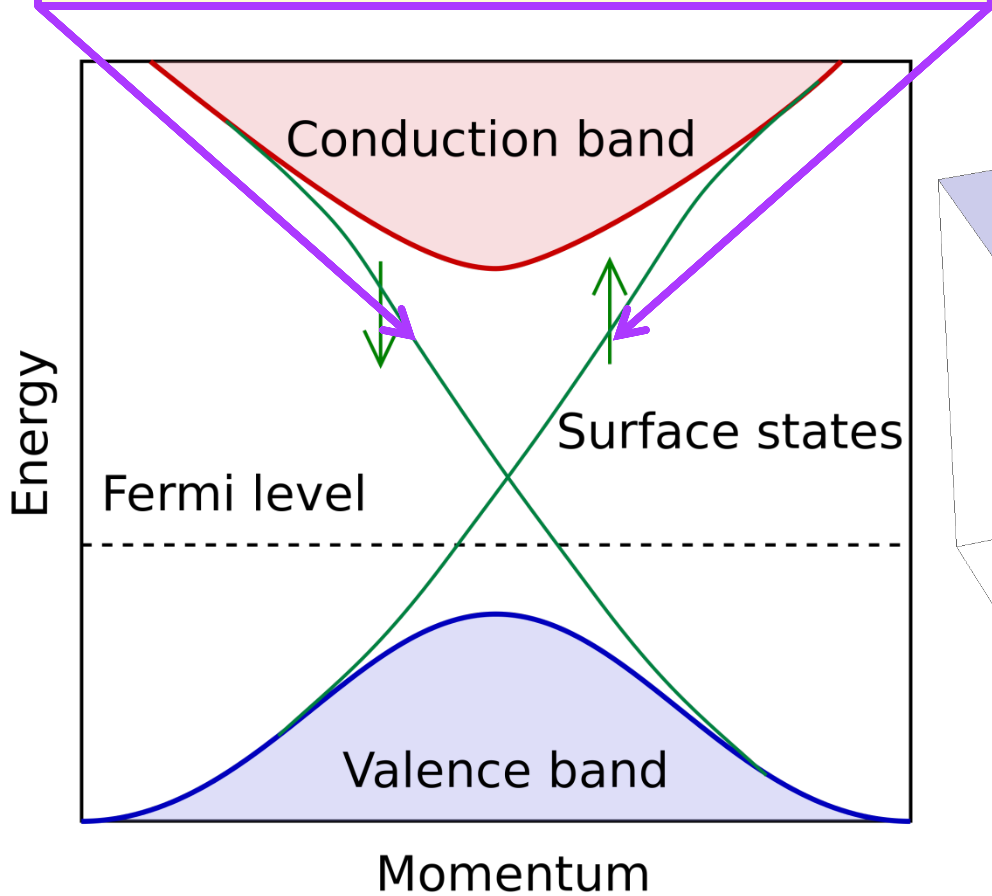
← the transport in the 1D exact model is nondissipative!

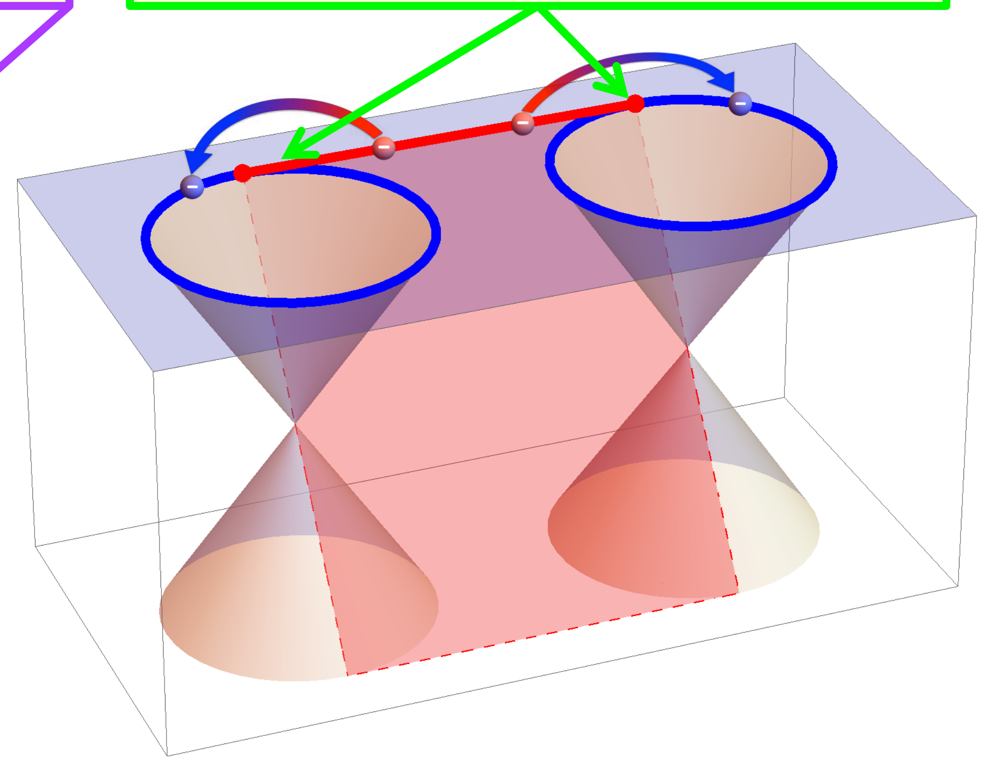
The result is correct, but is the 1D model sufficient to describe Fermi arc transport?

Topological insulator vs. Weyl semimetal

The bulk states are gapped, but the surface states are gapless.

Both bulk and surface states are gapless.





Signs of dissipation in the full model

The full 3D model: $H(\mathbf{k}) = \gamma(k_z^2 - m)\sigma_z + v_F(k_x\sigma_x + k_y\sigma_y),$

$$S^R(\omega,\mathbf{r},\mathbf{r}') = \int \frac{d^2\mathbf{k}_\parallel\, e^{i\mathbf{k}_\parallel(\mathbf{r}_\parallel - \mathbf{r}'_\parallel)}}{(2\pi)^2} S_s^R(\omega,\mathbf{k}_\parallel; y, y') + \int \frac{d^3\mathbf{k}\, e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}}{(2\pi)^3} S_b^R(\omega,\mathbf{k}).$$

The surface Green's function: $\boxed{S_s^R(\omega,\mathbf{k}_\parallel; y, y') = ip(k_z)\frac{(1+\sigma_x)\, e^{-(y+y')p(k_z)}}{\omega - v_F k_x + i0},}$

The bulk Green's function: $\boxed{S_b^R(\omega,\mathbf{k}) = i\frac{\omega + \gamma(k_z^2 - m)\sigma_z + v_F(k_x\sigma_x + k_y\sigma_y)}{\omega^2 - E(k)^2 + i0\,\mathrm{sign}(\omega)}.}$

The Lippman-Schwinger equation:

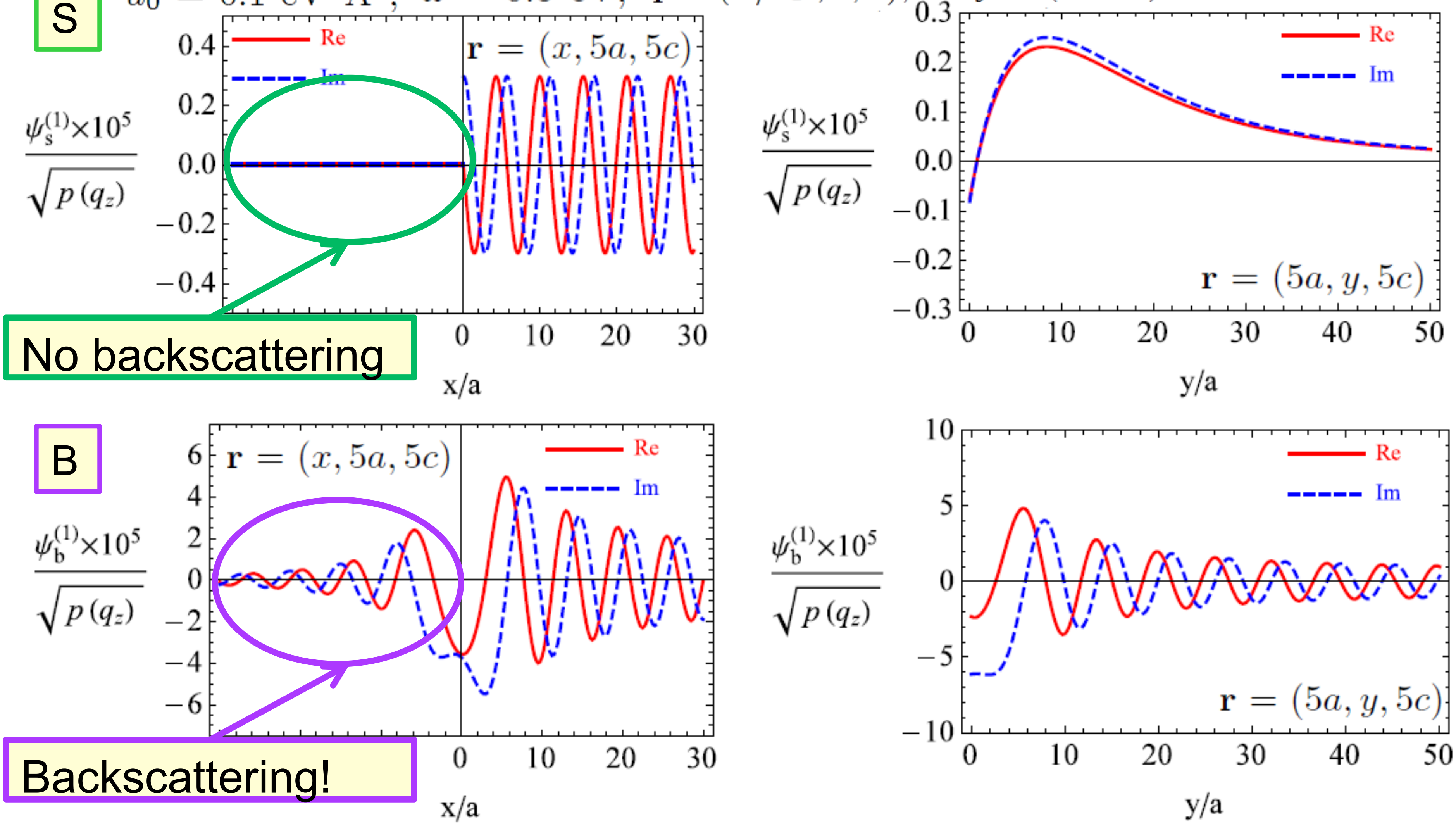
$$\psi_s(\mathbf{r}) = \psi_s^{(0)}(\mathbf{r}) \boxed{- i\int d^3\mathbf{r}' S(\mathbf{r},\mathbf{r}')U(\mathbf{r}')\psi_s(\mathbf{r}'),} = \boxed{\psi^{(1)}(\mathbf{r}) = \underline{\psi_s^{(1)}(\mathbf{r})} + \underline{\psi_b^{(1)}(\mathbf{r})},}$$

$$\psi_s^{(1)}(\mathbf{r}) \simeq -i\int d^3\mathbf{r}' S_s^R(\mathbf{r},\mathbf{r}')U(\mathbf{r}')\psi_s^{(0)}(\mathbf{r}') \simeq -iu_0\sum_j S_s^R(\mathbf{r},\mathbf{r}_j)\psi_s^{(0)}(\mathbf{r}_j),$$

$$\psi_b^{(1)}(\mathbf{r}) \simeq -i\int d^3\mathbf{r}' S_b^R(\mathbf{r},\mathbf{r}')U(\mathbf{r}')\psi_s^{(0)}(\mathbf{r}') \simeq -iu_0\sum_j S_b^R(\mathbf{r},\mathbf{r}_j)\psi_s^{(0)}(\mathbf{r}_j).$$

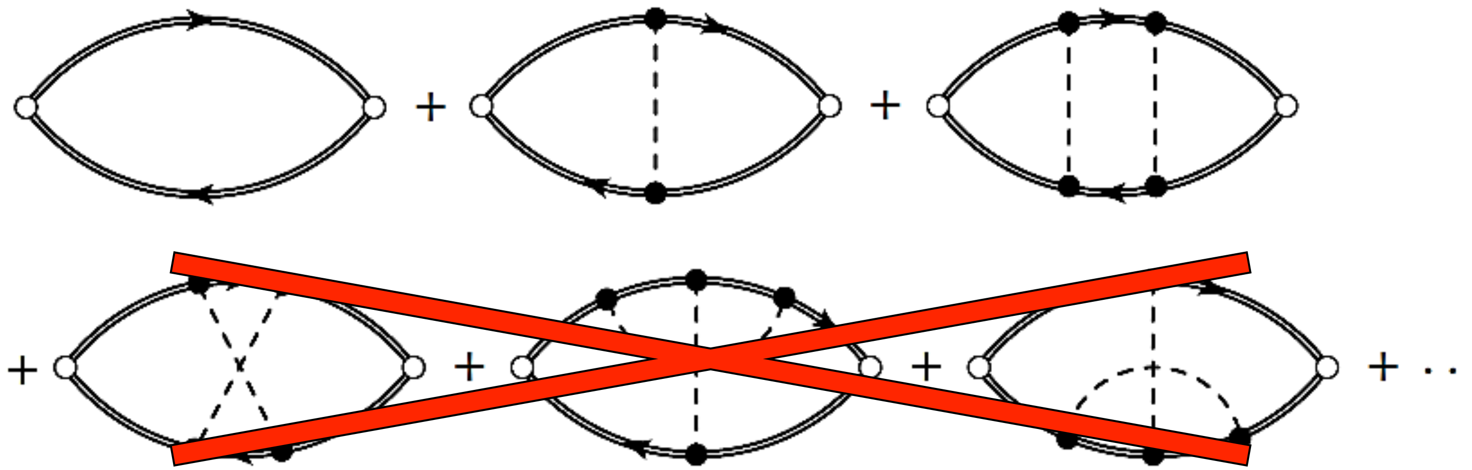
Scattered wave function

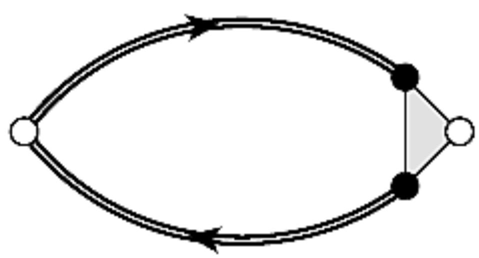
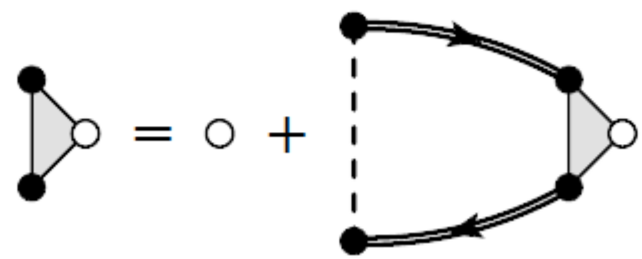
$u_0 = 0.1$ eV Å^3, $\omega = 0.5$ eV, $\mathbf{q} = (\omega/v_F, 0, 0)$, $\mathbf{r}_j = (0,0,0)$.



Polarization tensor in the full model

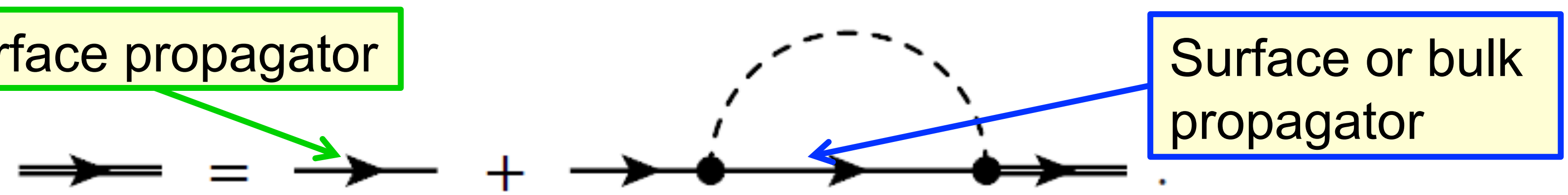
Fermi arcs are **not** purely 1D states → the diagrams with crossing impurity lines can be omitted.

$\Pi_{xx}(\mathbf{r}) =$ 

$\Pi_{xx} =$  where the vertex is 

Self energy and quasiparticle width

$\langle G \rangle_{\text{dis}} \equiv$



$$\langle G(\omega, \mathbf{k}_{\|}; y, y') \rangle_{\text{dis}} \simeq \frac{2ip(k_z)e^{-(y+y')p(k_z)}}{\omega + \mu - v_F k_x + i\Gamma(\omega, k_z)} \frac{1+\sigma_x}{2}.$$

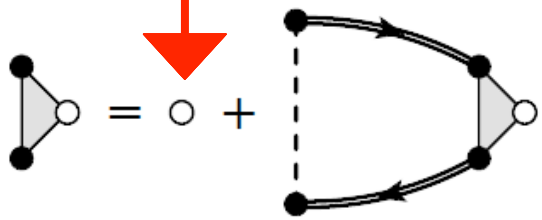
$$\boxed{\Gamma(\omega; k_z) = \Gamma_s(k_z) + \Gamma_b(\omega).}$$

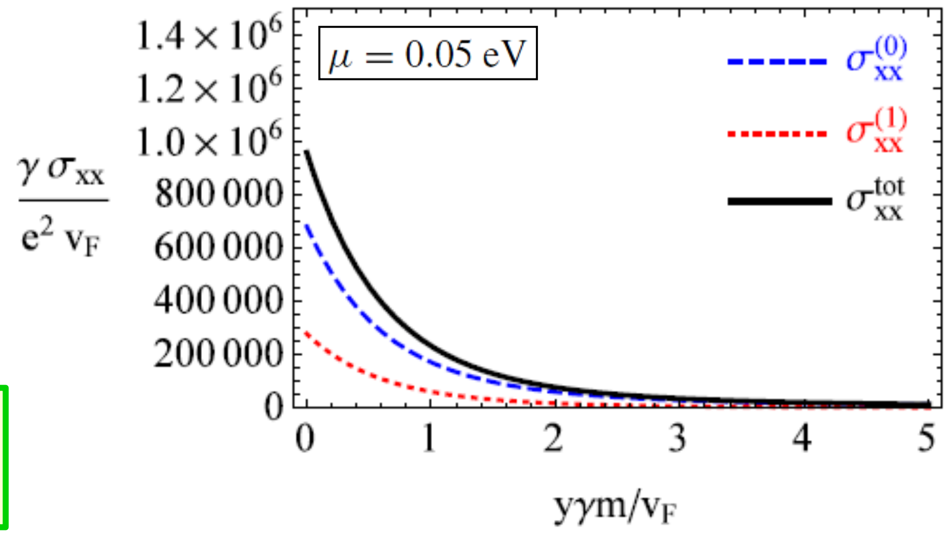
$$\Gamma_s(k_z) = \frac{\gamma(m - k_z^2)}{\pi v_F^2} n_{\text{imp}} u_0^2 \left[\sqrt{m} - \frac{(k_z^2 - m)}{\sqrt{k_z^2 - 2m}} \arctan\left(\frac{\sqrt{m}}{\sqrt{k_z^2 - 2m}} \right) \right],$$

$$\Gamma_b(\omega) \equiv n_{\text{imp}} u_0^2 \frac{|\omega + \mu|}{4 v_F^2 \pi} \left[\sqrt{m + \frac{|\omega + \mu|}{\gamma}} - \theta\left(m - \frac{|\omega + \mu|}{\gamma} \right) \sqrt{m - \frac{|\omega + \mu|}{\gamma}} \right].$$

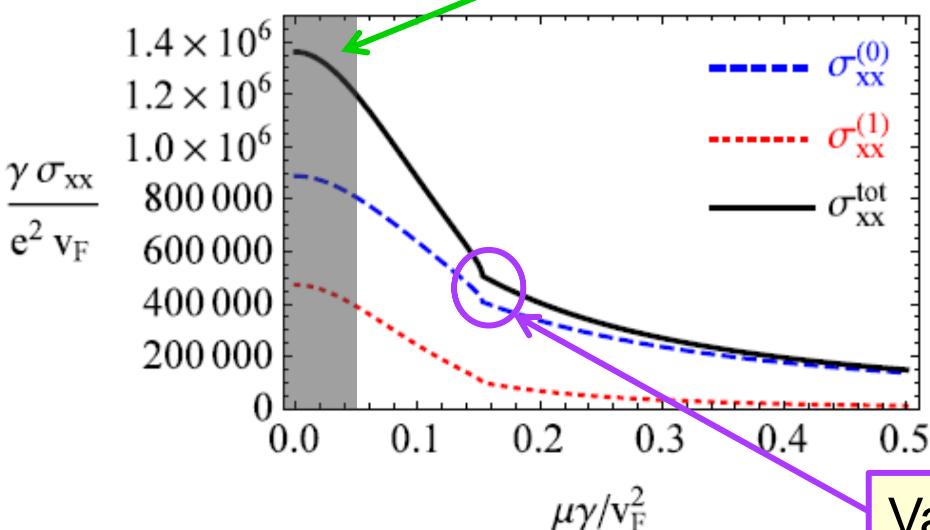
Conductivity, T=0

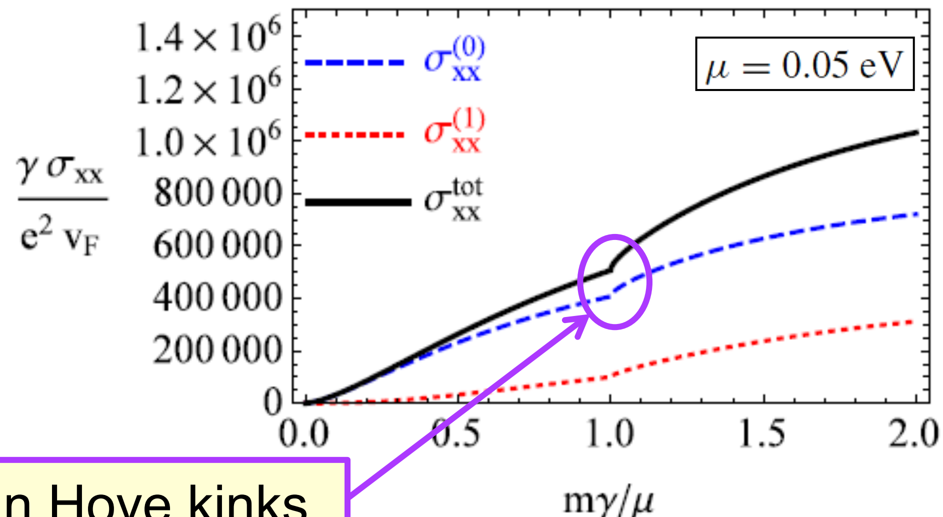
$$\mathfrak{Re}\,\sigma_{xx}^{\mathrm{tot}}(y) \equiv \mathfrak{Re}\,\sigma_{xx}^{(0)}(y) + \mathfrak{Re}\,\sigma_{xx}^{(1)}(y).$$





Nonperturbative effects, e.g. rare region effects, are important here

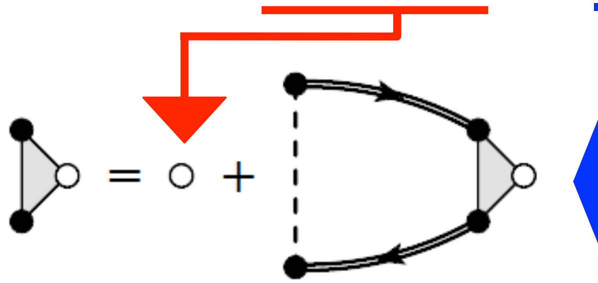


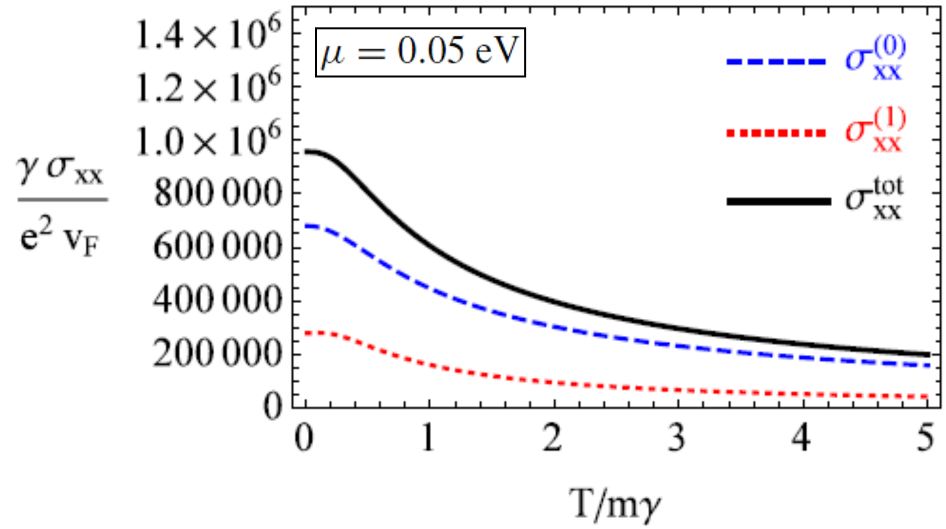


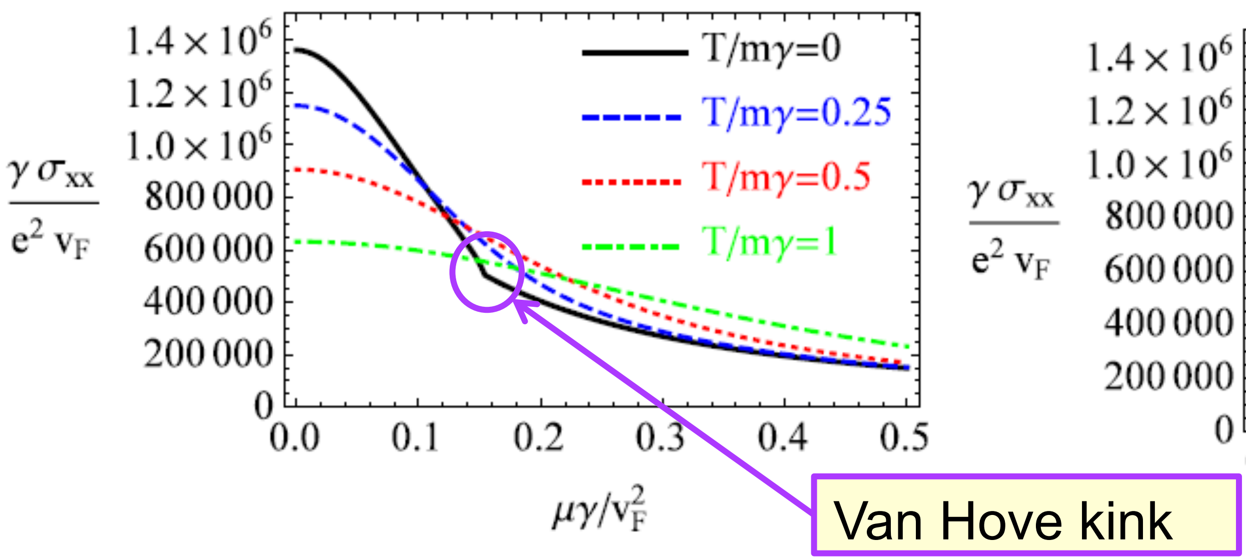
Van Hove kinks

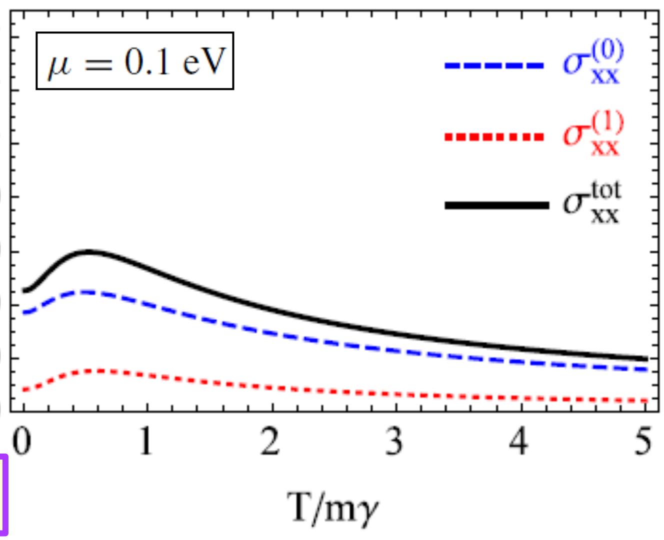
Conductivity, T≠0

$$\mathfrak{Re}\,\sigma^{\mathrm{tot}}_{xx}(y) \equiv \mathfrak{Re}\,\sigma^{(0)}_{xx}(y) + \mathfrak{Re}\,\sigma^{(1)}_{xx}(y).$$









Summary

- In the simplest model of a short-ranged quenched disorder, it was shown that the Fermi arc quasiparticles can scatter into the bulk states and into other surface Fermi arc states.
  - Fermi arcs transport is **dissipative**.
  - There is **no well-defined effective theory of Fermi arcs** in Weyl semimetals **in the presence of quenched disorder**.
- Fermi arc quasiparticle width that consist of (i) the intra-arc scattering and (ii) **the bulk dephasing** of the Fermi arc states was calculated.
- Using the ladder approximation **the Fermi arc conductivity** was calculated.
  - Conductivity decreases with increasing chemical potential μ.
  - At larger value of μ, the temperature dependence of the conductivity is nonmonotonic with a local maximum at a temperature that scales approximately as the chemical potential.