STUDIUM Workshop:

Classical and Quantum Black holes

Tours, Thursday 29th May, 2016

Black Hole Theory, where is it going?

Gary Gibbons
STUDIUM RESEARCH PROFESSOR

LMPT, Tours


LE STUDIUM
WORKSHOP
30-31 May 2016
Classical and quantum black holes
LOCATION
CONVENORS
Pr Gary Gibbons
Pr Sergey Solodukhin
LE STUDIUM

The likely observation at 9.45 UT Sept 14th last year by the LIGO team is just one more spectactular confirmation of the the central place of black hole theory in astrophysics. Observations of X-ray binary systems such as Cygnus X1, of SgA*, at the centre of the milky way, and the quasar phenomenon have all been successfully accommodated within the basic theoretical paradigm which emerged in an almost complete form around the time of the 1972 Les Houches Summer School.

That standard model of black hole physics was the result of many years of theoretical investigations of Einstein's Theory of Relativity and incorporates many cherished Fundamental Physical Principles. The validity of these fundamental principles has received support from many extremely high precision experiments and observations.

By *Fundamental Principles* I mean general statements expected to be true of all viable theories and which follow as consequences of a precise mathematical model within any given mathematically well defined theory. Such principles may have a heuristic value in motivating and formulating a theory, but cannot be used in themselves to define the theory.

For example Heisenberg's Uncertainty Principle is an elementary theorem in Wave Mechanics but is insufficient in itself to define Wave Mechanics. Moreover it rests heavily on translation invariance and may not be true in more general quantum mechanical theories such as Relativistic Quantum Field theory in which the notion of a position observable is problematic. Examples in General Relativity include Mach's Principle and various Equivalence Principles of which more later.

Today, that fundamental framework is under attack from the pressure of observations of the galaxies and galactic clusters indicating the the apparent existence of dark matter, not incorporated within the standard model of particle physics and of the CMB and distant supernovae showing the the existence of a cosmic repulsive force, possibly due to some sort of dark energy, on the one hand and the need to construct a quantum theory of gravity on the other.

In what follows I shall provide a brief account of how these fundamental principles are incorporated into our current black hole paradigm and how they are the modifications brought about by current work.

The standard model starts with Classical General Relativity coupled minimally to the standard model of particle physics. Minimal coupling is fairly unambiguous and I take it to exclude $R\Phi^2$ term for the Higgs. As such, incorporates a number of Fundamental Principles including

The Weak Equivalence Principle and The Strong Equivalence Principle together called

The Universality of Free Fall

Predictability from initial Data

Einstein Causality

It does not incorporate most formulations of Mach's Principle

The theory admits a Lagrangian and Hamiltonian structure

Thus allowing Definitions of Total Energy and Momentum P^μ for Isolated Systems and Positive Energy Theorems: P^μ is future directed timelike based on The Dominant Energy Condition.The singularity theorems are based on the The Strong Energy Condition.

Almost all of these are due to the fact that General Relativity is a single metric theory whose equations of motion and those of the matter are semi-linear p.d.e.'s of at most second order in derivatives.

This is not true of many classical theories currently receiving attention in the literature

In detail: all free particle motion is modelled by time like or null geodesics

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma_\alpha{}^\mu{}_\beta \frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0 \tag{1}$$

where $\Gamma_\alpha{}^\mu{}_\beta$ are components of the (Torsion free) Levi-Civta connection of $g_{\mu\nu}$. This is Universality of the Projective Structure given by free fall. This may derived by the WKB approximation

$$\Psi \approx Ae^{iS}\,, \qquad g^{\mu\nu}\partial_\mu S \partial_\nu S = -m^2\,, \qquad m\frac{dx^\mu}{d\lambda} = g^{\mu\nu}\partial_\nu S \tag{2}$$

to semi-linear wave equations for generic fields Ψ and masses m of the form

$$g^{\mu\nu}\partial_\mu\partial_\nu\Psi = \text{lower}\,,\text{derivative}\,,\text{terms} \tag{3}$$

which thus incorporates Wave Particle Duality.

The wave front surfaces $S =$ costant define spacelike ($m^2 > 0$) or $m^2 = 0$ *null hypersurfaces*. The latter define the characteristic surfaces of the wave equation for Φ and hence its causal cone. These causal cones are the same for all particles and hence we have a Universal Causal structure which is common to both $g_{\mu\nu}$ and $\Omega^2 g_{\mu\nu}$ and hence depends only on the conformal class of the metric. Thus we get Universal Conformal Structure and the theory incorporates Einstein Causality: there is a universal maximum speed for all forms of matter.

Roughly speaking the intersection of the Universal Conformal Structure and the Universal Projective Structure is the Einstein metric and and its (torsion-free) Levi-Civita connection.

The standard informal definition of a black hole : something from which nothing can escape or the more precise definitions of Absolute Event Horizon or Killing Horizon depend crucially on the this universality, as indeed the the very notions of timelike or null vector, depend on this conformal class. The same is true of the Weak Energy Energy Condition, Null Energy Energy Condition, Dominant Energy Energy Condition. and Strong Energy Energy Condition

$$T_{\mu\nu}V^{\mu}V^{\mu} \geq 0 \,\forall \{V^{\mu} | g_{\mu\nu}V^{\mu}V^{\nu} < 0\} \tag{4}$$

$$T_{\mu\nu}V^{\mu}V^{\nu} \geq 0 \,\forall \{V^{\mu} | g_{\mu\nu}V^{\mu}V^{\nu} = 0\} \tag{5}$$

$$T_{\mu\nu}V^{\mu}W^{\nu} \geq 0 \forall \{V^{\mu}, W^{\mu} | g_{\mu\nu}V^{\mu}V^{\nu},\, g_{\mu\nu}W^{\mu}W^{\nu},\, g_{\mu\nu}W^{\mu}V^{\nu} \leq 0\} \tag{6}$$

$$R_{\mu\nu}V^{\mu}V^{\nu} \geq 0 \,\forall \{V^{\mu} | g_{\mu\nu}V^{\mu}V^{\nu} \leq 0\} \tag{7}$$

By a Theorem of Hawking, The Dominant Energy Condition prevents matter appearing and disappearing a-causally. It is also a key assumption in positive energy theorems.

The Strong Energy Theorem may be paraphrased as Gravity is always attractive and plays is a key assumption in the singularity theorems and Hawking's area increase theorem for black hole event horizons, and hence the Second Law of Thermodynamics for black holes. By a related piece of theory which we will use later, Hawking's boundary conditions for stationary event horizons are

$$R^{\alpha\beta} l_\alpha l_\beta \Big|_{\text{Horizon}} = 0 = C_{\alpha\beta\gamma\delta} l^\alpha m^\beta l^\gamma m^\delta \Big|_{\text{Horizon}} . \tag{8}$$

As first shown my Choquet-Bruhat, the Vacuum Einstein equations satisfy the The Predictability Principle because in harmonic or wave coordinates defined by

$$\left\{x^{\mu}\middle|\Gamma_{\alpha}{}^{\mu}{}_{\beta} = 0\right\}, \tag{9}$$

they become

$$g^{\mu\nu}\partial_{\mu}\partial_{\nu}\, g_{\alpha\beta} = F_{\mu\nu}(g_{\alpha\beta}, \partial g_{\alpha\beta}) \tag{10}$$

Thus any spacelike surface $\{t = \text{constant}|g^{tt} \neq 0\}$ we can determine $\ddot{g}_{\alpha\beta}$ *uniquely* in terms of $g_{\alpha\beta}$ and $\dot{g}_{\alpha\beta}$. This allows us to determine *uniquely* all higher time derivatives of $g_{\alpha\beta}$ for sufficiently smooth data and hence to obtain a solution of (10) for sufficiently small times. *Moreover solution of equation (10) will satisfy the gauge condition (9) and hence satisfy the full Einstein equations.*

Thus we obtain an unambiguous prediction of the future, at least for sufficiently small times.

All of the Fundamental Principles and Energy conditions continue to hold for the Einstein equations coupled the matter of the standard model of particle physics, with $\Lambda = 0$ at least at the classical level. They also hold for all the Un-gauged Supergravity Theories.

The introduction of a Positive Cosmological Constant changes some things. In particular the Strong Energy condition no longer holds. As far as horizons are concerned much is unchanged but the violation of the Strong Energy condition allows a new type of Cosmological Horizon which behave very much like an inside out black hole with Newton's attraction overwhelming De-Sitter's repulsion at large distances. In general these two types of horizon have different surface gravities and thus different Hawking temperatures

In the case of Negative Cosmological Constant and Gauged Supergravity Theories the weak, and dominant energy conditions no longer hold but there are nevertheless positive energy theorems provided certain conditions of Breitenlohner and Freedman hold. There is no analogue of cosmological horizons. The issue of Predictability is complicated by the timelike boundary at conformal infinity.

Recent alternative theories appear to be based on the No Higher Derivatives than 2 Principle because of concerns about energy and stability which go back to the fundamental paper of Pais and Uhlenbeck. In classical theories excitations whose kinetic energies are negative are often known as "ghost's" or "phantom's" Quantum-mechanically the former term is also applied to quantum states $|\psi>$ with negative norm $<\psi|\psi><0$. For clarity I shall refer to excitations in classical theories whose kinetic energies are negative as poltergeists. A general result, but not due to Ostrogradsky, is that non-degenerate Lagrangians with derivatives higher than 2 violate the No Poltergeist Principle. In general poltergeists may be quantised such that $< poltergeist|poltergeist >> 0$ but some negative energy states remain.

Poltergeists first entered cosmology with the Steady State Theory and Hoyle and Narlikar's C-field which, since it was a poltergeist it was able to provide a mechanism for the continuous creation of standard model matter required in that theory, thus evading Hawking's theorem.

More recently, they have been invoked to account for cosmic acceleration

Poltergeists may be minimally coupled to a metric and like tachyons (in the sense of fields with $m^2 < 0$) satisfy Einstein Causality. If $m^2 \geq 0$, solutions of the Klein Gordan equation satisfy a No-Hair theorem regardless of the over-all sign of the energy. Therefore static black holes admit no poltergeist hair.

Since poltergeists violate the energy conditions needed to prove topology censorship theorems one is not surprised to find that the resultant Einstein equations (with $m^2 = 0$) admit globally static complete and non-singular Einstein-Rosen bridge solutions *.

*often called wormholes but Misner and Wheeler were originally thinking of solitions without horizons with non-simply connected space sections. There are no such solutions of four-dimensional ungauged supergravity but many in 5 dimensions where they are known as fuzzballs.

The most famous is the Bronnikov-Ellis ultra-static solution

$$ds^2 = -dt^2 + dr^2 + (r^2 + a^2)(d\theta^2 + \sin^2\theta d\phi^2) \tag{11}$$

Using solution generating techniques one may construct similar solutions with $g_{tt} \neq$ constant.

Poltergeists can anti-gravitate. Recently static multi-bridge solutions have been constructed. A simple, albeit singular static antigravitating solution is given by

$$ds^2 = -e^{2H}dt^2 + e^{-2H}d\mathbf{x}^2 \qquad \nabla^2 H = 0\,. \tag{12}$$

Higher derivatives in quantum corrections to the Einstein Hilbert Lagrangian, Indeed the theory can be renormalised at the expense of introducing poltergeists or ghosts. These theories are quite complicated. Scalar poltergeists can offer insights with much less calculational effort.

If scared of poltergeist and one adopts the No higher derivatives than 2 Principle one is in for a much harder time. In 4 dimensions there is, by a celebrated result of Lovelock, nothing to be done beyond Einstein-Hilbert as long as one maintains minimal coupling. As realised by Lovelock and Horndeski long ago if one admits a scalar ϕ as well are many theories of a metric $g_{\mu\nu}$ (but not *the* metric) coupled to ϕ. However almost all of these theories can lead to violations of Universality of Free Fall, Einstein Causality, Predictability. Define a field $\Phi^a = (g_{\alpha\beta}, phi)$ One imposes 4 gauge conditions and seeks to cast the equations of motion in the form

$$M^a{}_b{}^{\mu\nu} \partial_\mu \partial \nu \Phi^b = F^a \tag{13}$$

where both $M^a{}_b = M^a{}_b(\Phi^c, \partial_\mu \Phi^c)$ and $F^a = F^a(\Phi^c, \partial_\mu \Phi^c)$. This entails solving a highly non-linear set of equations for $\partial_\mu \partial \nu \Phi^b$. Generically there may be many solutions or even none. As one moves in spacetime the number of solutions may jump.

Pick one branch. On a $S = constant$ surface one seeks to find $\ddot{\Phi}^a$ where $\dot{\Phi}=\frac{\partial \Phi^a}{\partial S}$ as functions of Φ^a and up to one time derivative and and two spatial derivatives. The analogue of null hypersurfaces are characteristic surfaces on which $M^a{}_b{}^{\mu\nu}\partial_\mu S \partial_\nu S$ has a kernel. In general one may not be able to extract any sort of universal light cone structure. If one linearises the equations around a background solution by setting $\Phi^a = \Phi^a_0 + \epsilon \Phi^a_1 + \dots$ the characteristics of the equations for the resulting gravitons and phions are given by

$$\det M^a{}_b{}^{\mu\nu}(g_0^{\alpha\beta}, \phi_0)\partial_\mu S \partial_\nu S \tag{14}$$

In general one expects bi-refringence with different mixtures of gravitons and phions propagating at different maximum speeds. The Einstein Causality breaks down and the very notion of a black hole problematic. Even in if a bi-metric structure were to emerge, one has to ask which one defines a black hole and for what particles?

The response implicit in the literature seems to assume that standard model matter couples minimally to the metric $g_{\mu\nu}$ and a variety of exact solutions have been found in various theories. That leaves unanswered the question can gravitons and phions escape from such black holes?

Interestingly Jacobson has found in the spherically symmetric case when the metrics admit coordinates in which the metric is diagonal, then the two Killing horizons coincide and their surfaces gravities and hence Hawking Temperatures coincide. Note that in general the surface gravities of two conformally related metrics coincide.