Chiral Waves and kinks in chiral fluids

M. N. Chernodub

Centre national de la recherche scientifique (CNRS)
University of Tours, France

Plan of the talk

- **Anomalous transport phenomena in chiral media (brief review):**
  Chiral Magnetic Effect (in magnetic field at finite chiral density)
  - Chiral Separation Effect (in magnetic field at finite fermion density)
  - Chiral Vortical Effects (in rotating medium at finite densities and temperature)
  - Transfer of energy (rotating hot system at finite densities and temperature)

- **Anomalous chiral waves:**
  - Chiral Magnetic Wave
  - Chiral Vortical Wave
  - ***Chiral Heat Wave***

- **Chiral kinks: shock waves in chiral media**

The Chiral Magnetic Effect (CME)

Electric current is induced by applied magnetic field:

$$\boldsymbol{J} = \tilde{\sigma} \boldsymbol{B}$$

Spatial inversion ($\boldsymbol{x} \rightarrow - \boldsymbol{x}$) symmetry (*P*-parity):

- Electric current is a vector (parity-even quantity);
- Magnetic field is a pseudovector (parity-odd quantity).

Thus, the CME-supporting medium should be parity-odd !

In other words, the spectrum of the medium which supports the CME should not be invariant under a spatial inversion transformation.

Non-dissipative nature of the current

- Time inversion:

$$t \to -t$$

Ohm conductivity (dissipative, loss of current):

$$\boldsymbol{j} = \sigma \boldsymbol{E} \quad \longrightarrow \quad \boldsymbol{j} = -\sigma \boldsymbol{E}$$

- Superconductivity (non-dissipative, no losses):

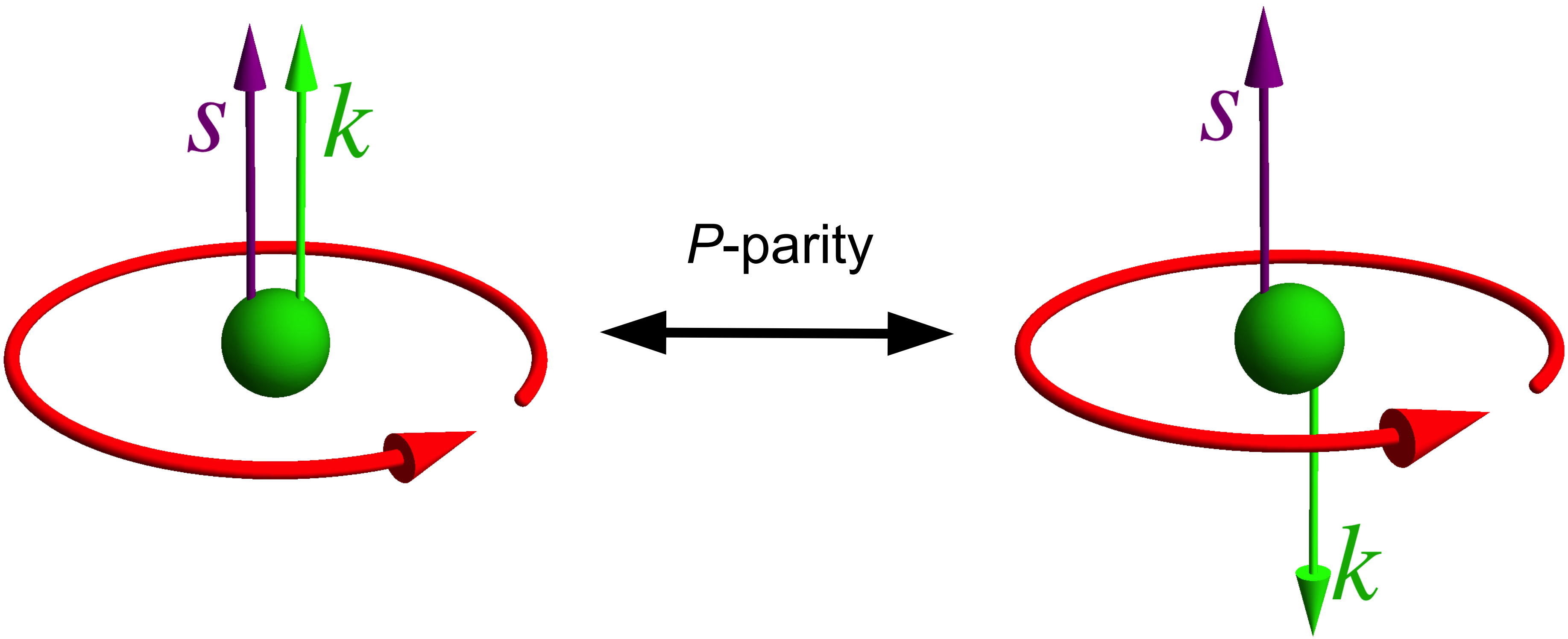
$$\frac{\partial \boldsymbol{j}}{\partial t} = \mu^2 \boldsymbol{E} \quad \longrightarrow \quad \frac{\partial \boldsymbol{j}}{\partial t} = \mu^2 \boldsymbol{E}$$

- Chiral Magnetic Effect (non-dissipative, no losses):

$$\boldsymbol{j} = \tilde{\sigma} \boldsymbol{B} \quad \longrightarrow \quad \boldsymbol{j} = \tilde{\sigma} \boldsymbol{B}$$

An example of a parity-odd system?

Consider a massless fermion:



Right handed **Left handed**

The chirality (left/right) is a conserved number.

How to make a parity-odd system out of fermions?

Systems with equal number of left- and right-handed particles are P-invariant.

Examples of P-invariant systems in the daily life: One right-handed particle and one left-handed particle with opposite momenta at IHEP, Beijing.

However, if we have a different number of left- and right-handed particles (for example, in a gas) then this system is not *P*-invariant. If these particles are electrically charged, then we may expect that this system exhibits the CME:

$$\boldsymbol{J} = \tilde{\sigma} \boldsymbol{B}$$

[A. Vilenkin, '80; K. Fukushima, D. E. Kharzeev, H. J. Warringa, '08; D. E. Kharzeev, L. D. McLerran and H. J. Warringa, '08]

Chiral Magnetic Effect – experiment (Quark Gluon Plasma)

- Signatures of the Chiral Magnetic Effect in heavy-ion collisions

PRL **113,** 052302 (2014) PHYSICAL REVIEW LETTERS week ending 1 AUGUST 2014

Beam-Energy Dependence of Charge Separation along the Magnetic Field in Au + Au Collisions at RHIC

L. Adamczyk,[1] J. K. Adkins,[23] G. Agakishiev,[21] M. M. Aggarwal,[35] Z. Ahammed,[53] I. Alekseev,[19] J. Alford,[22] C. D. Anson,[32] A. Aparin,[21] D. Arkhipkin,[4] E. C. Aschenauer,[4] G. S. Averichev,[21] A. Banerjee,[53] D. R. Beavis,[4] R. Bellwied,[49] A. Bhasin,[20] A. K. Bhati,[35] P. Bhattarai,[48] H. Bichsel,[55] J. Bielcik,[13] J. Bielcikova,[14] L. C. Bland,[4]

....

(STAR Collaboration)

Local parity-odd domains are theorized to form inside a quark-gluon plasma which has been produced in high-energy heavy-ion collisions. The local parity-odd domains manifest themselves as charge separation along the magnetic field axis via the chiral magnetic effect. The experimental observation of charge separation has previously been reported for heavy-ion collisions at the top RHIC energies. In this Letter, we present the results of the beam-energy dependence of the charge correlations in Au + Au collisions at midrapidity for center-of-mass energies of 7.7, 11.5, 19.6, 27, 39, and 62.4 GeV from the STAR experiment. After background subtraction, the signal gradually reduces with decreased beam energy and tends to vanish by 7.7 GeV. This implies the dominance of hadronic interactions over partonic ones at lower collision energies.

Chiral Magnetic Effect – experiment (Solid State)

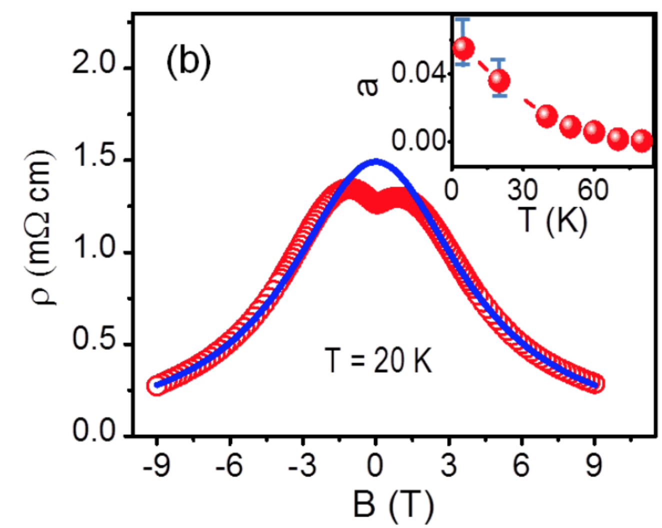
- Signatures of the Chiral Magnetic Effect in Dirac Semimetals

Observation of the chiral magnetic effect in $ZrTe_5$

Qiang Li,[1] Dmitri E. Kharzeev,[2,3] Cheng Zhang,[1] Yuan Huang,[4] I. Pletikosić,[1,5]
A. V. Fedorov,[6] R. D. Zhong,[1] J. A. Schneeloch,[1] G. D. Gu,[1] and T. Valla[1]

Abstract

The chiral magnetic effect is the generation of electric current induced by chirality imbalance in the presence of magnetic field. It is a macroscopic manifestation of the quantum anomaly[1,2] in relativistic field theory of chiral fermions (massless spin 1/2 particles with a definite projection of spin on momentum) – a dramatic phenomenon arising from a collective motion of particles and antiparticles in the Dirac sea. The recent discovery[3–5] of Dirac semimetals with chiral quasi-particles opens a fascinating possibility to study this phenomenon in condensed matter experiments. Here we report on the first observation of chiral magnetic effect through the measurement of magneto-transport in zirconium pentatelluride, $ZrTe_5$. Our angle-resolved photoemission spectroscopy experiments show that this material's electronic structure is consistent with a 3D Dirac semimetal. We observe a large negative magnetoresistance when magnetic field is parallel with the current. The measured quadratic field dependence of the magnetoconductance is a clear indication of the chiral magnetic effect. The observed phenomenon stems from the effective transmutation of Dirac semimetal into a Weyl semimetal induced by the parallel electric and magnetic fields that represent a topologically nontrivial gauge field background.



ArXiv:1412.6543

Examples of anomalous transport effects (I)

1) **Chiral Magnetic Effect –** the electric current is induced in the direction of the magnetic field due to the chiral imbalance:

$$\boldsymbol{j} = \frac{\mu_5}{2\pi^2} e\boldsymbol{B}$$

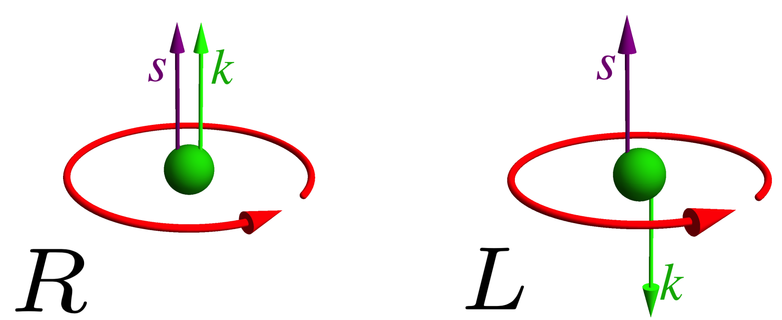
[all formulae are written for one flavor of fermions]

$$\boldsymbol{j} = \boldsymbol{j}_R + \boldsymbol{j}_L$$

$$\boldsymbol{j}_5 = \boldsymbol{j}_R - \boldsymbol{j}_L$$

2) **Chiral Separation Effect –** the axial current is induced in the direction of the magnetic field:

$$\boldsymbol{j}_5 = \frac{\mu}{2\pi^2} e\boldsymbol{B}$$



Notes:

A) This effect is realized even if the plasma is chirally-trivial.

B) μ is the quark chemical potential (the plasma is dense)

C) The axial (chiral) current = the difference “right current”–“left current”.

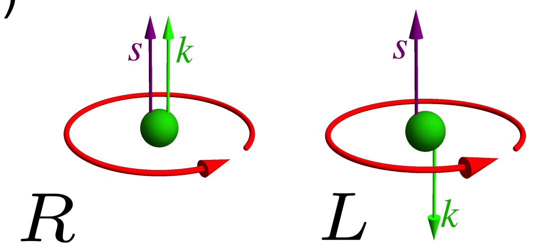
Examples of anomalous transport effects (II)

3) **Chiral Vortical Effect** – the axial current is induced in the rotating quark-gluon plasma along the axis of rotation:

$$\boldsymbol{j}_5 = \frac{T^2}{6}\boldsymbol{\Omega}$$

angular velocity (vorticity)

$$\boldsymbol{\Omega} = \frac{1}{2}\boldsymbol{\partial} \times \boldsymbol{v}$$



4) **Dissipationless energy transfer** – the energy flow is generated along the axis of magnetic field:

$$\boldsymbol{j}_E = \frac{\mu\mu_5}{2\pi^2} e\boldsymbol{B}$$

Energy current:

$$j^i_E \equiv T^{0i} = \frac{i}{2}\bar{\psi}\left(\gamma^0\partial^i + \gamma^i\partial^0\right)\psi$$

Note: All these effects will become more complicated (much richer) in a rotating, chirally-imbalanced dense matter subjected to background magnetic fields.

Chiral Waves

- The chiral waves are massless excitations in a fluid or plasma of chiral fermions.

- There are four types of waves

  - Chiral Magnetic Wave [D. Kharzeev and H.-Y. Yee, 2011]
  - Chiral Vortical Wave [Y. Jiang, X.-G. Huang and J. Liao, 2015]
  - Chiral Alfvén Wave [N. Yamamoto, 2015] – not reviewed here
  - ***Chiral Heat Wave*** [M.Ch. 2015]

  These sound-like excitations correspond to coherent propagation of charge density, axial (chiral) charge density, and energy density waves in different combinations.

Chiral Magnetic Wave: currents and chemical potentials

- Chiral fluid/plasma (massless fermions of both chiralities)

- Currents:

$$j^\mu \equiv j_V = (\rho, \boldsymbol{j}) = \langle \bar{\psi}\gamma^\mu\psi \rangle$$ vector

$$j_5^\mu \equiv j_A = (\rho_5, \boldsymbol{j}_5) = \langle \bar{\psi}\gamma^\mu\gamma^5\psi \rangle$$ axial

- Right-handed and left-handed currents:

$$\boldsymbol{j}_V = \boldsymbol{j}_R + \boldsymbol{j}_L$$

$$\boldsymbol{j}_A = \boldsymbol{j}_R - \boldsymbol{j}_L$$

- Chemical potentials:

$$\mu_V \equiv \mu = \frac{1}{2}(\mu_R + \mu_L)$$ vector

$$\mu_A \equiv \mu_5 = \frac{1}{2}(\mu_R - \mu_L)$$ axial

Chiral Magnetic Wave: currents and density fluctuations

- Anomalous transport laws:

$$\boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B}$$

chiral magnetic effect (CME)

$$\boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}$$

chiral separation effect (CVE)

- Consider a neutral chiral system:
  - $\rightarrow$ on average, all densities are zero
  - $\rightarrow$ globally, all chemical potentials are zero
  - $\rightarrow$ no equilibrium CME and CVE

$$\langle \rho_V \rangle = \langle \rho_A \rangle = 0 \qquad \overline{\mu}_V = \overline{\mu}_A = 0$$

… but local fluctuations of densities may exist:

$$\delta\rho_V(x) = \rho_V(x) - \langle \rho_V \rangle = \rho_V(x)$$

$$\delta\rho_A(x) = \rho_A(x) - \langle \rho_A \rangle = \rho_A(x)$$

Chiral Magnetic Wave: couple currents and densities

- Combine:

$$\boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B} \qquad \boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}$$

and

$$\rho_V(x) = \chi \mu_V(x) \qquad \rho_A(x) = \chi \mu_A(x)$$

with the result:

$$\boldsymbol{j}_V(x) = \frac{e\boldsymbol{B}}{2\pi^2 \chi} \rho_A(x)$$

Axial density induces vector current.

$$\boldsymbol{j}_A(x) = \frac{e\boldsymbol{B}}{2\pi^2 \chi} \rho_V(x)$$

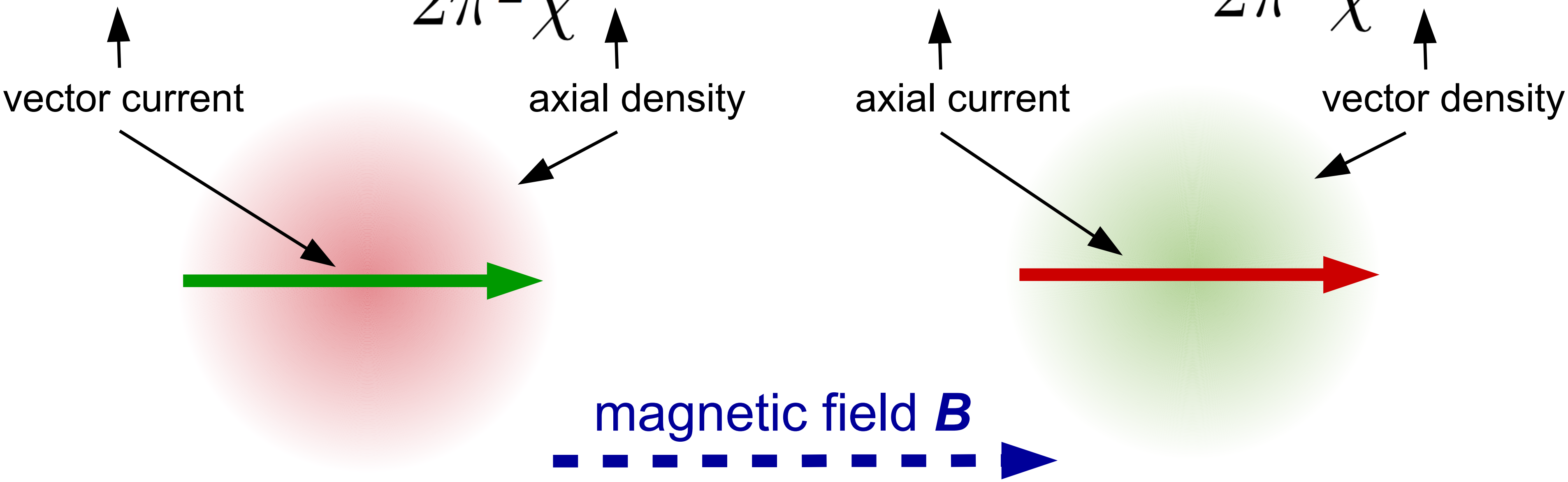
Vector density induces axial current.

Question: Are they ~~married~~ chained to each other?

Chiral Magnetic Wave: two current-density relations

- Anomalous current-density relation:

$$\boldsymbol{j}_V(x) = \frac{e\boldsymbol{B}}{2\pi^2\chi}\rho_A(x) \qquad \boldsymbol{j}_A(x) = \frac{e\boldsymbol{B}}{2\pi^2\chi}\rho_V(x)$$



- Usual current-density relation (conservation of charge):

$$\partial_\mu j^\mu_A \equiv \partial_t \rho_V + \boldsymbol{\partial j}_V = 0$$
$$\partial_\mu j^\mu_A \equiv \partial_t \rho_A + \boldsymbol{\partial j}_A = 0$$

[We do not consider external electric field so that we have no anomaly for the axial current.]

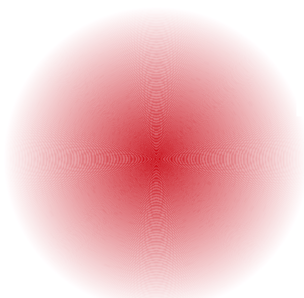
Magnetic Wave: couple densities and chemical potentials

- Anomalous transport laws:

$$\boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B} \qquad\qquad \boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}$$

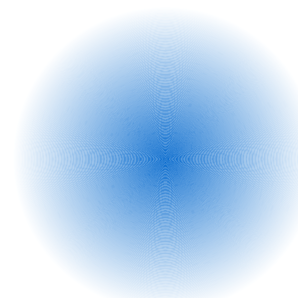
What are these chemical potentials?

It is better to work with densities rather than with chemical potentials!



$$\rho(x)>0 \;\rightarrow\; \mu(x)>0$$

$$\rho(x)<0 \;\rightarrow\; \mu(x)<0$$



For small perturbations:

$$\rho_V(x) = \chi\mu_V(x) \qquad\qquad \rho_A(x) = \chi\mu_A(x)$$

"susceptibility" (calculable)

Chiral Magnetic Wave: relations between all densities

- Anomalous current-density relation

$$\boldsymbol{j}_V(x) = \frac{e\boldsymbol{B}}{2\pi^2\chi}\rho_A(x) \qquad \boldsymbol{j}_A(x) = \frac{e\boldsymbol{B}}{2\pi^2\chi}\rho_V(x)$$

- Let ***B*** be along the *z*-axis, ***B***=(0,0,*B*):

$$j_V^z(x) = \frac{eB}{2\pi^2\chi}\rho_A(x)\,, \qquad j_A^z(x) = \frac{eB}{2\pi^2\chi}\rho_V(x)$$

- Charge conservation:

$$\partial_t\rho_V + \partial_z j_V^z = 0\,, \qquad \partial_t\rho_A + \partial_z j_A^z = 0$$

- Exclude currents:

$$-\partial_t\rho_V(x) = \frac{eB}{2\pi^2\chi}\rho_A(x)\,, \qquad -\partial_t\rho_A(x) = \frac{eB}{2\pi^2\chi}\rho_V(x)$$

Chiral Magnetic Wave – wave equations

- Relations between densities:

$$\partial_t \rho_V(x) + \frac{eB}{2\pi^2\chi}\partial_z \rho_A(x) = 0\,, \qquad \partial_t \rho_A(x) + \frac{eB}{2\pi^2\chi}\partial_z \rho_V(x) = 0$$

- Two first-order equations for two variables.
- Diagonalize them and get second order equations:

$$\left(\partial_t^2 - v_{\rm CMW}^2 \partial_z^2\right) \rho_V(x) = 0$$

$$\left(\partial_t^2 - v_{\rm CMW}^2 \partial_z^2\right) \rho_A(x) = 0$$

- Sound-like ("gapless") wave propagation with velocity

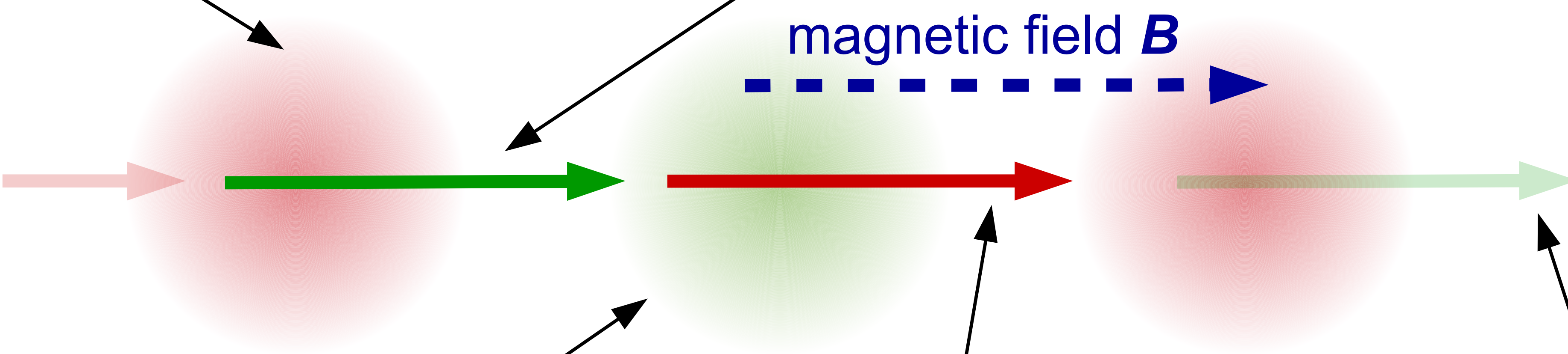
$$v_{\rm CMW} = \frac{eB}{2\pi^2\chi}$$

- This is the Chiral Magnetic Wave

Chiral Magnetic Wave – physical picture

(1) fluctuation in axial density

(2) creates fluctuation in vector current



(3) which induces vector density fluctuation at the next point

(4) which creates fluctuation of the vector current

(5=1) which generates fluctuation of the axial density at the next point

(6=2) creates fluctuation in vector current

(7=3) etc etc etc.

$$\boldsymbol{j}_V(x) = \frac{e\boldsymbol{B}}{2\pi^2\chi}\rho_A(x)$$

$$\boldsymbol{j}_A(x) = \frac{e\boldsymbol{B}}{2\pi^2\chi}\rho_V(x)$$

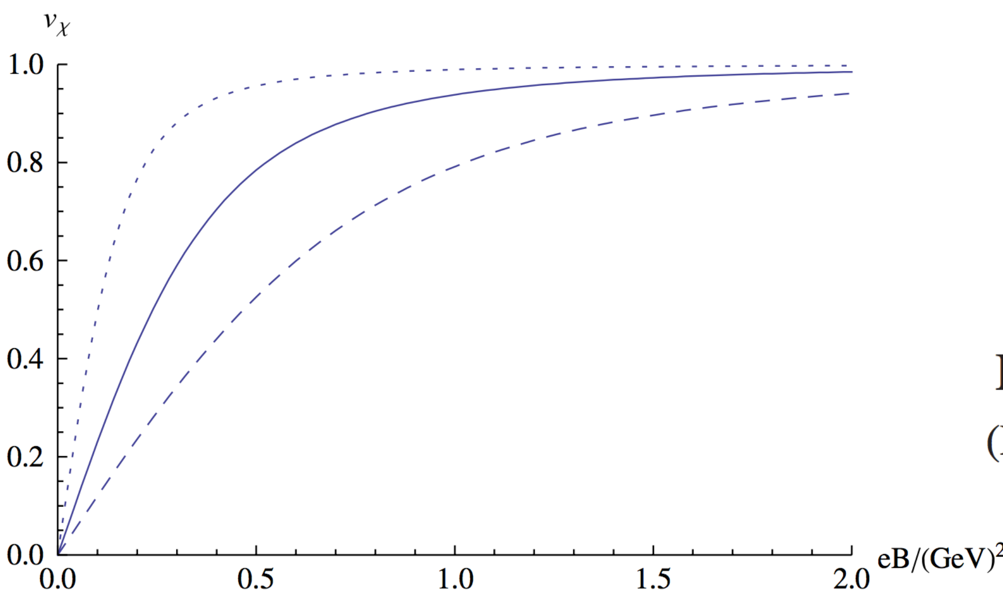
The Chiral Magnetic Wave is a chain-like process.

Chiral Magnetic Wave – velocity

- The wave propagates with velocity:

$$v_{\mathrm{CMW}} = \frac{eB}{2\pi^2\chi}$$

- At weak magnetic field χ is constant.
- At arbitrary field strength:



PHYSICAL REVIEW D **83,** 085007 (2011)

Chiral magnetic wave

Dmitri E. Kharzeev[1,2,*] and Ho-Ung Yee[1,†]

(Received 26 January 2011; published 6 April 2011)

Chiral Magnetic Wave – experiment

- Signatures of the Chiral Magnetic Wave in heavy-ion collisions

PRL **114,** 252302 (2015) PHYSICAL REVIEW LETTERS week ending 26 JUNE 2015

Observation of Charge Asymmetry Dependence of Pion Elliptic Flow and the Possible Chiral Magnetic Wave in Heavy-Ion Collisions

L. Adamczyk,[1] J. K. Adkins,[20] G. Agakishiev,[18] M. M. Aggarwal,[30] Z. Ahammed,[47] I. Alekseev,[16] J. Alford,[19] A. Aparin,[18] D. Arkhipkin,[3] E. C. Aschenauer,[3] G. S. Averichev,[18] A. Banerjee,[47] R. Bellwied,[43] A. Bhasin,[17] A. K. Bhati,[30] P. Bhattarai,[42] J. Bielcik,[10] J. Bielcikova,[11] L. C. Bland,[3] I. G. Bordyuzhin,[16] J. Bouchet,[19] A. V. Brandin,[26] I. Bunzarov,[18]
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(STAR Collaboration)

We present measurements of π^- and π^+ elliptic flow, v_2, at midrapidity in Au + Au collisions at $\sqrt{s_{NN}} = 200, 62.4, 39, 27, 19.6, 11.5$, and 7.7 GeV, as a function of event-by-event charge asymmetry, A_{ch}, based on data from the STAR experiment at RHIC. We find that π^- (π^+) elliptic flow linearly increases (decreases) with charge asymmetry for most centrality bins at $\sqrt{s_{NN}} = 27$ GeV and higher. At $\sqrt{s_{NN}} = 200$ GeV, the slope of the difference of v_2 between π^- and π^+ as a function of A_{ch} exhibits a centrality dependence, which is qualitatively similar to calculations that incorporate a chiral magnetic wave effect. Similar centrality dependence is also observed at lower energies.

Chiral Magnetic Wave – simplest signatures

- Can we see ("feel") the existence of the Wave just from anomalous transport without making all those steps?

- Yes, we see the wave just from anomalous transport laws:

$$\boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B} \qquad \boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}$$

- Signature: **linear** cross-coupling of chemical potentials and the corresponding currents

- In this particular example:

Vector current – Axial chemical potential
Axial current – Vector chemical potential

Other waves?

- We just discussed vector and axial currents for chiral fermions in the background of magnetic field:

$$\boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B} \qquad \boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}$$

- There are also anomalous effects due to rotation! Our plan:
  - ➔ Switch off magnetic field, ***B*** = 0.
  - ➔ Rotate the fluid or gas of chiral fermions!

- Anomalous currents in a rotating system:

$$\boldsymbol{j}_V = \sigma_V^{\mathcal{V}} \boldsymbol{\Omega}\,, \qquad \boldsymbol{j}_A = \sigma_A^{\mathcal{V}} \boldsymbol{\Omega}$$

$$\sigma_V^{\mathcal{V}} = \frac{\mu_V \mu_A}{\pi^2}\,, \qquad \sigma_A^{\mathcal{V}} = \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}$$

Chiral Vortical Wave – feeling of existence

- Non-zero density, zero temperature, no magnetic field, rotation:

$$\delta \boldsymbol{j}_V = \frac{\bar{\mu}_V}{\pi^2} \delta\mu_A \, \boldsymbol{\Omega} \equiv \frac{\bar{\mu}_V \boldsymbol{\Omega}}{\pi^2 \chi} \delta\rho_A$$

$$\delta \boldsymbol{j}_A = \frac{\bar{\mu}_V}{\pi^2} \delta\mu_V \, \boldsymbol{\Omega} \equiv \frac{\bar{\mu}_V \boldsymbol{\Omega}}{\pi^2 \chi} \delta\rho_V$$

- Here δ is used explicitly to stress that we consider fluctuations only.

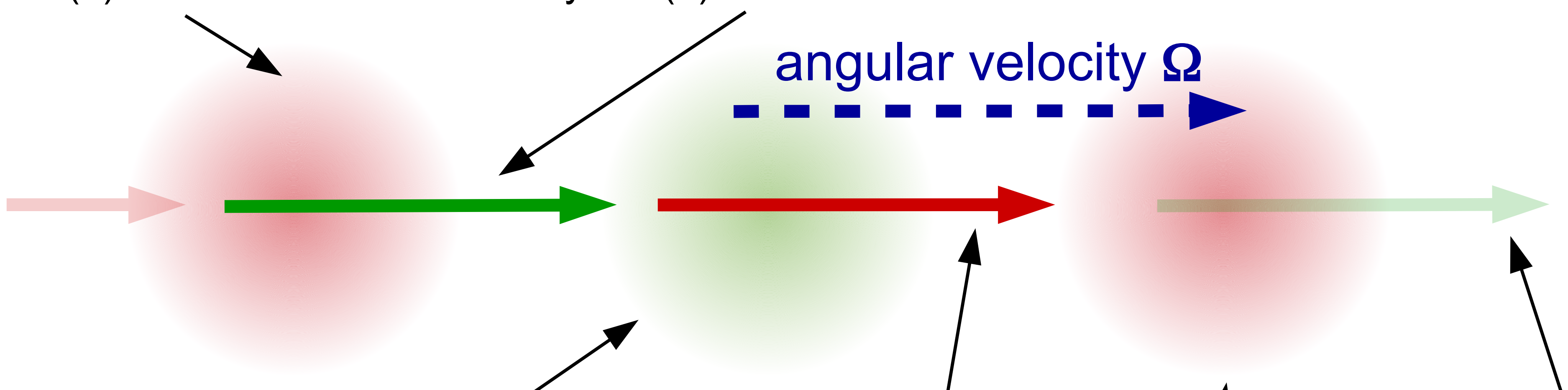
➔ Coupling between vector/axial currents/densities!

➔ Chiral Vortical Wave!

Chiral Vortical Wave – physical picture

(1) fluctuation in axial density

(2) creates fluctuation in vector current



(3) which induces vector density fluctuation at the next point

(4) which creates fluctuation of the vector current

$$\delta \boldsymbol{j}_V = \frac{\bar{\mu}_V \boldsymbol{\Omega}}{\pi^2 \chi} \delta \rho_A$$

$$\delta \boldsymbol{j}_A = \frac{\bar{\mu}_V \boldsymbol{\Omega}}{\pi^2 \chi} \delta \rho_V$$

(5=1) which generates fluctuation of the axial density at the next point

(6=2) creates fluctuation in vector current

(7=3) etc etc etc.

The Chiral Vortical Wave is also a chain-like process.

Chiral Vortical Wave – results

- Velocity of wave propagation depends on angular velocity Ω:

$$v_{\mathrm{CVW}} = \frac{\bar{\mu}_V \Omega}{\pi^2 \chi}$$

Chiral vortical wave and induced flavor charge transport in a rotating quark-gluon plasma

Yin Jiang,[1,*] Xu-Guang Huang,[2,†] and Jinfeng Liao[1,3,‡]

[1] *Physics Department and Center for Exploration of Energy and Matter, Indiana University, 2401 N Milo B. Sampson Lane, Bloomington, IN 47408, USA.*
[2] *Physics Department and Center for Particle Physics and Field Theory, Fudan University, Shanghai 200433, China.*
[3] *RIKEN BNL Research Center, Bldg. 510A, Brookhaven National Laboratory, Upton, NY 11973, USA.*
(Dated: April 14, 2015)

We show the existence of a new gapless collective excitation in a rotating fluid system with chiral fermions, named as the Chiral Vortical Wave (CVW). The CVW has its microscopic origin at the quantum anomaly and macroscopically arises from interplay between vector and axial charge fluctuations induced by vortical effects. The wave equation is obtained both from hydrodynamic current equations and from chiral kinetic theory and its solutions show nontrivial CVW-induced charge transport from different initial conditions. Using the rotating quark-gluon plasma in heavy ion collisions as a concrete example, we show the formation of induced flavor quadrupole in QGP and estimate the elliptic flow splitting effect for Λ baryons that may be experimentally measured.

ArXiv:1504.03201

Chiral Heat Wave – anomalous transport for energy

- Dissipationless energy transfer flow (energy current):

$$\boldsymbol{j}_E = \sigma_E^{\mathcal{B}} e\boldsymbol{B} + \sigma_E^{\mathcal{V}} \boldsymbol{\Omega}$$

- Energy flow is defined by the stress-energy tensor:

$$j_E^i \equiv T^{0i} = \frac{i}{2}\bar{\psi}\left(\gamma^0\partial^i + \gamma^i\partial^0\right)\psi$$

- Anomalous conductivities:

$$\sigma_E^{\mathcal{B}} = \frac{1}{2\pi^2}\mu_V\mu_A\,,$$

$$\sigma_E^{\mathcal{V}} = \frac{\mu_A}{3}\left[\frac{1}{\pi^2}\left(3\mu_V^2 + \mu_A^2\right) + T^2\right]$$

Temperature fluctuations:

$$\delta\epsilon = c_V\,\delta T$$

We must consider thermal fluctuations!

Chiral Heat Wave – finding its traces

- Finite temperature, zero density, no magnetic field, rotation:

$$\delta \boldsymbol{j}_V = 0\,,$$

fluctuations in vector (electric) current are negligible

$$\delta \boldsymbol{j}_A = \frac{T\boldsymbol{\Omega}}{3}\,\delta T,$$

fluctuations in axial current depend on temperature fluctuations

$$\delta \boldsymbol{j}_E = \frac{T^2\boldsymbol{\Omega}}{3}\delta \mu_A$$

fluctuations in energy (thermal) current depend on fluctuations of the axial charge

- Conclusion: axial charge density and energy density are together, the vector charge density does not participate.

- We have a wave of axial charge density and energy density!

Chiral Heat Wave

- Coupled axial and energy fluctuations:

$$\delta \boldsymbol{j}_A = \frac{T\boldsymbol{\Omega}}{3c_V}\,\delta\epsilon, \qquad \delta \boldsymbol{j}_E = \frac{T^2\boldsymbol{\Omega}}{3\chi}\delta\rho_A$$

- Sound waves of axial charge density and energy density:

$$\left(\partial_t^2 - v_{\rm CHW}^2 \partial_z^2\right)\delta\epsilon(t,z) = 0$$
$$\left(\partial_t^2 - v_{\rm CHW}^2 \partial_z^2\right)\delta\rho_A(t,z) = 0$$

- Velocity of propagation:

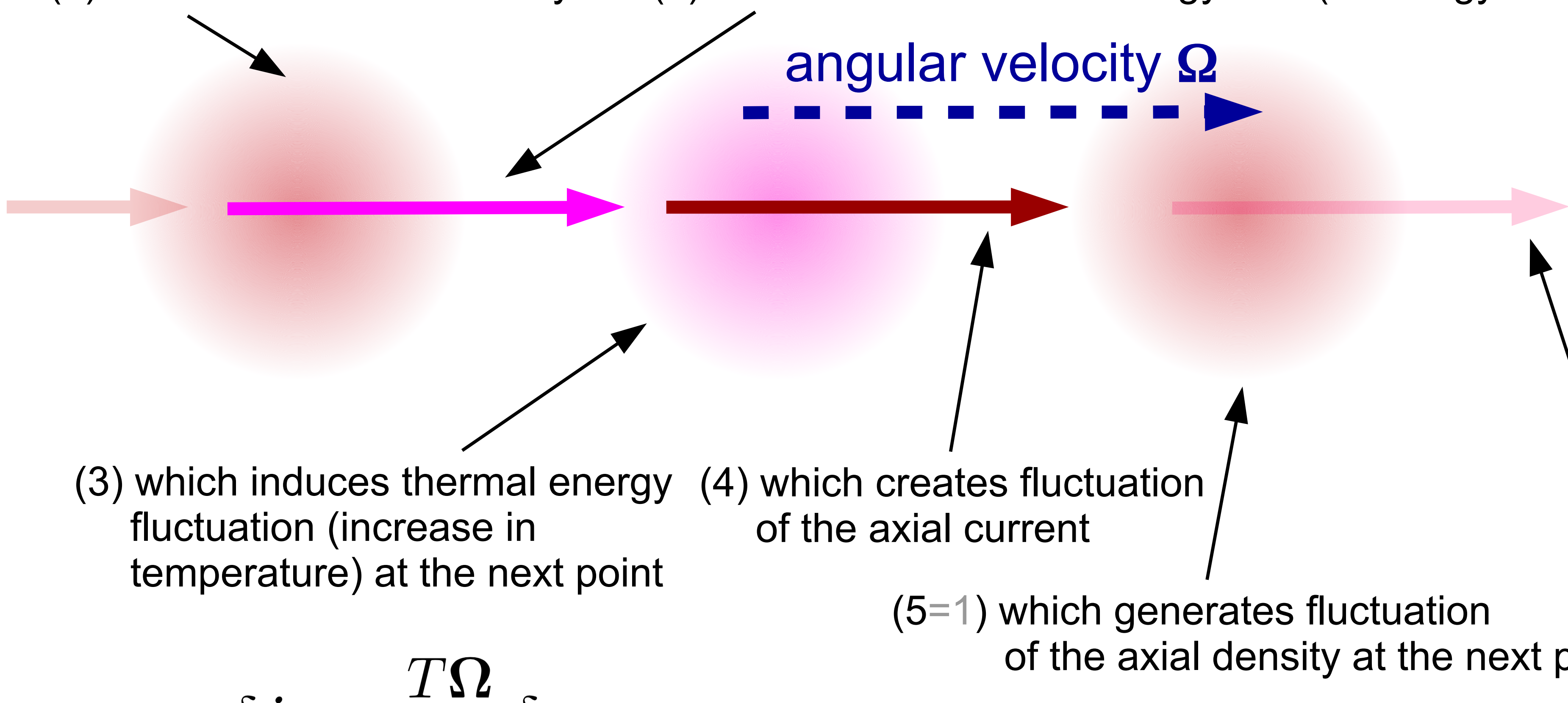
$$v_{\rm CHW} = \sqrt{\frac{T^3}{c_V\chi}\frac{\Omega}{3}}$$

- This is the Chiral Heat Wave [M.Ch. JHEP01(2016)100]
  + confirmed in kinetic approach [D. Frenklakh, arXiv:1603.08971]

Chiral Heat Wave – physical picture

(1) fluctuation in axial density

(2) creates fluctuation in energy flow (in energy current)



(3) which induces thermal energy fluctuation (increase in temperature) at the next point

(4) which creates fluctuation of the axial current

(5=1) which generates fluctuation of the axial density at the next point

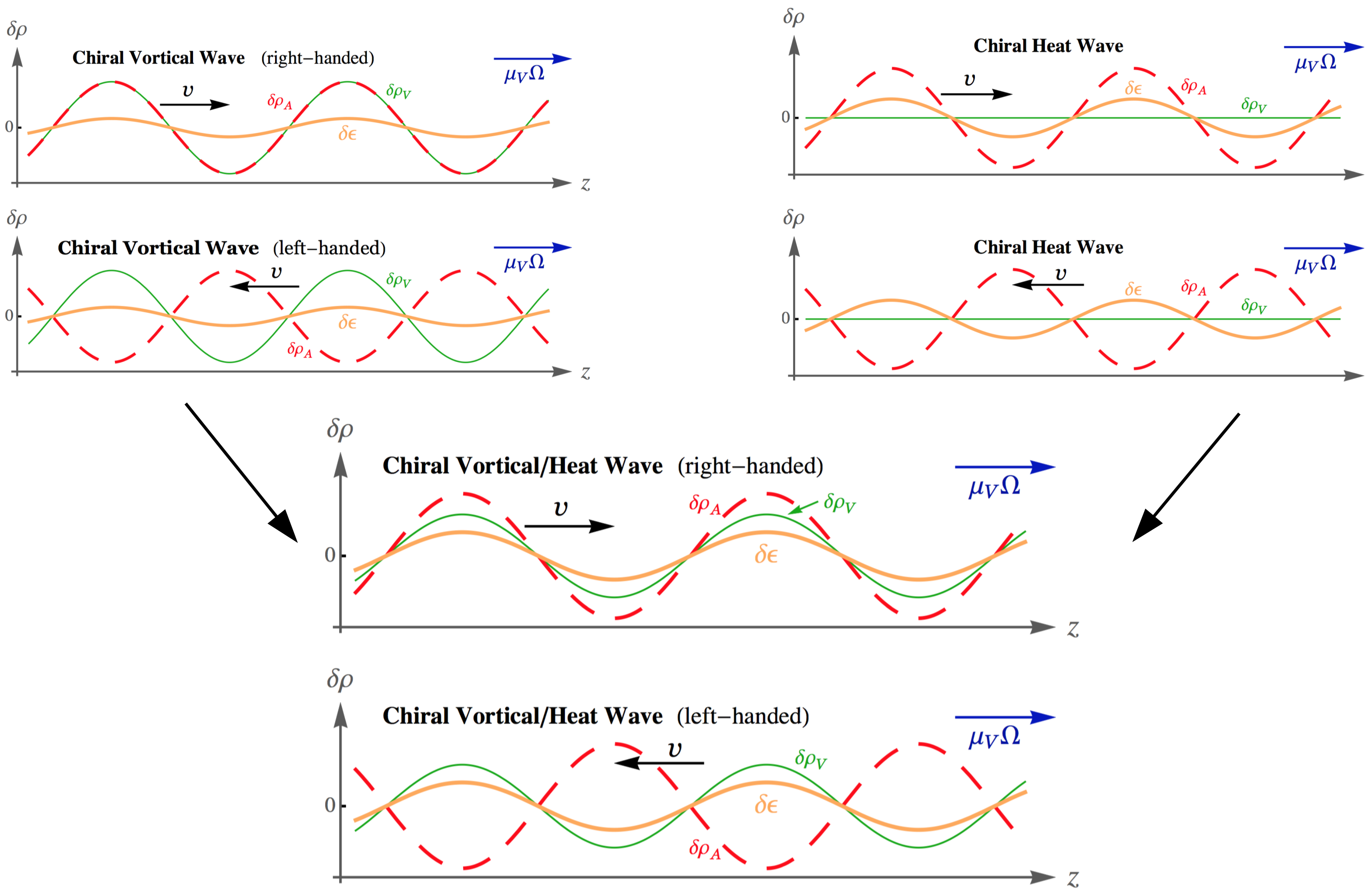
(6=2) creates fluctuation in energy flow
(7=3) etc etc etc.

$$\delta \boldsymbol{j}_A = \frac{T\boldsymbol{\Omega}}{3c_V}\,\delta\epsilon$$

$$\delta \boldsymbol{j}_E = \frac{T^2\boldsymbol{\Omega}}{3\chi}\,\delta\rho_A$$

The Chiral Heat Wave is again a chain-like process.

Mixing of chiral waves: example



$$v_{\rm VH} = \Omega \sqrt{\frac{\mu_V^2}{\pi^4 \chi^2} + \frac{T}{3 c_V \chi} \left(\frac{T^2}{3} + \frac{\mu_V^2}{\pi^2} \right)} \equiv \sqrt{v_{\rm CVW}^2 + \left[1 + 3 \left(\frac{\mu_V}{\pi T} \right)^2 \right] v_{\rm CHW}^2} \,.$$

[M.Ch. JHEP01(2016)100]

One-chirality media: waves

Simplest example: chiral fluid in a weak magnetic field at finite temperature and density

Currents and chemical potentials:

$$\boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B}, \quad \boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}, \quad \boldsymbol{j}_E = \frac{\mu_V \mu_A}{2\pi^2} e\boldsymbol{B}$$

$$\longrightarrow$$

$$\boldsymbol{j}_R = \frac{\mu_R}{4\pi^2} e\boldsymbol{B}\,, \qquad \boldsymbol{j}_L = -\frac{\mu_L}{4\pi^2} e\boldsymbol{B}\,,$$

$$\boldsymbol{j}_{E_R} = \frac{\mu_R^2}{8\pi^2} e\boldsymbol{B}\,, \qquad \boldsymbol{j}_{E_L} = -\frac{\mu_L^2}{8\pi^2} e\boldsymbol{B}\,.$$

$$\mu_V = \frac{1}{2}\left(\mu_R + \mu_L\right), \qquad \mu_A = \frac{1}{2}\left(\mu_R - \mu_L\right)$$

$$\boldsymbol{j}_V = \boldsymbol{j}_R + \boldsymbol{j}_L\,, \qquad \boldsymbol{j}_A = \boldsymbol{j}_R - \boldsymbol{j}_L\,, \qquad \boldsymbol{j}_E = \boldsymbol{j}_{E,R} + \boldsymbol{j}_{E,L}$$

One-chirality media: waves

Conservation of energy and charges (separately, for R- and L-handed)

$$\frac{\partial j_\ell^0}{\partial t} + \nabla \boldsymbol{j}_\ell = 0 \,, \qquad \ell = R, L, E_R, E_L$$

Energy and charge densities:

$$n_\ell \equiv j_\ell^0 = \frac{\mu_\ell}{6}\left(T^2 + \frac{\mu_\ell^2}{\pi^2}\right) ,$$

$$\varepsilon_\ell \equiv j_{E,\ell}^0 = \frac{7\pi^2 T^4}{120} + \frac{\mu_\ell^2 T^2}{4} + \frac{\mu_\ell^4}{8\pi^2} \,, \qquad \ell = R, L$$

One-chirality media: waves

Conservation relations in terms of chemical potentials and temperature:

$$\frac{\mu_\ell T_\ell}{3}\frac{\partial T_\ell}{\partial t} + \left(\frac{T_\ell^2}{6} + \frac{\mu_\ell^2}{2\pi^2}\right)\frac{\partial \mu_\ell}{\partial t} + \frac{eB_\ell}{4\pi^2}\frac{\partial \mu_\ell}{\partial z} = 0$$

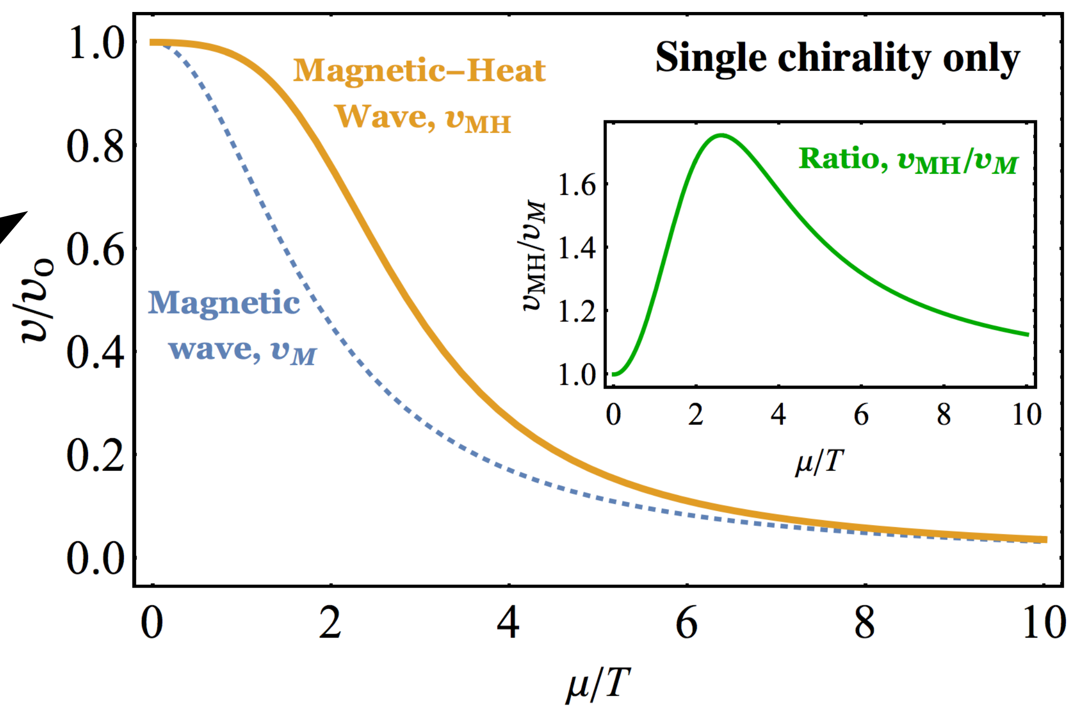
$$\left(\frac{7\pi^2 T_\ell^3}{30} + \frac{\mu_\ell^2 T_\ell}{2}\right)\frac{\partial T_\ell}{\partial t} + \left(\frac{\mu_\ell T_\ell^2}{2} + \frac{\mu_\ell^3}{2\pi^2}\right)\frac{\partial \mu_\ell}{\partial t} + \frac{eB_\ell \mu_\ell}{4\pi^2}\frac{\partial \mu_\ell}{\partial z} = 0$$

with right-handed/left-handed magnetic field:

$$B_\ell = (-1)^\ell B\,, \quad \ell = R, L$$

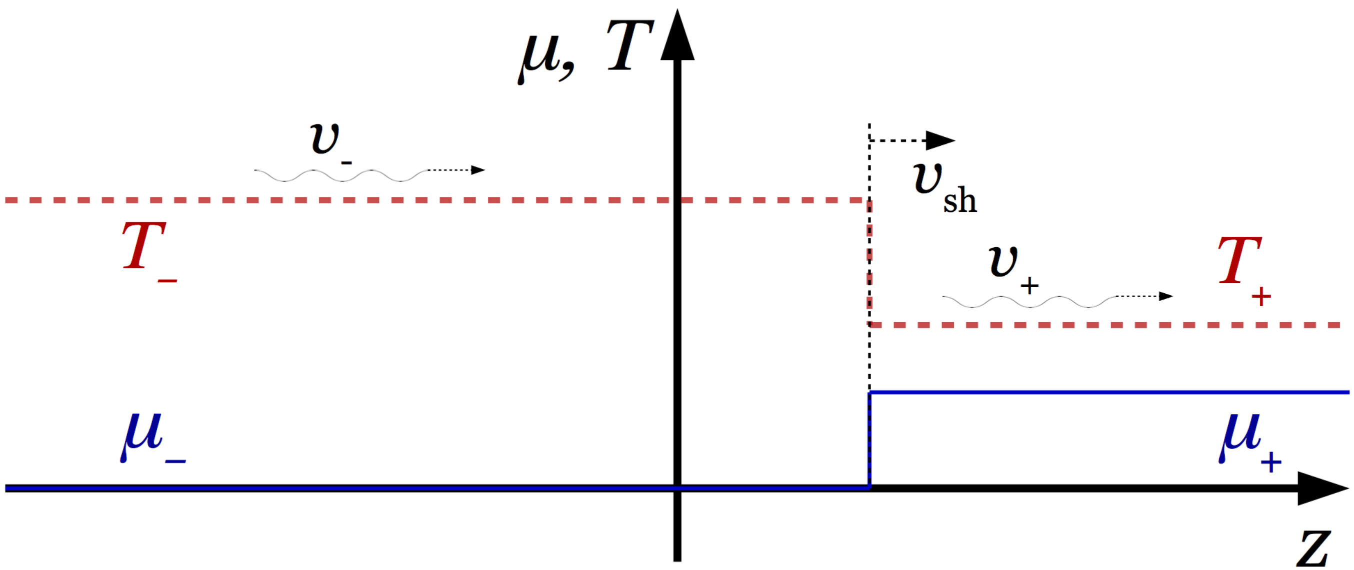
$$B_R = +B\,, \qquad B_L = -B$$

Smooth solution: Mixed Chiral Magnetic-Heat Wave



One-chirality media: shocks (kinks)

Idealized kink configuration:



We do not take into account finite viscosity, (thermal) conductivity and dissipation (relaxation) of the chiral charge:

→ nondissipative sharp shock front

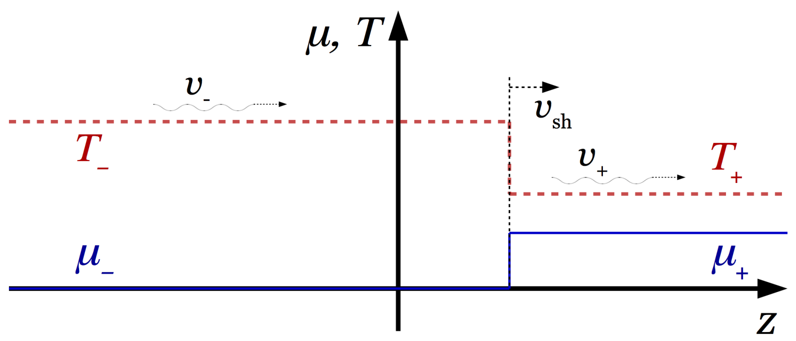
→ otherwise: finite-width (but still narrow) finite-propagation-time shock

One-chirality media: shocks (kinks)

Velocity of the shock can be calculated using the Rankine–Hugoniot conditions

$$v_{\rm sh}^Q = \frac{\mu_+ - \mu_-}{\mu_+(\pi^2 T_+^2 + \mu_+^2) - \mu_-(\pi^2 T_-^2 + \mu_-^2)} \frac{3eB}{2},$$

$$v_{\rm sh}^E = \frac{\mu_+^2 - \mu_-^2}{\left(\frac{7\pi^4}{15} T_+^4 + 2\pi^2 \mu_+^2 T_+^2 + \mu_+^4\right) - \left(\frac{7\pi^4}{15} T_-^4 + 2\pi^2 \mu_-^2 T_-^2 + \mu_-^4\right)} eB$$



Coming from the conservation equations:

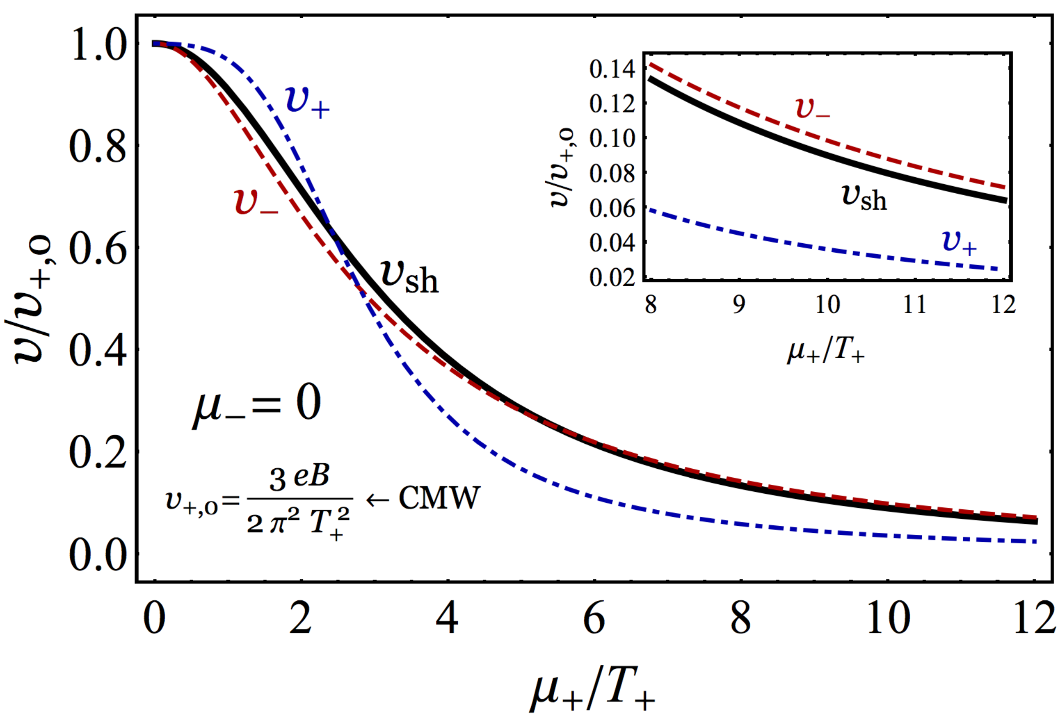
$$\frac{\partial}{\partial t}\left(\frac{\mu T^2}{6} + \frac{\mu^3}{6\pi^2}\right) + \frac{eB}{4\pi^2}\frac{\partial \mu}{\partial z} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{7\pi^2 T^4}{120} + \frac{\mu^2 T^2}{4} + \frac{\mu^4}{8\pi^2}\right) + \frac{eB}{8\pi^2}\frac{\partial \mu^2}{\partial z} = 0$$

Self-consistency condition:

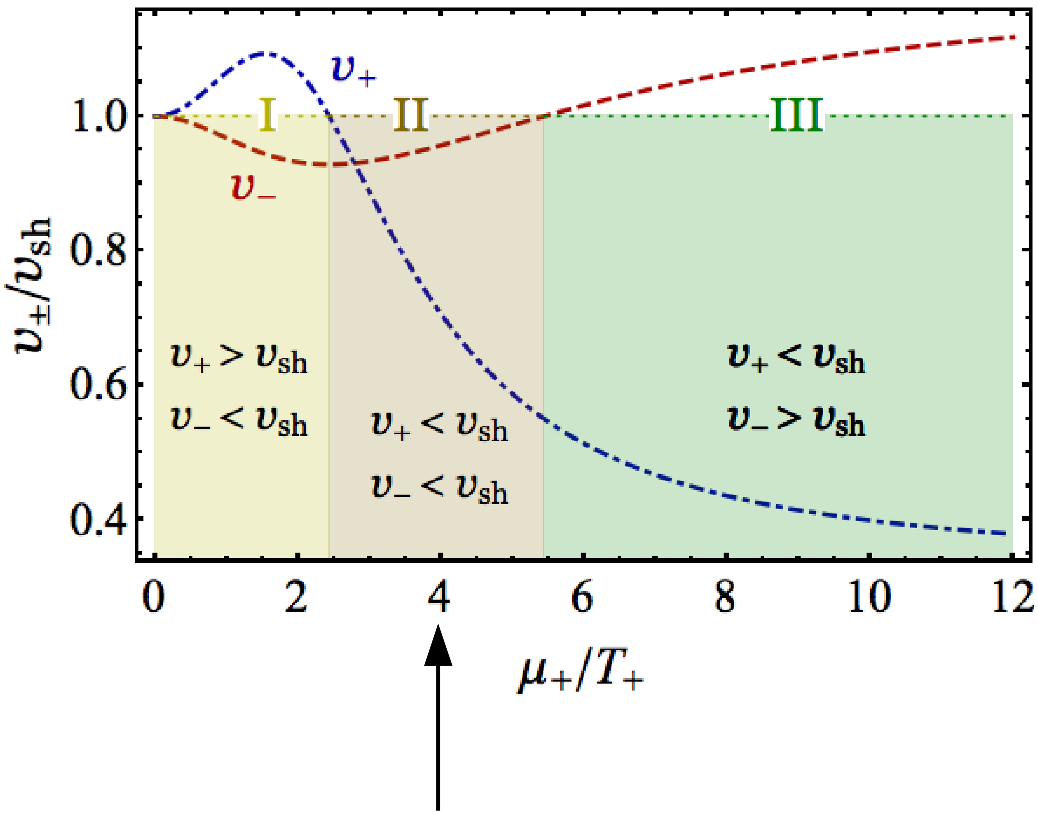
$$T_-(T_+, \mu_+) = \sqrt[4]{T_+^4 + \frac{5\mu_+^2}{7\pi^4}\left(4\pi^2 T_+^2 + \mu_+^2\right)}$$

To simplify illustration we set $\mu_- = 0$



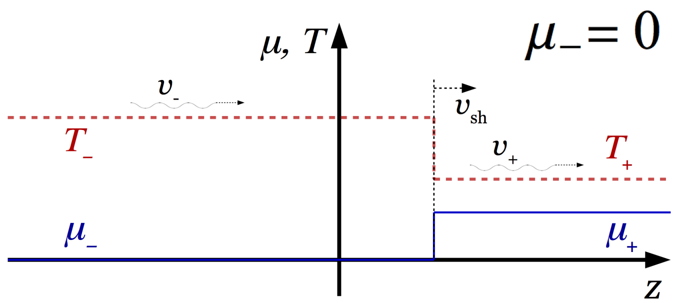
One-chirality media: shocks (kinks)

Shock velocity vs. speed of sound



Normal region: in usual fluids shocks are faster than sound.

Regions I and III are exotic.



Properties of shocks/kinks:

1. Propagate along magnetic field
2. May be slower than speed of sound
3. Shock converts low-density hot fluid into high-density cold fluid and vice-versa
4. Weak shock waves propagate with velocity of chiral magnetic heat wave
5. Fast small shocks may coalescence into slower bigger shocks.

$$\frac{\partial y}{\partial t} + y\frac{\partial y}{\partial x} = 0,$$

Chiral Burgers Kinks!

Chiral Burgers Kinks!

$$\frac{\partial y}{\partial t} + y\frac{\partial y}{\partial x} = 0$$

Inviscid Burgers' equation

Summary: chiral waves and kinks

- **Anomalous transport phenomena in chiral media (brief review)**

- **Anomalous chiral waves:**
  - coherent propagation of axial/vector/energy densities
  - along direction of magnetic field/axis of rotation (for pure waves)
  - velocity may be higher/lower than the speed of sound.

- **Examples:**
- Chiral Magnetic Wave (zero density, finite temperature, magnetic field)
- Chiral Vortical Wave (zero density, zero temperature, rotating fluid)
- Chiral Heat Wave (nonzero density, nonzero temperature, rotating fluid)
- All waves may mix with each other ("Chiral Magnetic-Vortical Wave" etc.).

- **Chiral kinks - shock waves in dense chiral media:**
  - exist in dense chiral media in magnetic field background
  - propagate along magnetic field
  - convert hot dilute fluid into cold dense fluid (and back)
  - exotic properties:
    - velocity may be lower than the speed of sound
    - higher density – lower velocity and vice versa etc