



Gary Gibbons, gravitational waves, and the LIGO observations of binary black holes.

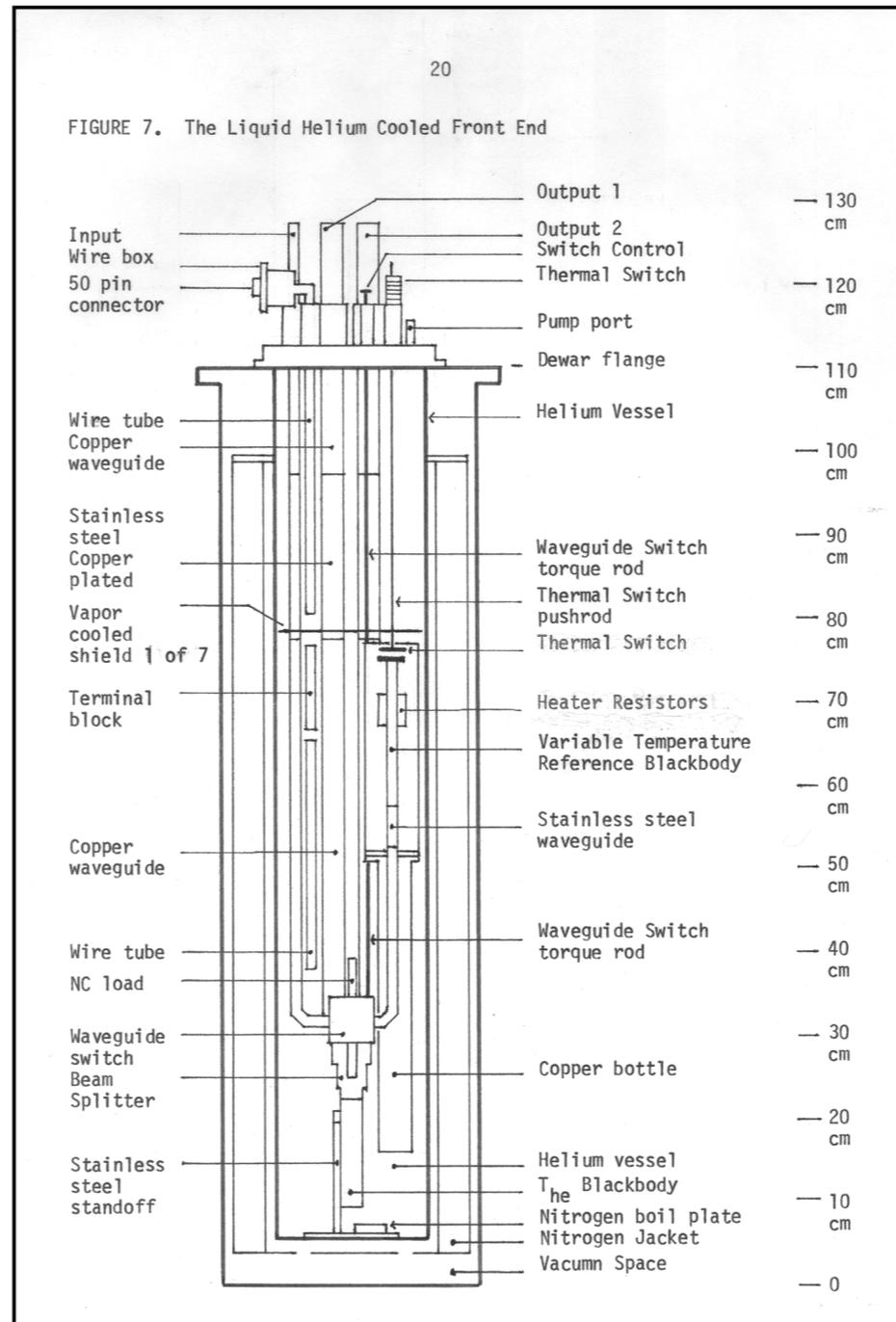
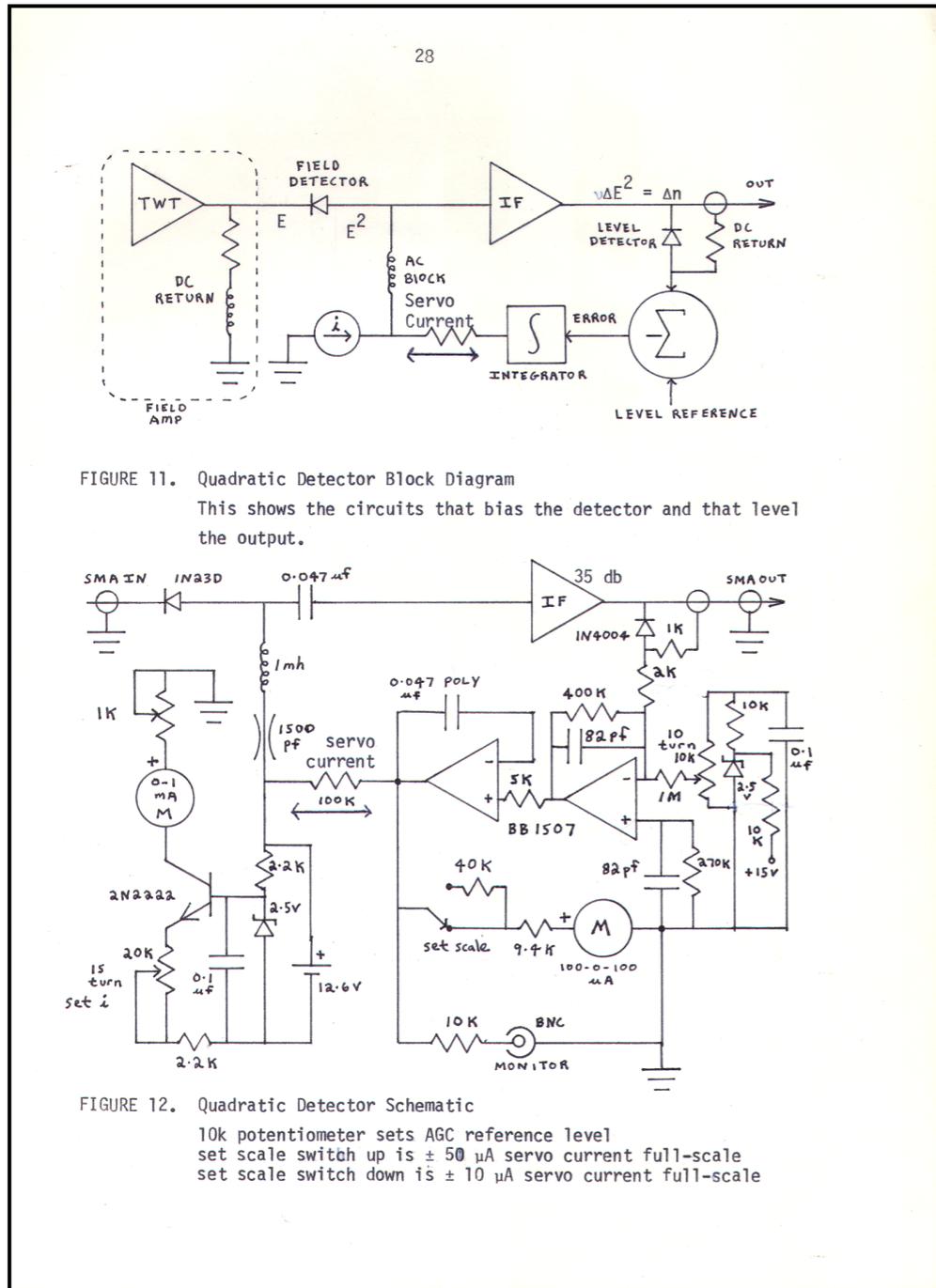


Bruce Allen

**Max Planck Institute for Gravitational Physics
(Albert Einstein Institute, AEI)**



MIT Undergraduate 1976-80

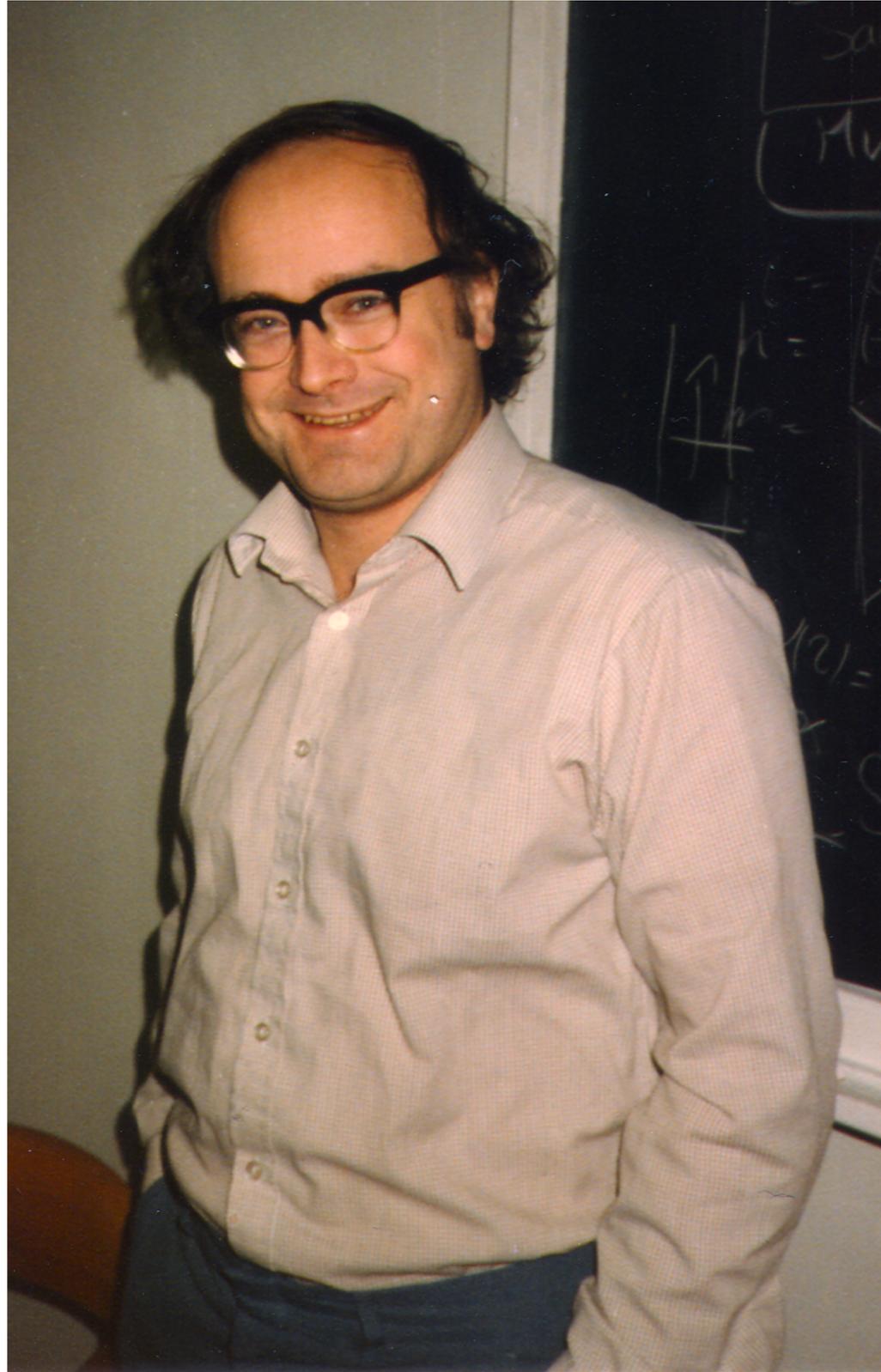


**DAMTP
October 1980**

Undergrad thesis, June 1980, CMB experiment



Cambridge 1980-83





EVIDENCE FOR DISCOVERY OF GRAVITATIONAL RADIATION*

J. Weber

Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

(Received 29 April 1969)

Coincidences have been observed on gravitational-radiation detectors over a base line of about 1000 km at Argonne National Laboratory and at the University of Maryland. The probability that all of these coincidences were accidental is incredibly small. Experiments imply that electromagnetic and seismic effects can be ruled out with a high level of confidence. These data are consistent with the conclusion that the detectors are being excited by gravitational radiation.

Some years ago an antenna for gravitational radiation was proposed.¹ This consists of an elastic body which may become deformed by the dynamic derivatives of the gravitational potentials, and its normal modes excited. Such an antenna measures, precisely, the Fourier transform of certain components of the Riemann curvature tensor, averaged over its volume. The theory has been developed rigorously, starting with Einstein's field equations to deduce² equations of motion. Neither the linear approximation nor the energy-flux relations are needed to describe these experiments, but their use enables discussion in terms of more familiar quantities. All aspects of the antenna response and signal-to-noise ratio can be written in terms of the curvature tensor. The theory was verified experimentally by developing a high-frequency source³ and producing and detecting dynamic gravitational fields in the laboratory.

Several programs of research are being con-

array is a new set of windows for studying the universe.

Search for gravitational radiation in the vicinity of 1660 Hz.—A frequency in the vicinity of 1660 Hz was selected because the dimensions are convenient for a modest effort and because this frequency is swept through during emission in a supernova collapse. It was expected that once the technology was refined, detectors could be designed for search for radiation from sources with radio or optical emission, such as the pulsars. A knowledge of the expected frequency and Q of a source enormously increases the probability of successful search.

However, occasional signals were seen at 1660 Hz and small numbers of coincidences were observed on detectors^{7, 8} separated by a few kilometers. To explore these phenomena further, larger detectors were developed. One of these is now operating at Argonne National Laboratory.

My definition of a coincidence is that the most

GRAVITATIONAL RADIATION EXPERIMENTS*

J. Weber

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(Received 8 September 1969)

A summary is given of the statistics and coincidences of the Argonne-Maryland gravitational-radiation-detector array. New experiments have been carried out. These include a parallel coincidence experiment in which one coincidence detector had a time delay in one channel and a second coincidence detector operated with no time delays. Other experiments involve observations to rule out the possibility that the detectors are being excited electromagnetically. These results are evidence supporting an earlier claim that gravitational radiation is being observed.

An earlier Letter¹ described an experiment involving coincidences of gravitational radiation detectors at Argonne National Laboratory and the University of Maryland. This is the first experiment which tests directly the dynamics of gravitational fields. The results may have significance for physics, astronomy, and cosmology. Further experiments were therefore carried out to verify claims that the coincidences were not all accidental and that neither seismic nor electromagnetic effects were causing them.

Statistics.— Each gravitational-radiation-detector voltage output consists primarily of the thermal fluctuations of the suspended cylinder's lowest frequency compressional mode. A coincidence is recorded if the output voltages of two detectors cross some arbitrarily set threshold in the positive direction within some small time interval Δt . A classification scheme is set up for the coincidences. For each class the number of observed coincidences is compared with the expected number of accidental ones. A signifi-



May 1971



The Detection of Gravitational Waves, by Joseph Weber

The existence of such waves is predicted by the theory of relativity. Experiments designed to detect them have recorded evidence that they are being emitted in bursts from the direction of the galactic center”





DAMTP, Silver Street, Cambridge





Theory of the Detection of Short Bursts of Gravitational Radiation

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Department of Applied Mathematics and Theoretical Physics, University of Cambridge, England

and

S. W. Hawking

Institute of Theoretical Astronomy, University of Cambridge, England

(Received 30 November 1970)

It is argued that the short bursts of gravitational radiation which Weber reports most probably arise from the gravitational collapse of a body of stellar mass or the capture of one collapsed object by another. In both cases the bulk of the energy would be emitted in a burst lasting about a millisecond, during which the Riemann tensor would change sign from one to three times. The signal-to-noise problem for the detection of such bursts is discussed, and it is shown that by observing fluctuations in the phase or amplitude of the Brownian oscillations of a quadrupole antenna one can detect bursts which impart to the system an energy of a small fraction of kT . Applied to Weber's antenna, this method could improve the sensitivity for reliable detection by a factor of about 12. However, by using an antenna of the same physical dimensions but with a much tighter electromechanical coupling, one could obtain an improvement by a factor of up to 250. The tighter coupling would also enable one to determine the time of arrival of the bursts to within a millisecond. Such time resolution would make it possible to verify that the radiation was propagating with the velocity of light and to determine the direction of the source.

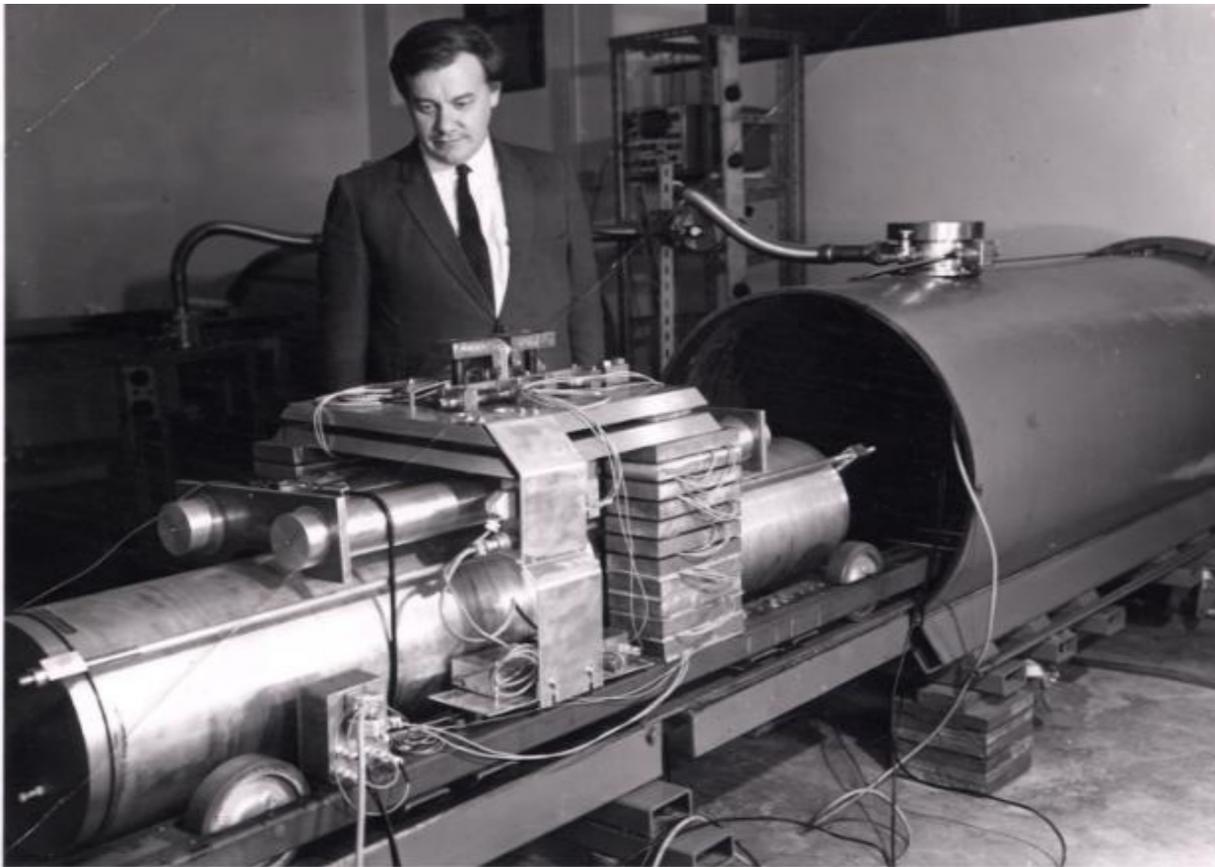
I. INTRODUCTION

In this paper we discuss the problem of detecting short bursts of gravitational radiation. This is rather different from the detection of continuous

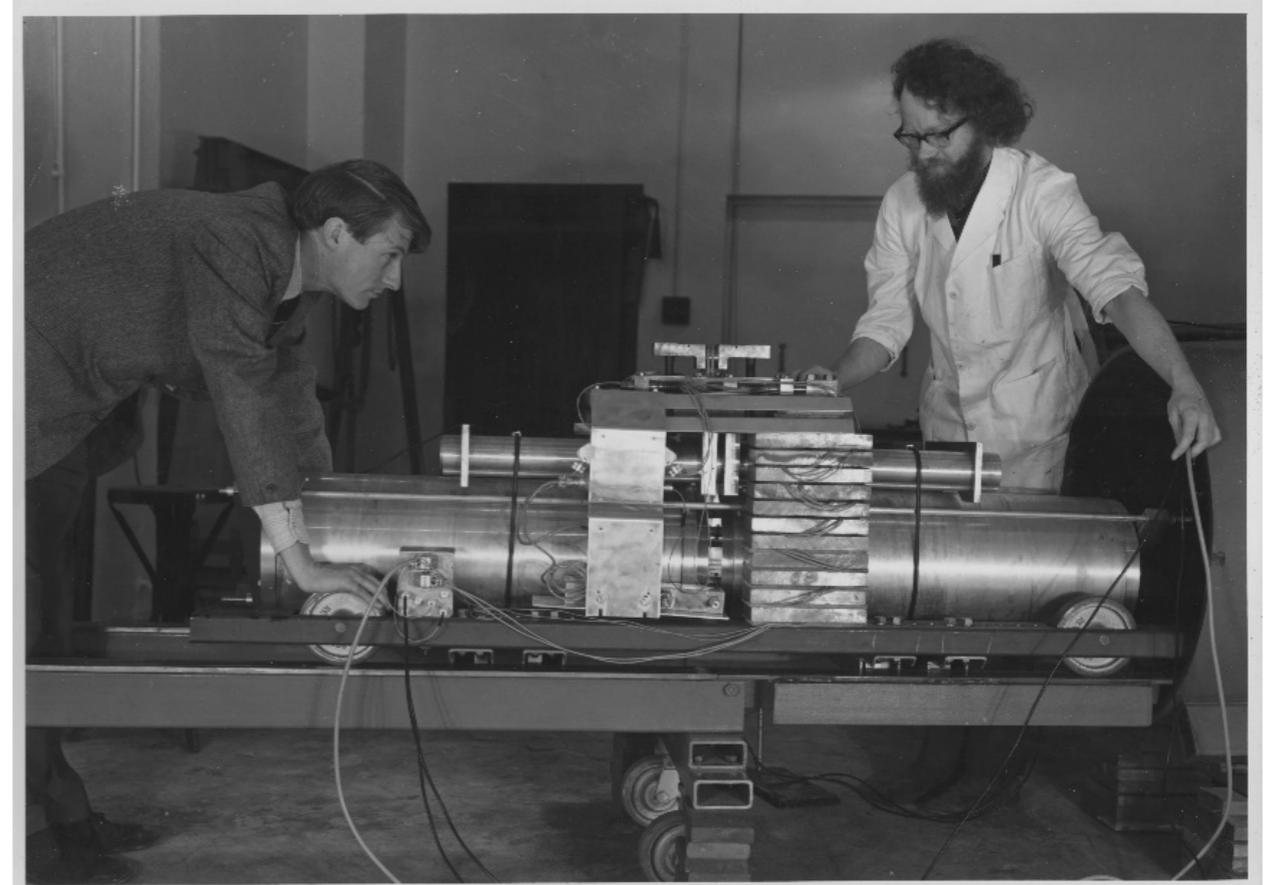
like Weber's, which has a very low electromechanical coupling, this method would improve the sensitivity for reliable detection by a factor of about 12. However, by using a detector consisting of two metal bars connected by a piezoelectric trans-



Glasgow, 1971



Ron Drever



Jim Hough (L) and Stuart Cherry (R)



Gary's first publications, PhD thesis

- Gibbons and Hawking, *Theory of the Detection of Short Bursts of Gravitational Radiation*, PRD4 2191–2197 (1971).
Cited 64 times, including Astone, Billing, Blair, Caves, Dewey, Drever, Hamilton, Hough, Isaacson, Lobo, Michelson, Misner, Pizzella, Press, Ruffini, Sathyaprakash, Saulson, Schutz, Thorne, Trimble, Vinet, Weber, Winkler
- Gibbons, *Detection of Long Period Gravitational Radiation*, Nature Physical Science 230, 113 (1971).
Cited 2 times
- Gibbons and Hawking, *Evidence for Black Holes in Binary Star Systems*, Nature 232, 465–466 (1971).
Cited 15 times
- PhD Thesis (1972): *Some Aspects of Gravitational Radiation and Gravitation Collapse*



- Focus on merger of compact objects (mentions “collapsed objects” and “neutron stars” but does not contain the words “black hole”)
- Correct time-scales and energy estimates (msec per solar mass) when objects approach $O(\text{Schwarzschild radius})$
- Concept of matched filtering (not by that name) to “dig into the noise”. Hence $\times 12$
- Precision of arrival-time determination, use of triangulation to determine direction to source
- Does not specifically discuss orbital behaviour (head-on collision?)
- Some amusing typos (“Earth orbiting around the sun radiates 1kW at a frequency of 3 cycles/year.”)

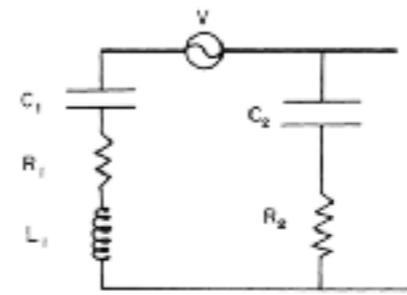


FIG. 4. The equivalent circuit of the detector and transducer.

$$L_1 = (\omega_0^2 \beta C_2)^{-1},$$

$$C_1 = \beta C_2,$$

$$R_1 = (Q \omega_0 \beta C_2)^{-1},$$

and

$$V = -c^2 I R_{1010} (2\omega_0)^{-1} m^{1/2} (\beta C_2)^{-1/2}.$$

The equivalent circuit of the detector and transducer is therefore given by Fig. 4.¹³

We shall assume that the output of this equivalent circuit is fed into a high-impedance amplifier with gain A and that the amplifier output is divided in the ratio Z_3/Z_4 (Fig. 5). In this figure Z_1 represents the impedance of the series L_1, C_1, R_1 and Z_2 represents the impedance of the series C_2, R_2 . The impedances Z_3 and Z_4 are chosen as $Z_3 = D(Z_1 + Z_2)^{-1}$ and $Z_4 = HZ_2^{-1}$, where D and H are constant with $D \gg H$. The Z_3, Z_4 circuit “undoes” the effect of the resonance of the detector and gives an output signal voltage

$$V_2 = AHD^{-1} V$$

$$= -AHD^{-1} c^2 I R_{1010} (2\omega_0)^{-1} m^{1/2} (\beta C_2)^{-1/2}.$$

It is not necessary to use such an inverse circuit but it is convenient for discussing the signal/noise ratio. The impedances Z_3 and Z_4 could be realized physically by a parallel LCR circuit and a parallel LR circuit respectively, though one might need to use a superconducting inductance in Z_3 to obtain a sufficiently high Q . It would probably be more convenient to simulate the Z_3, Z_4 circuit electronically.

Superimposed on the output signal voltage will be the Johnson noise produced by R_1 which represents the Brownian noise of the detector. This will produce at the output a flat noise spectrum with a mean-square voltage

$$V_n^2 = (AHD^{-1})^2 2kT\tau^{-1} (Q\omega_0\beta C_2)^{-1}$$

per unit bandwidth. The transducer noise produced by the resistance R_2 will give at the output a mean-

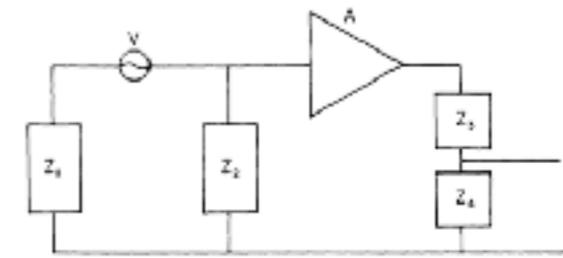


FIG. 5. The equivalent circuit for discussing the signal/noise ratio.

square voltage of

$$V_p^2 = (AHD^{-1})^2 2kT\tau^{-1} \tan \delta (\omega_0 C_2)^{-2} |Z_3|^2 |Z_2|^{-2}$$

$$= (AHD^{-1})^2 2kT\tau^{-1} \tan \delta (\omega_0 C_2)^{-1} (\beta \omega_0)^{-2}$$

$$\times [\omega^2 Q^{-2} + \omega_0^{-2} (\omega^2 - \omega_0^2)^2]$$

per unit bandwidth. This has a sharp minimum at the resonant frequency ω_0 . The noise produced by the amplifier will have a rather similar spectrum at the output. Using modern techniques, it seems possible to reduce the amplifier noise below the transducer noise and it will be neglected.

Suppose now that the output of the circuit in Fig. 5 is fed into a filter of bandwidth $\Delta\omega$. If the signal is of the form suggested in Sec. 2, i.e., a burst of one to three cycles, its Fourier transform will have a maximum at a frequency ω_1 of the order of $2\pi\tau^{-1}$ and a half-width of the same order. Therefore, if the filter pass band is centered at ω_1 , the amplitude of the transmitted signal will be $V_2 \Delta\omega \omega_1^{-1}$ and the power will be $V_2^2 (\Delta\omega)^2 \omega_1^{-2}$ for $\Delta\omega \ll \omega_1$. This behavior distinguishes short bursts from continuous incoherent radiation where the power is proportional to $\Delta\omega$. It is the reason why it is desirable to use a fairly large value of $\Delta\omega$, i.e., good time resolution.

If the resonant frequency ω_0 is chosen to be equal to ω_1 ,¹⁴ the filter will transmit a Brownian noise power approximately equal to

$$(AHD^{-1})^2 2kT\tau^{-1} (Q\omega_0\beta C_2)^{-1} \Delta\omega$$

and a transducer noise power

$$(AHD^{-1})^2 kT(3\pi)^{-1} \tan \delta \beta^{-2} C_2^{-2} (\Delta\omega)^2 \omega_0^{-2}.$$

The optimum value of $\Delta\omega$ will be the smaller of ω_0 and the value for which the transmitted noise power equals the transmitted transducer noise power. This gives

$$(\Delta\omega \omega_0^{-1})^2 = 6\beta(Q \tan \delta)^{-1}$$

which agrees almost exactly with Eq. (13). With this value of $\Delta\omega$ one could detect against the noise a short burst in which the amplitude of R_{1010} was



**Fast-forward 45 years,
from 1971 to 2016...**

February 12, 2016

Gravitational waves: the discovery which shows that Einstein was right





- On 14 September 2015, 4 days *before* starting its first observational run O1, Advanced LIGO recorded a strong gravitational wave burst
- Source unambiguous. In source frame: **merger of a 29 and 36 solar mass BH**
- What did we see?
How can we be sure it is real?
What was going on “behind the scenes”?
What do we learn?
Other discoveries from O1
- Status of O2, prospects for the future

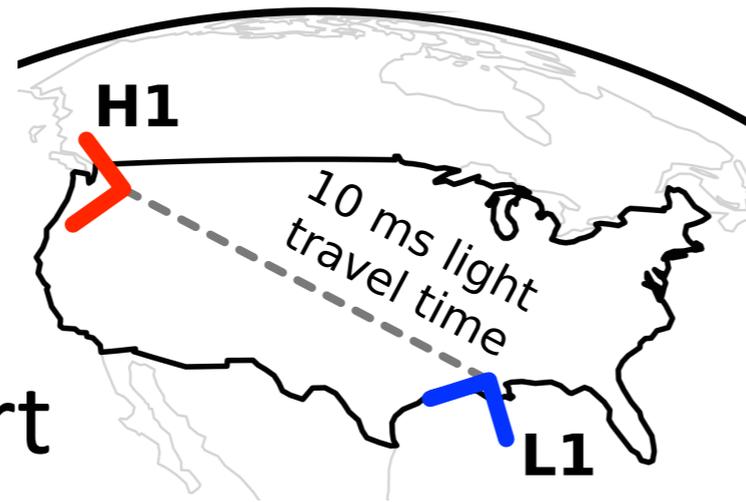
References: PRL 116, 061102 (2016);
PRX 6, 041015 (2016); Annalen der
Physik, 529, 1600209 (2017).



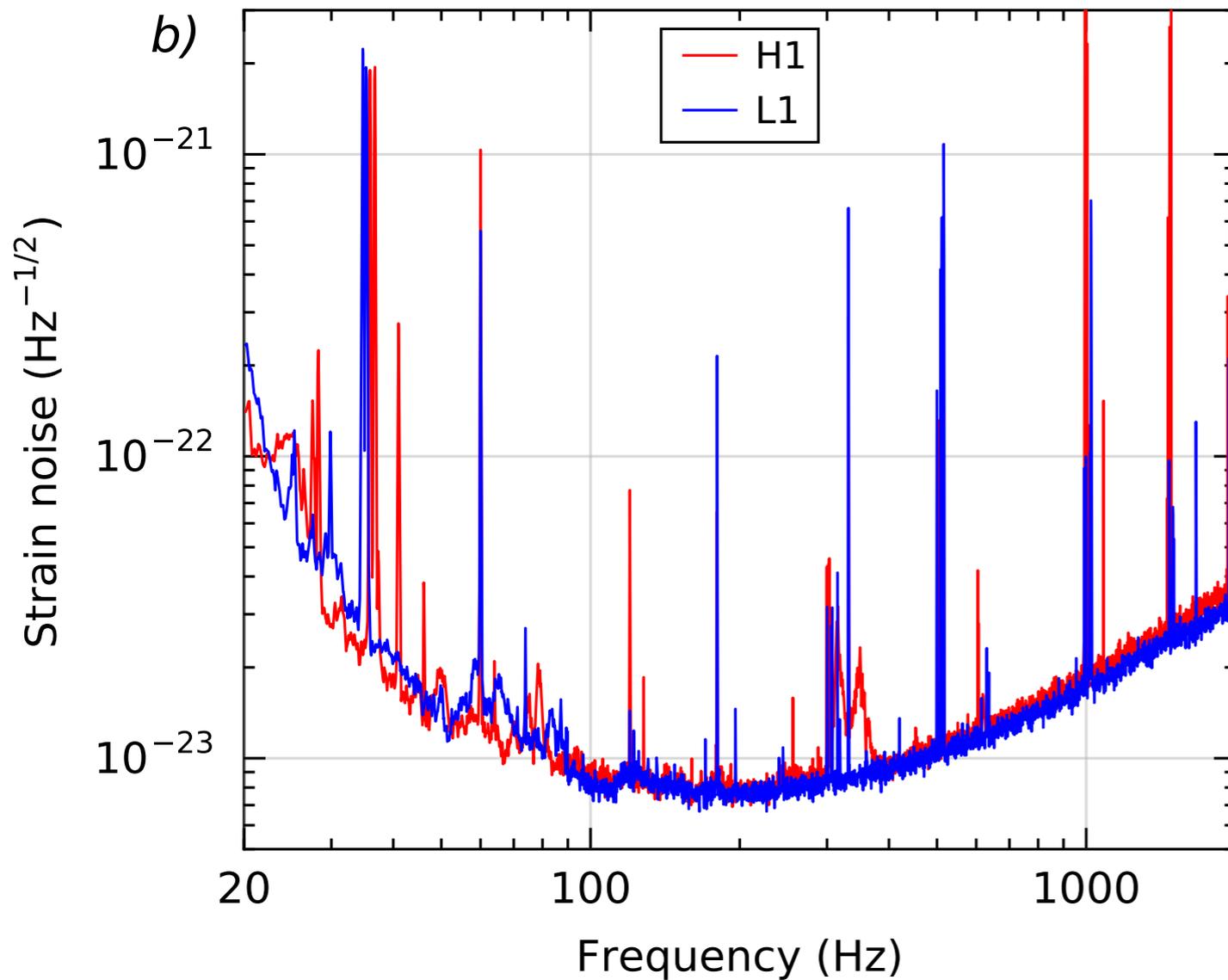


Advanced LIGO Detectors

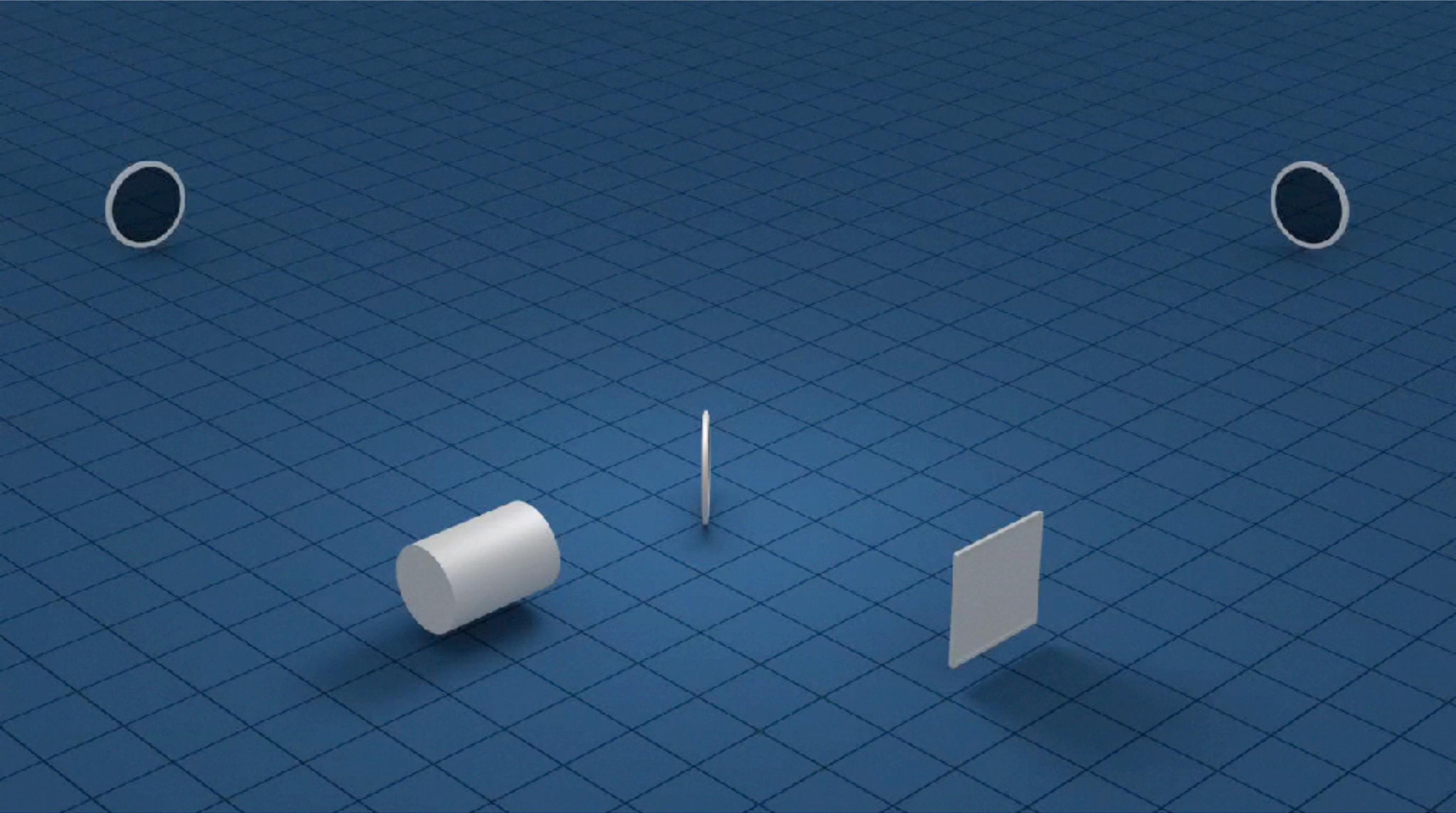
Livingston &
Hanford
3000 km apart



$\Delta L/L$



- Sensitive from 30 to 2000 Hz
- Strain $h = \Delta L/L$
- In 100 Hz band at minimum, r.m.s. noise $h \sim 10^{-22}$
- O1 noise a factor ~ 3 above design sensitivity



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GW150914

- Engineering run had begun 17 August 2015, for tuning, calibration, injection tests, and noise characterisation studies.
- First observing run O1 (“science operations mode”) scheduled to start on **18 September 2015**
- **VIRGO** not operating (under construction)
GEO-600 lost lock 10 minutes before (AND not sufficiently sensitive at low frequencies)
- **Event at 09:50 UTC on 14 September 2015, four days before planned O1 start**
02:50 at LIGO Hanford, WA
04:50 at LIGO Livingston, LA
11:50 in Germany



AEI Hannover, September 14, 2015



Marco Drago



Andrew Lundgren

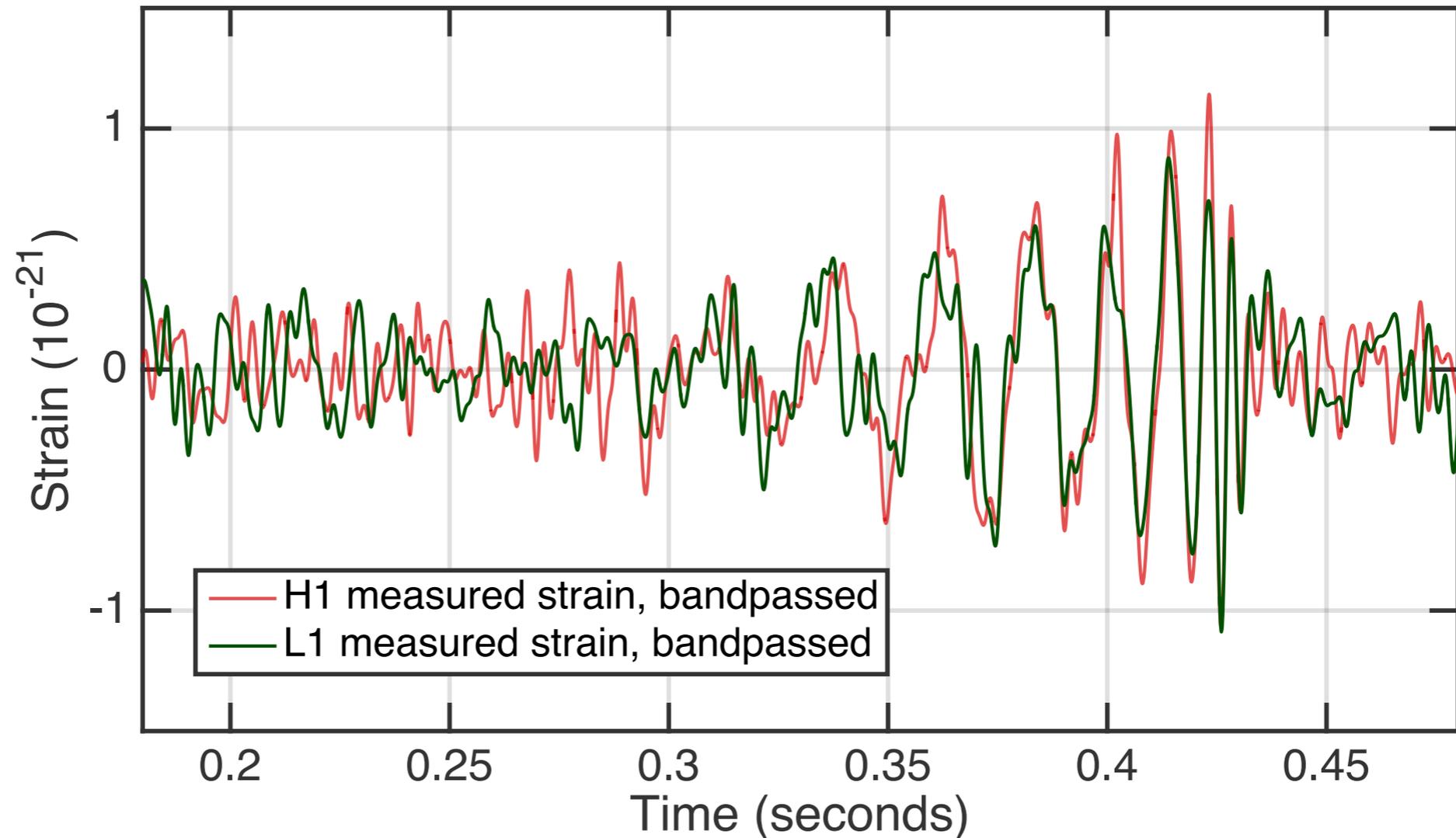
- Monday morning 11:50
- Coherent waveburst pipeline running at Caltech, event database had ~1000 entries
- Marco and Andy checked injection flags and logbooks, data quality, made Qscans of LHO/LLO data.
- Called LIGO operators: “everyone’s gone home”

- At 12:54, Marco sent an email to the collaboration, asking for confirmation that it’s not a hidden test signal (hardware injection)
- Next hours: flurry of emails, decision to lock down sites, freeze instrument state



The Chirp

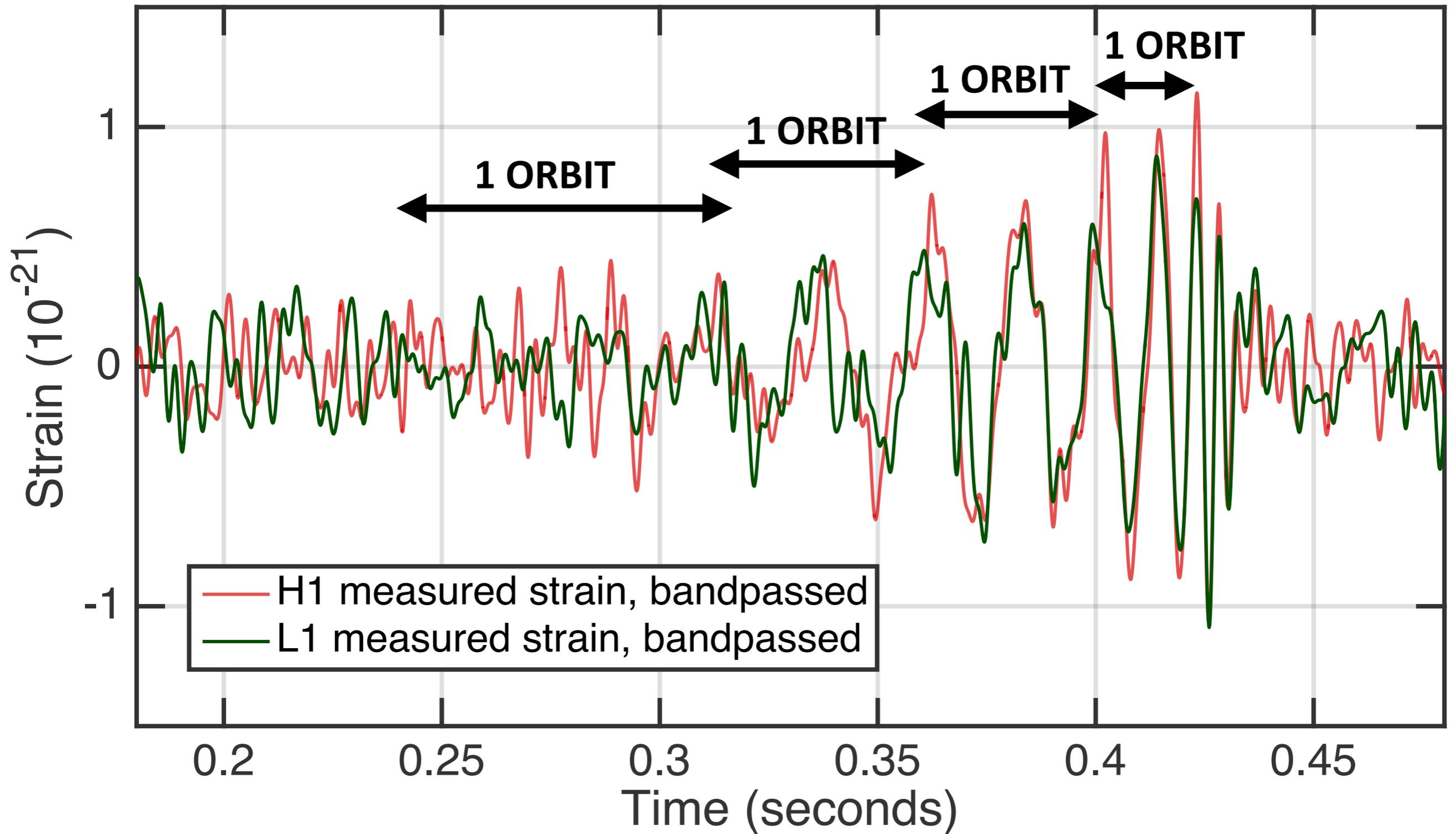
$\Delta L/L$



- Bandpass filtered 35-350 Hz, some instrumental and calibration lines removed
- Hanford inverted, shifted 7.1 ms earlier
- Signal visible to the naked eye: ~ 200 ms
- “Instantaneous” SNR ~ 5 , optimal filter SNR ~ 24

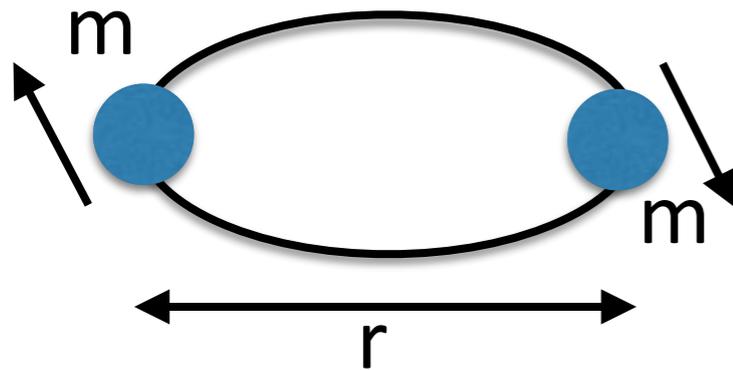


The Chirp





Gravitational waves from orbiting masses



orbital angular
frequency ω

$$\text{Newton : } \frac{Gm^2}{r^2} = m\omega^2 \left(\frac{r}{2}\right) \Rightarrow r^3 = \frac{2Gm}{\omega^2}$$

$$E_{\text{mechanical}} = \frac{1}{2}m\left(\frac{\omega r}{2}\right)^2 + \frac{1}{2}m\left(\frac{\omega r}{2}\right)^2 - \frac{Gm^2}{r} = -\frac{Gm^2}{2r} = -\frac{G^{2/3}m^{5/3}}{2^{4/3}}\omega^{2/3}$$

$$\text{GW Luminosity} = \frac{G}{5c^5} \left(\frac{d^3}{dt^3} Q_{ab}\right) \left(\frac{d^3}{dt^3} Q_{ab}\right) = \frac{8G}{5c^5} m^2 r^4 \omega^6 = \frac{2^{13/3} G^{7/3} m^{10/3}}{5c^5} \omega^{10/3}$$

$$\text{GW Luminosity} = -\frac{d}{dt} E_{\text{mechanical}} = \frac{G^{2/3} m^{5/3}}{3 \cdot 2^{1/3}} \omega^{-1/3} \frac{d\omega}{dt}$$

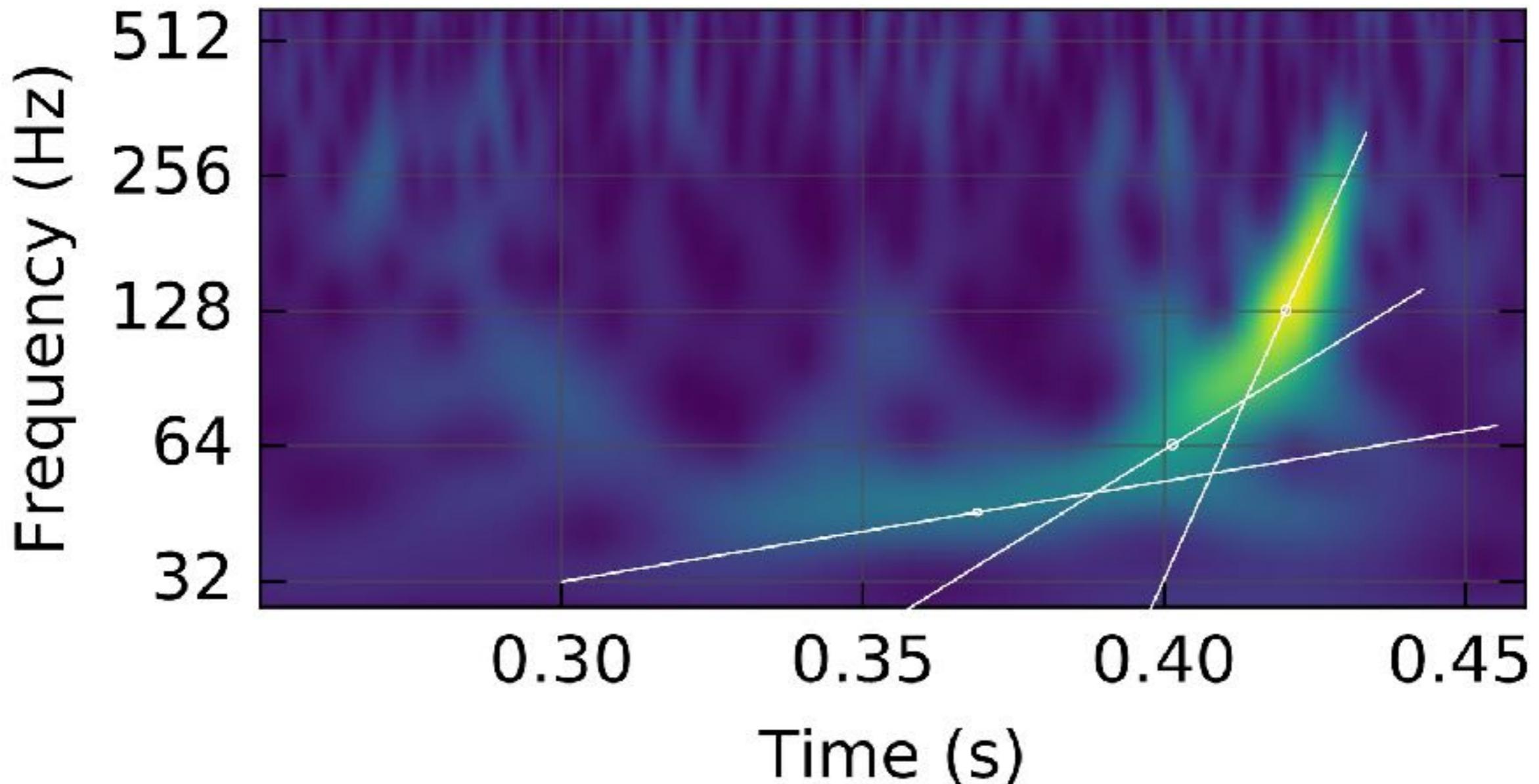
$$\frac{d\omega}{dt} = \frac{3 \cdot 2^{14/3} G^{5/3} m^{5/3}}{5c^5} \omega^{11/3}$$

get mass from
frequency and its
rate of change!



Masses from the rate of frequency increase

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G} \left[\frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right]^{3/5} = 30 M_{\odot}$$





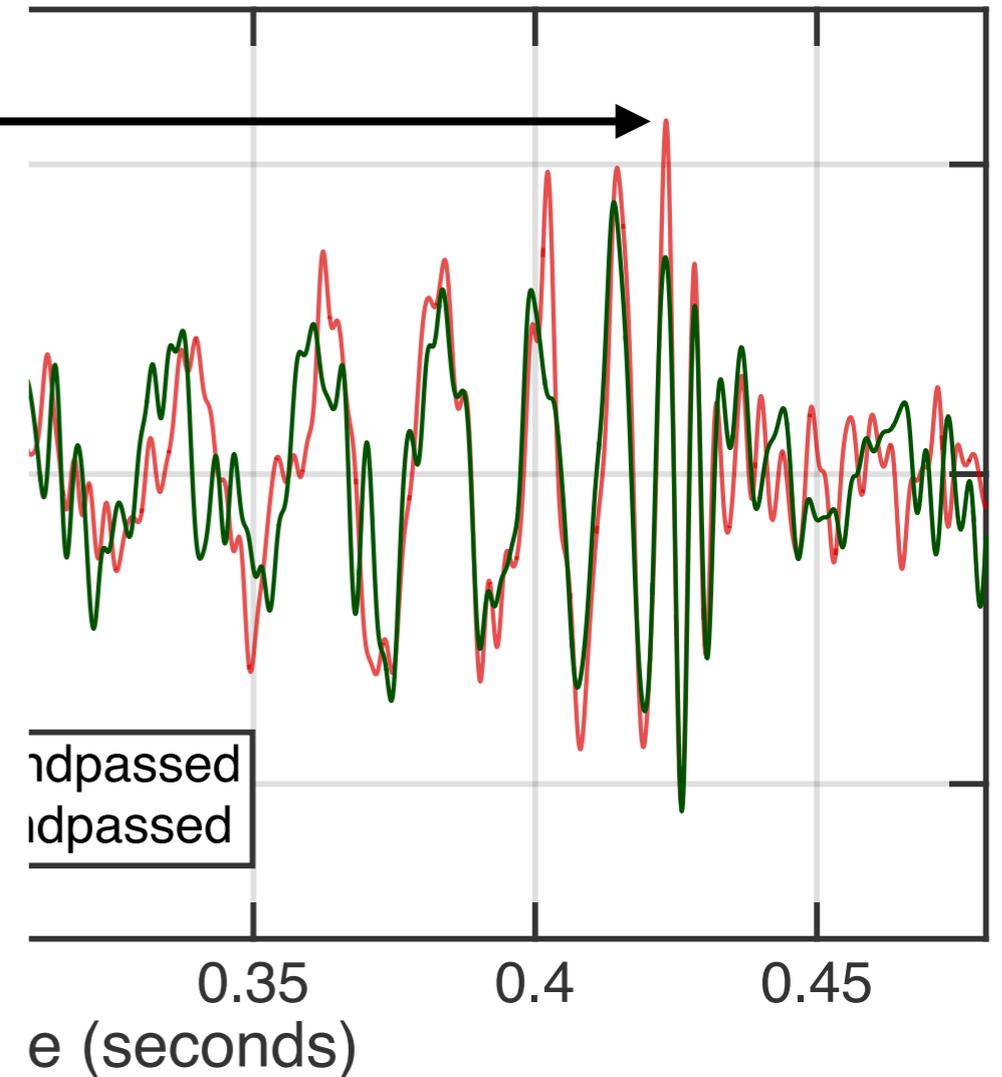
Can only be two black holes!

- Chirp mass $\mathcal{M} \sim 30 M_{\odot} \Rightarrow$

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G} \left[\frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right]^{3/5}$$

Sum of Schwarzschild radii $\geq 206 \text{ km}$

- At peak $f_{\text{GW}} = 150 \text{ Hz}$, orbital frequency = 75 Hz separation of Newtonian point masses 346 km
- **Ordinary stars** are 10^6 km in size (merge at mHz). **White dwarfs** are 10^4 km (merge at 1 Hz). They are too big to explain this!
- **Neutron stars** are also not possible:
 $m_1 = 4 M_{\odot} \Rightarrow m_2 = 600 M_{\odot}$
 \Rightarrow Schwarzschild radius $1800 \text{ km} \Rightarrow$ too big!



Only black holes are heavy enough and small enough!



Real? Or a detector artifact?

- Instruments in normal operation and stable since September 12th, 2015 (apart from deliberate intervention)
- aLIGO can see such sources at 6 times the distance => $6 \times 6 \times 6 \sim 200$ times the rate at initial LIGO instruments
- Last scientists left sites 2 hours (LHO) and 15 minutes (LLO) before the event. Operators only.
- Waveform does not resemble instrumental glitches or artefacts
- Susceptibility to radio, acoustic, magnetic, seismic and other external disturbances measured. These external disturbances are monitored: can not explain more than 6% of the observed GW amplitude
- Was not an accidental or malicious hardware injection: recorded control loop signals permit reconstruction of the actuators: no fake signal was added



Stefan Ballmer and Evan Hall, departed the LHO site soon after midnight, **2 hours before the event**



Robert Schofield and Anamaria Effler, departed the LLO site at 04:35am **15 minutes before the event**



Random Noise?



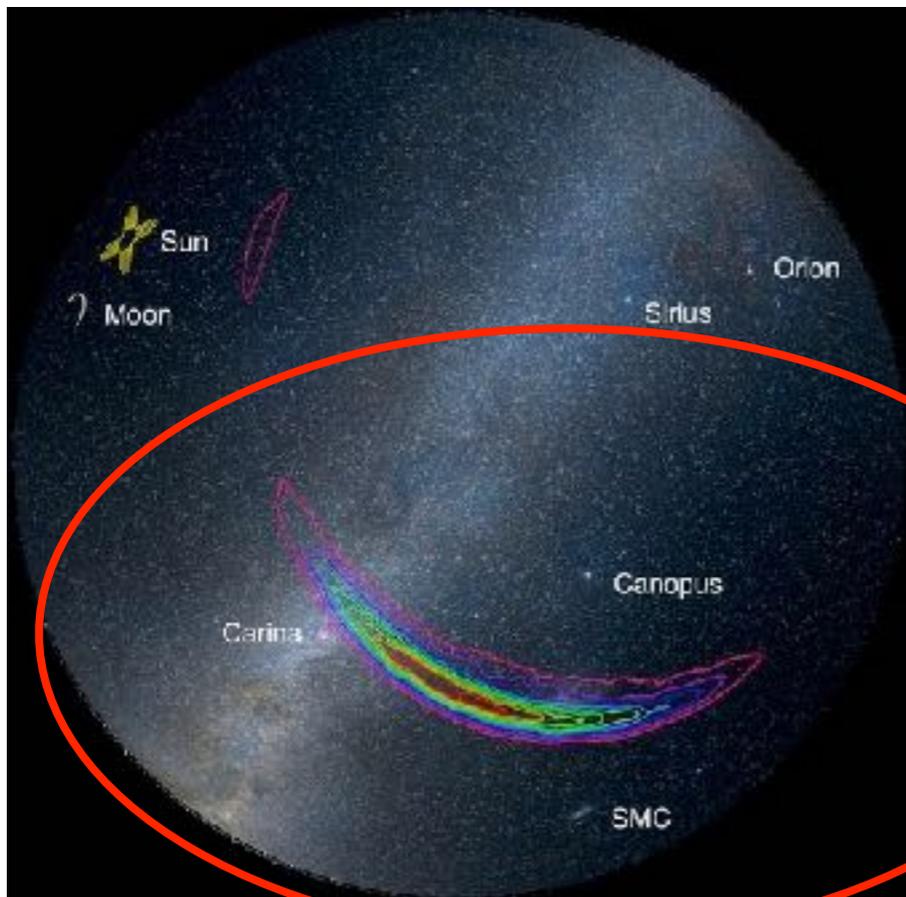
Much longer than 200,000 years before noise in the detector would mimic this signal, or a similar signal of the types that we search for.



Parameters from fitting (in source frame)

Primary black hole mass	$36_{-4}^{+5} M_{\odot}$
Secondary black hole mass	$29_{-4}^{+4} M_{\odot}$
Final black hole mass	$62_{-4}^{+4} M_{\odot}$
Final black hole spin	$0.67_{-0.07}^{+0.05}$
Luminosity distance	$410_{-180}^{+160} \text{ Mpc}$
Source redshift, z	$0.09_{-0.04}^{+0.03}$

- Radiated energy: $3M_{\odot}$ (± 0.5)
- Peak luminosity: $3.6 \times 10^{56} \text{ erg/s}$ ($\pm 15\%$): 200 solar masses per second! (About $1 \mu\text{W}/\text{cm}^2$ at detector, $\sim 10^{12}$ millicrab!)
- Spins s_1 and s_2 only weakly constrained: not extreme. Consistent with merger of two non-spinning black holes.
- Final spin of 0.67 is about 6000 rpm

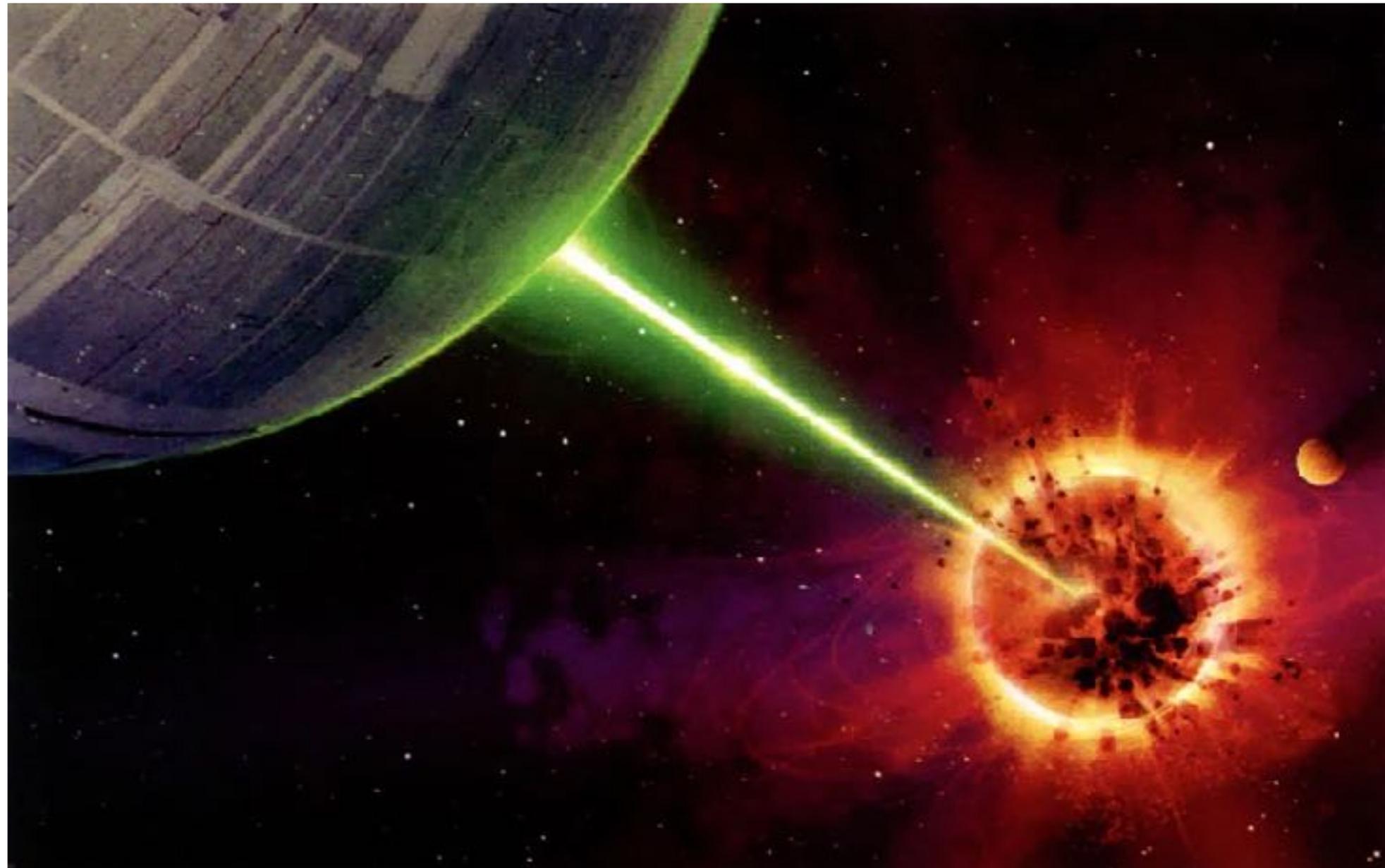




Estimate of Radiated Energy in GWs

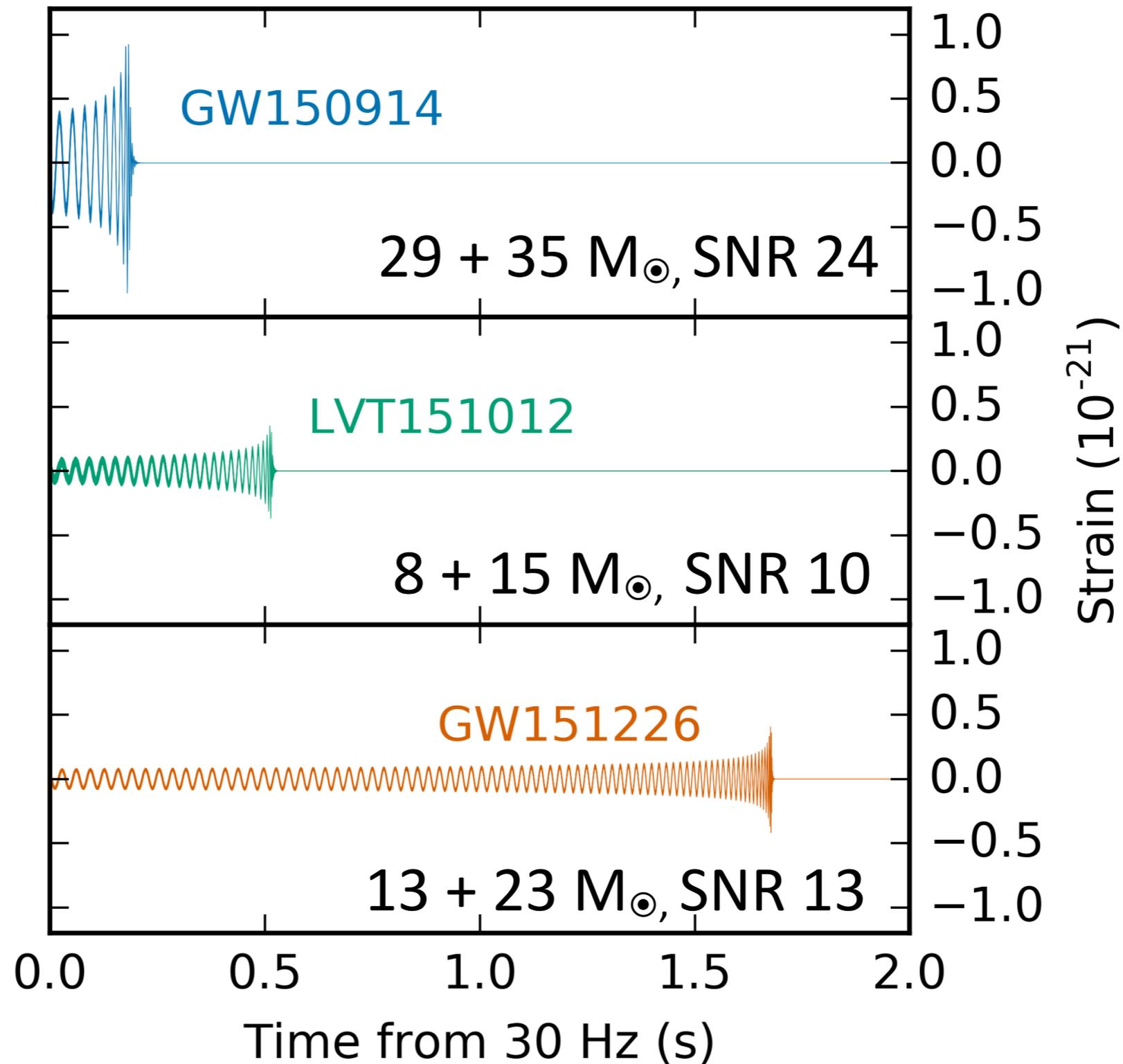
$$E_{\text{mechanical}} = -\frac{Gm^2}{2r}$$

Set $m = 35 M_{\odot}$ and $r=346 \text{ km}$, obtain $E_{\text{mechanical}} \sim 3 M_{\odot}c^2$





Binary Black Holes in O1





Hawking's Area Theorem PRL 21, 1344 (1971)

- Plug in m_1 , m_2 , m_f and s_f : it's satisfied!
- Problem: most of the SNR is before the merger, so only values of m_1 , m_2 are determined independently. The value of m_f and s_f are determined by numerical relativity (which gives the matching waveforms). GUARANTEED to satisfy the area theorem, because the numerical solution satisfies Einstein's equations.

For this event, the area theorem is being tested by the code that's solving Einstein equations, not by Nature.

Gravitational Radiation from Colliding Black Holes

S. W. Hawking

Institute of Theoretical Astronomy, University of Cambridge, Cambridge, England

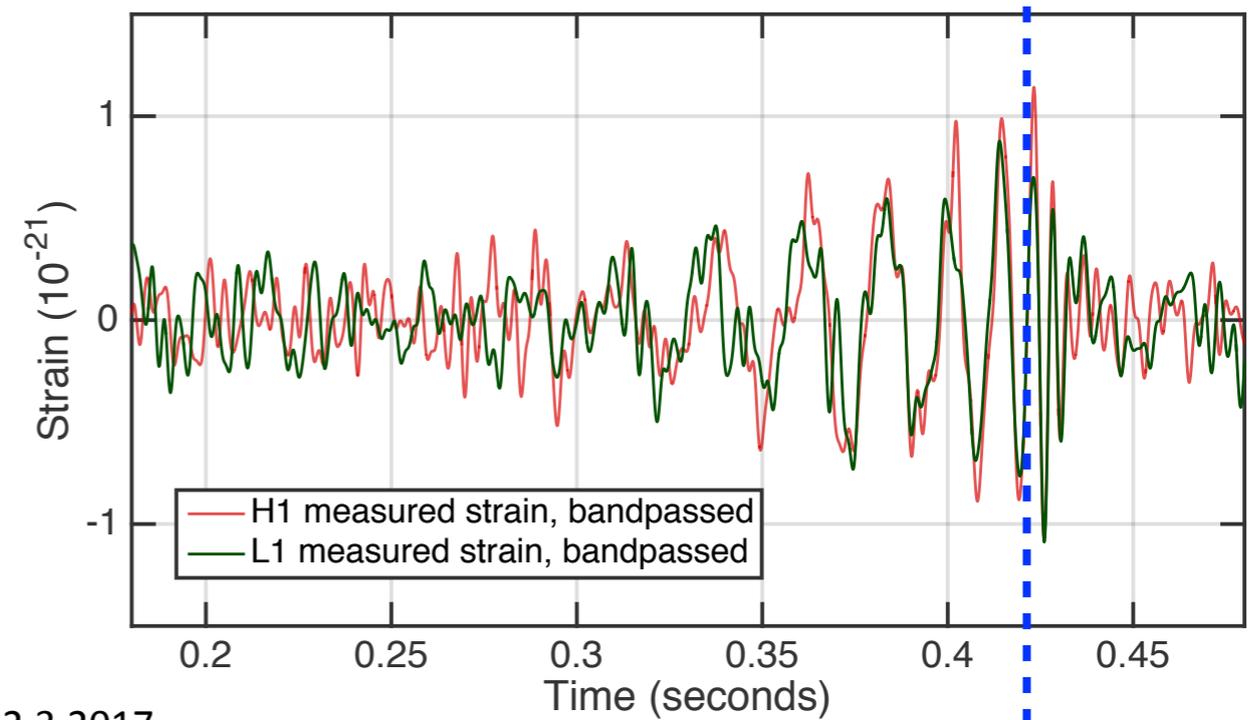
(Received 11 March 1971)

It is shown that there is an upper bound to the energy of the gravitational radiation emitted when one collapsed object captures another. In the case of two objects with equal masses m and zero intrinsic angular momenta, this upper bound is $(2-\sqrt{2})m$.

Weber¹⁻³ has recently reported coinciding measurements of short bursts of gravitational radiation at a frequency of 1660 Hz. These occur at a rate of about one per day and the bursts appear to be coming from the center of the galaxy. It seems likely^{3,4} that the probability of a burst causing a coincidence between Weber's detectors

collapsed objects. Up to now no limits on the efficiency of the processes have been known. The object of this Letter is to show that there is a limit for the second process. For the case of two colliding collapsed objects, each of mass m and zero angular momentum, the amount of energy that can be carried away by gravitational or

$$m_f^2 \left(1 + \sqrt{1 - s_f^2} \right) > m_1^2 \left(1 + \sqrt{1 - s_1^2} \right) + m_2^2 \left(1 + \sqrt{1 - s_2^2} \right)$$





LIGO Collaboration statement (March 9) on status of O2

The second Advanced LIGO run began on November 30, 2016 and is currently in progress. As of February 23 **approximately 30 days of Hanford-Livingston coincident science data have been collected**, with a scheduled break between December 22, 2016 and January 4, 2017. **So far, 3 event candidates, identified by online analysis using a loose false-alarm-rate threshold of one per month, have been identified and shared with astronomers** who have signed memoranda of understanding with LIGO and Virgo for electromagnetic followup. A thorough investigation of the data and offline analysis are in progress; results will be shared when available.



Things I didn't talk about

- Testing GR: everything consistent. New ability to test GR in the strong field dynamic regime.
- Astrophysical implications: metallicity during star formation that led to these BH could not have been too large.
- Broad limit of 9-240 events per cubic Gpc/year on binary black hole merger rate.
- Stochastic “background” from more distant weaker sources: potentially detectable when we reach design sensitivity
- Other potential LIGO sources of gravitational waves
- Coming new GW detectors: VIRGO, KAGRA, LIGO India
- Searches for gravitational waves with other instruments and in other frequency bands: LISA, Pulsar Timing Arrays, CMB, ...



Conclusions

- We can detect gravitational waves **directly** (tracking amplitude and phase)
- Existence of stellar mass black hole binaries established (not visible any other way!). Will be our dominant source.
- A golden age for GW astronomy is coming. We will go from 2 detections to 10 to 100 in the next few years.
- Other signal sources (NS/NS, NS/BH, CW, or the unexpected). Please sign up for Einstein@Home

