

# Clues on gravitational flow dynamics from seismic inversion

Pauline Bonnet, 2018



European Research Council  
ERC  
EUROPEAN RESEARCH COUNCIL  
EUROPEAN COMMISSION

Anne Mangeney<sup>1,1'</sup>

with specialists in seismology (C. Hibert, E. Stutzmann, Y. Capdeville, C. Levy, etc.), mathematics (F. Bouchut, E. Fernandez-Nieto, G. Narbona-Reina, J. Sainte-Marie), acoustics (J. De Rosny, X. Jia; R. Toussaint)

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<sup>1'</sup> Equipe ANGE INRIA-Laboratoire Jacques Louis Lions

# Outline

I – Introduction : landslides and seismic waves

II – Inversion/modelling of *low frequency* forces generated by landslides

III – Insight from *high frequency* seismic data

IV – Monitoring landslide activity in link with volcanic activity

V – Conclusion

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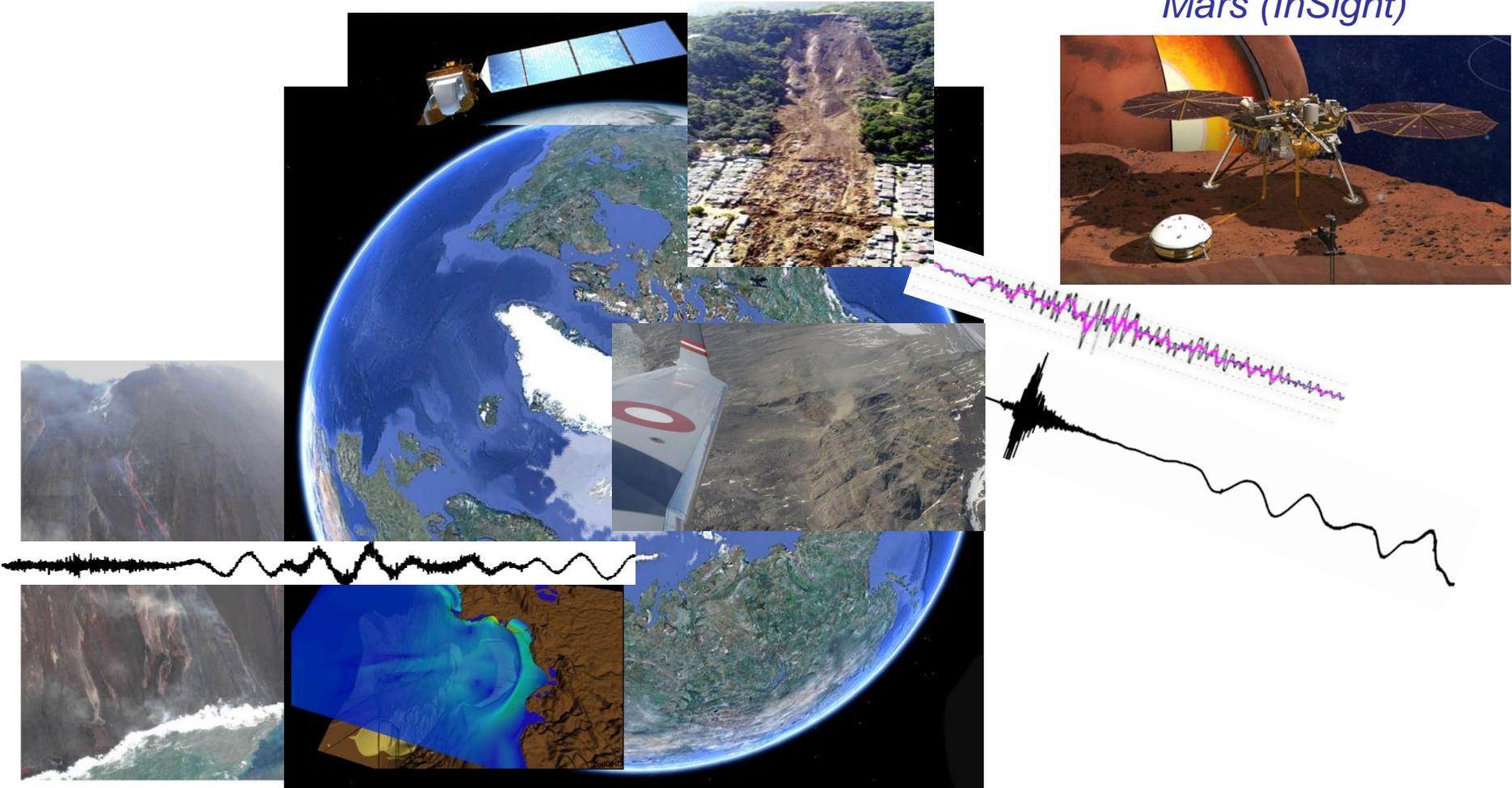
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# Landslides and seismic waves

- Erosion processes at the **Earth** surface and on telluric **planets**
- Risk assessment on Earth in relation with **seismic, volcanic, climate forcing**

*Mars (InSight)*



# Challenges

- Predict **velocity and runout extent**

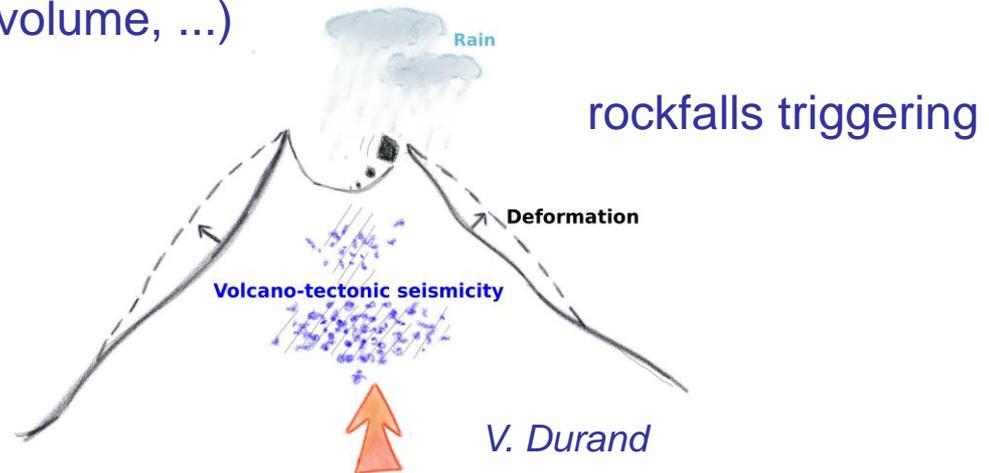
explain and quantify the **high mobility** of natural landslides ...

Lack of field measurements of their dynamics



- Understand and quantify their **occurrence/properties in link with external forcing**

detection, localization, **characterization** (volume, ...)



Seismic data ↔ Granular flow mechanics and modelling

# Seismic signal generated by rockfalls

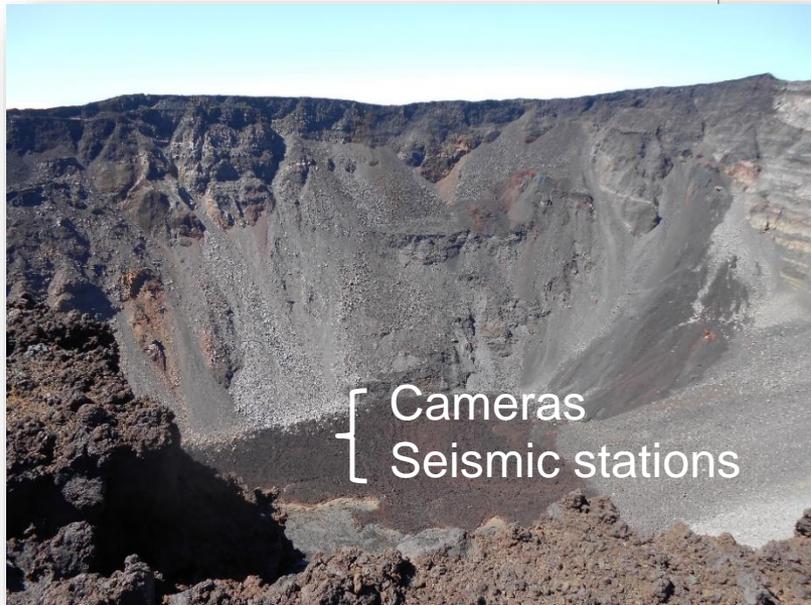
*Piton de la Fournaise volcano, La Réunion*



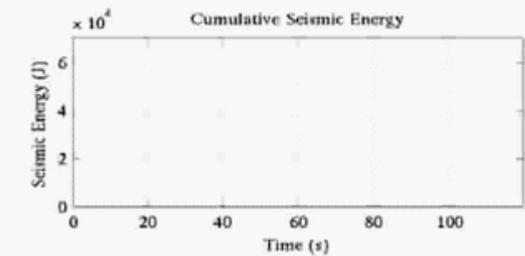
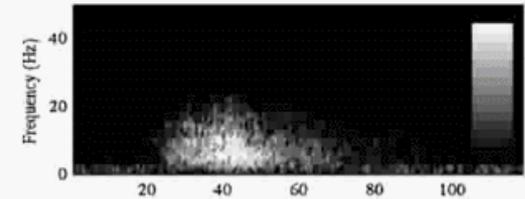
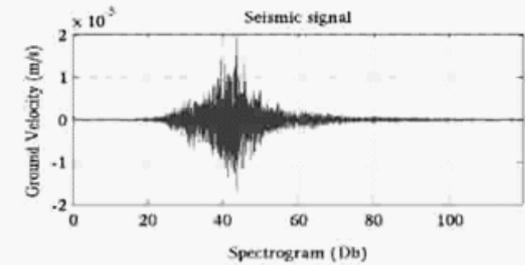
**erc SLIDEQUAKES**

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*Hibert, et al. 2011, 2014, 2017, Durand et al., 2018*



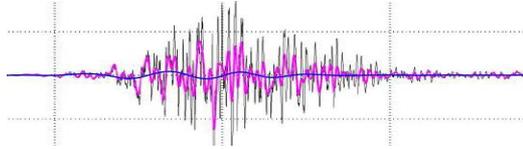
SFRC video - 23/09/2011 : 12-10-00-15



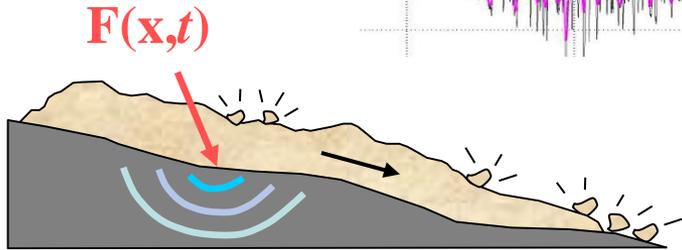
$$t_{\text{seismic}} \approx t_{\text{flow}}$$

# Decrypt processes impacting seismic waveform ?

$$0.01 < f < 20 \text{ Hz}$$



Convolution of :



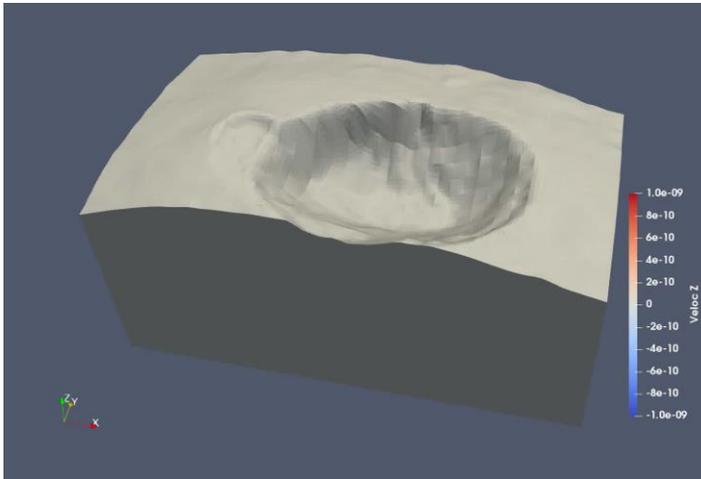
- Source

Mass, geometry, rheology, topography, particle agitation, fluid content ...

- Wave path from source to station

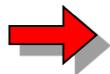
Ok at low frequencies  $f < 0.1-0.2 \text{ Hz}$  (5-10 s)  
Challenging at high frequencies  $f > 1 \text{ Hz}$  !

Earth heterogeneity, topography



Kühnert et al., 2018

How can we separate the processes ?



Physical simulation of field scale granular flows

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# Force at the origin of seismic waves

Deconvolution of long period ( $T > 5-10$  s) seismograms

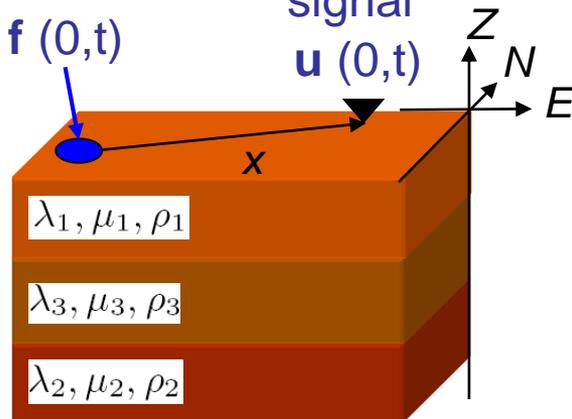
$$\mathbf{u}_i(x, t) = \mathbf{g}_{ij}(x, t) * \mathbf{f}_j(0, t)$$

Green's function for signal component  $i$  and force direction  $j$

Point source  
landslide force

$\mathbf{f}(0, t)$

Seismic  
signal  
 $\mathbf{u}(0, t)$



Frequency domain :  $U, G, F$  Fourier transform of  $\mathbf{u}, \mathbf{g}, \mathbf{f}$

$$U(\omega) = G(\omega)F(\omega)$$

→ Force :  $F = (G'G)^{-1}G'U$

→ Inverse Fourier transform  $\mathbf{f}(0, t)$

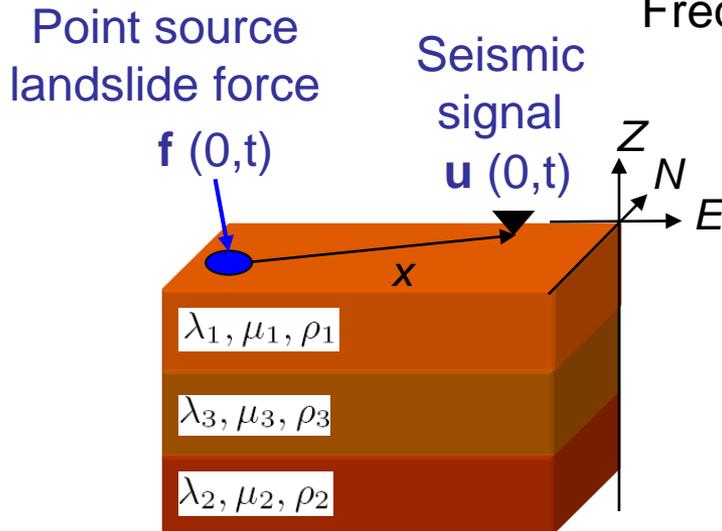
*Kanamori and Given, 1982; Kanamori et al., 1984*

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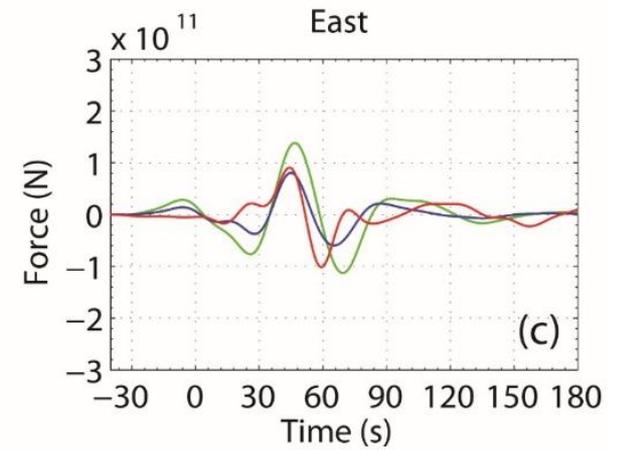
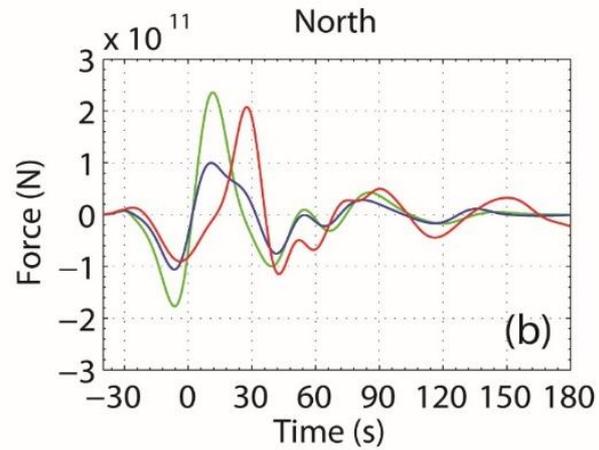
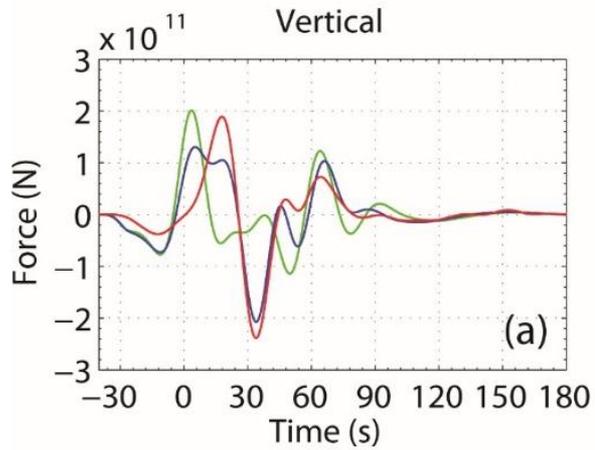
**Recovered with good accuracy using one seismic station !**

*Kawakatsu, 1989*

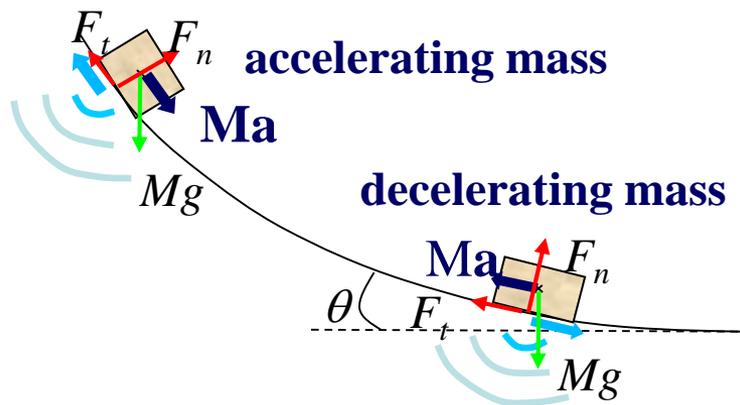
*Zhao, Moretti, Mangeney, Stutzmann, Kanamori, Capdeville, Calder, Hibert et al., 2015*

*Sergeant, Mangeney, Stutzmann, Montagner, Walter, Moretti, and Castelnau, 2016*

# Force inverted from seismic data



*Moretti, Mangeney, Capdeville, Stutzmann, Huggel, Schneider, Bouchut 2012*



*Brodsky et al. 2003*

# Mt-Steller rock-ice avalanche

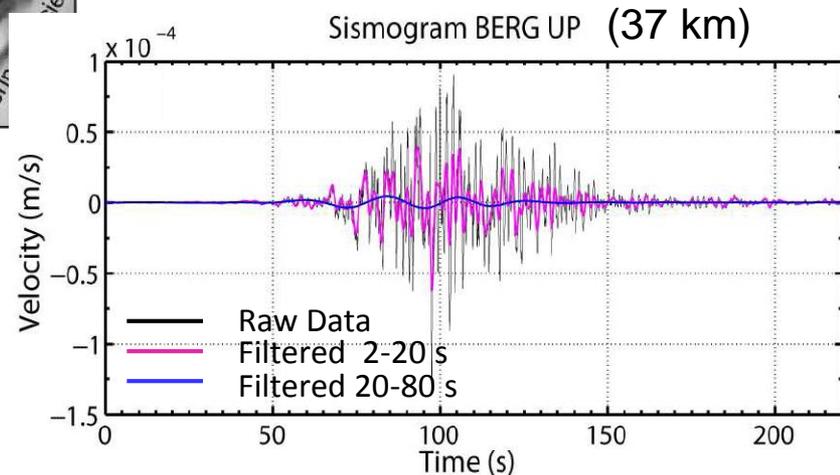
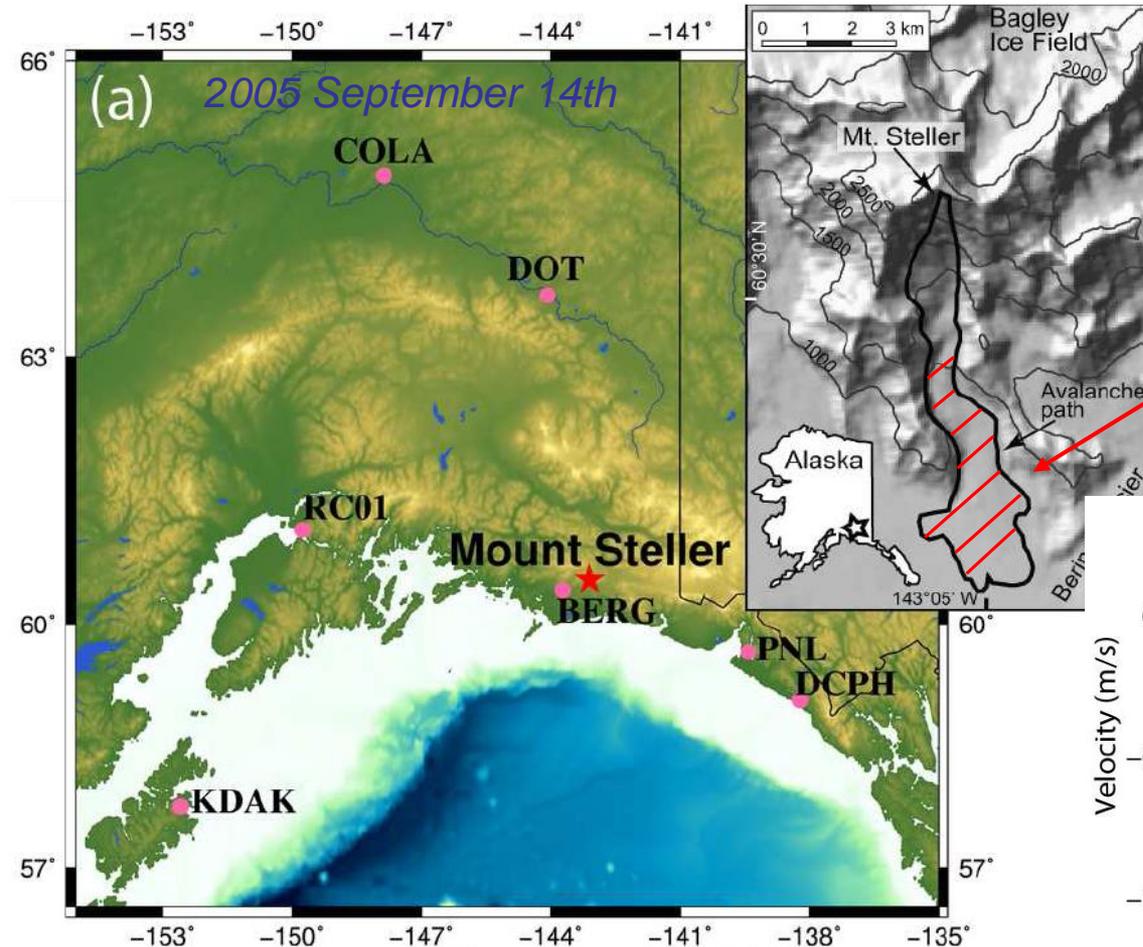
Simulation of landslide and force history

Quantify the role of **erosion in landslide dynamics ?**

Traveled distance ~9 km

Volume :  $40-60 \times 10^6 \text{ m}^3$

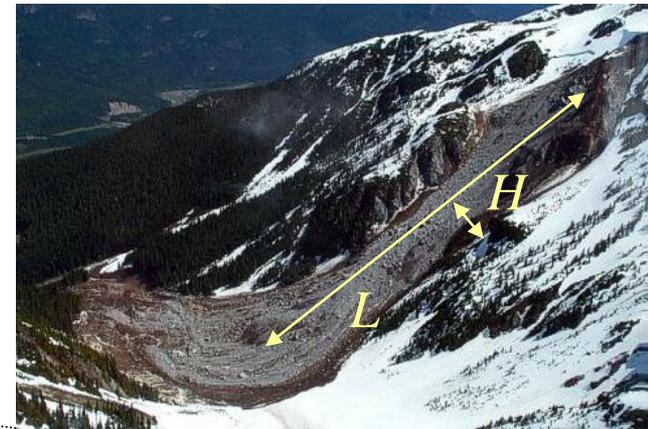
Material eroded along the path:  
 $\sim 20 \times 10^6 \text{ m}^3$



# Thin layer models for natural landslides

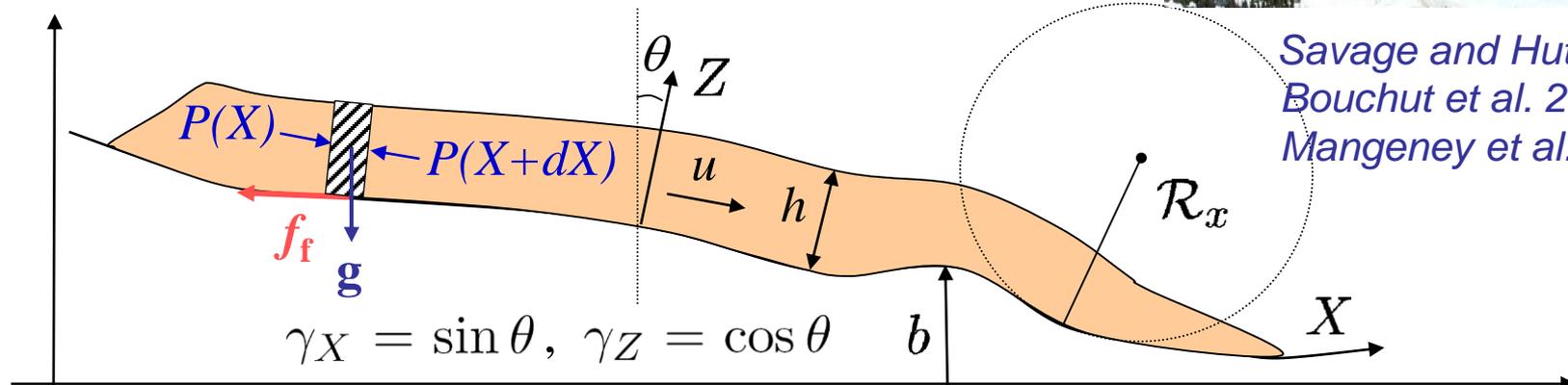
- Flow on **complex natural topography**

high computational cost  $\Rightarrow$   $\epsilon = \frac{H}{L} \ll 1$  small aspect ratio



- Depth-averaged thin layer model**

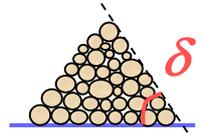
$$h = \mathcal{O}(\epsilon) \quad \nabla b = \mathcal{O}(\epsilon) \quad u^x = \overline{u^x}(t, \mathbf{x}) + \mathcal{O}(\epsilon^2)$$



Savage and Hutter, 1989  
Bouchut et al. 2003,  
Mangeney et al. 2005, 2007

$$\underbrace{\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial X}}_{\text{inertia}} = \underbrace{\gamma_X g}_{\text{gravity}} - \underbrace{K \frac{\partial}{\partial X} (g \gamma_Z h)}_{\text{pressure gradient}} - \underbrace{\mu \left( g \gamma_Z + \frac{u^2}{R_x} \right)}_{\text{Coulomb friction Empirical law}} \frac{u}{|u|}$$

$$\mu = \tan \delta$$

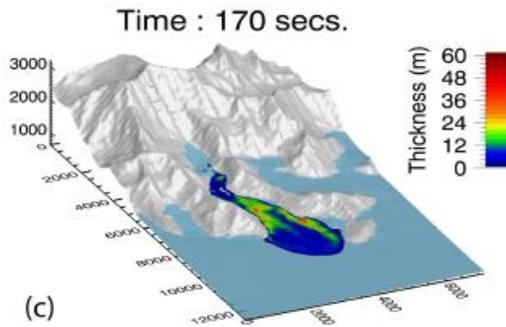
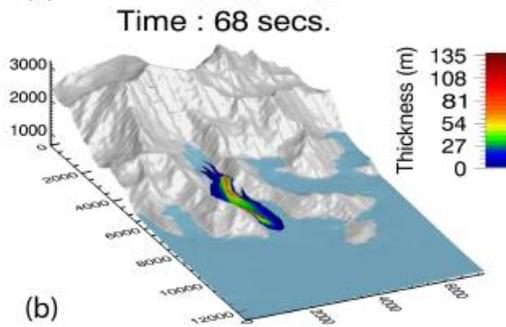
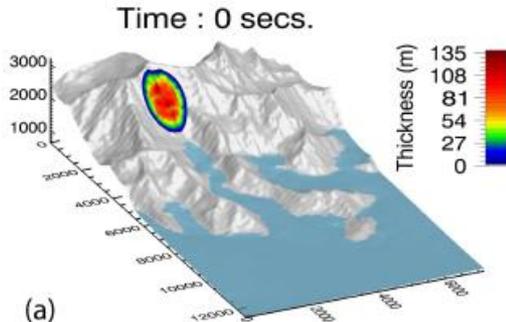


**3D topography** (code SHALTOP)

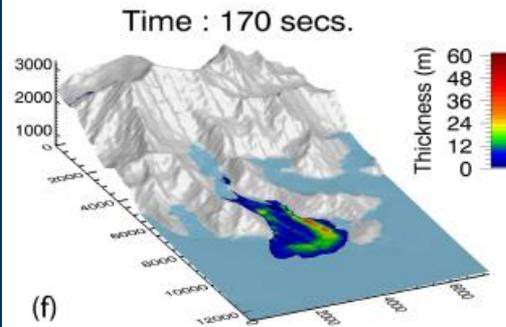
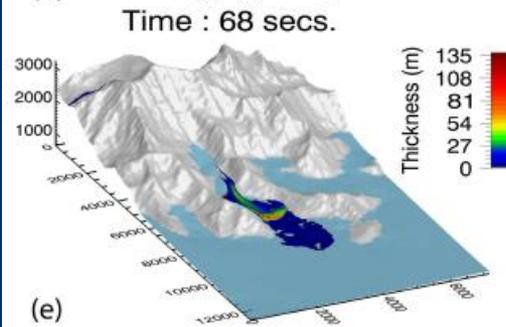
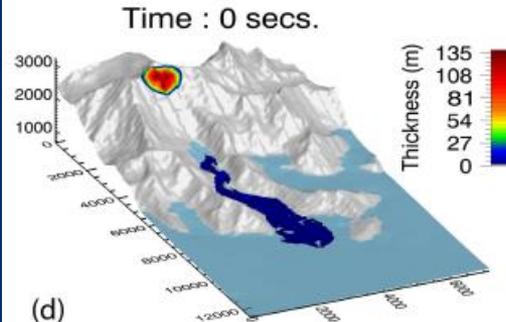
$\delta \approx 25^\circ$  (glass beads),  $35^\circ$  (debris)

# Simulation of the Mt-Steller landslide

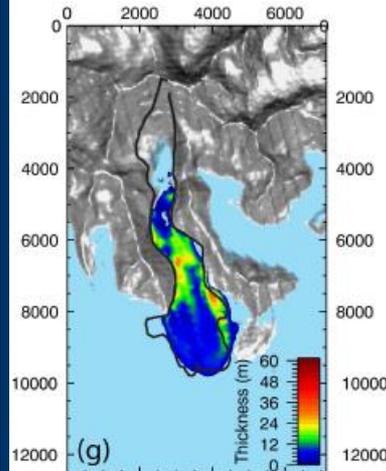
**no erosion**



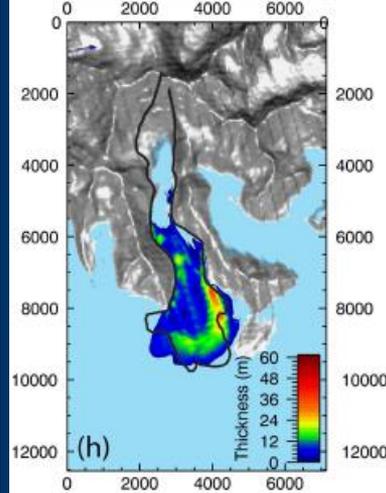
**with erosion**



Comparison of deposits



**no erosion**

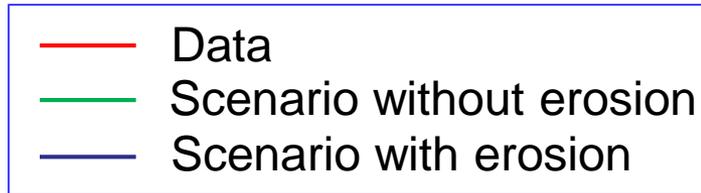
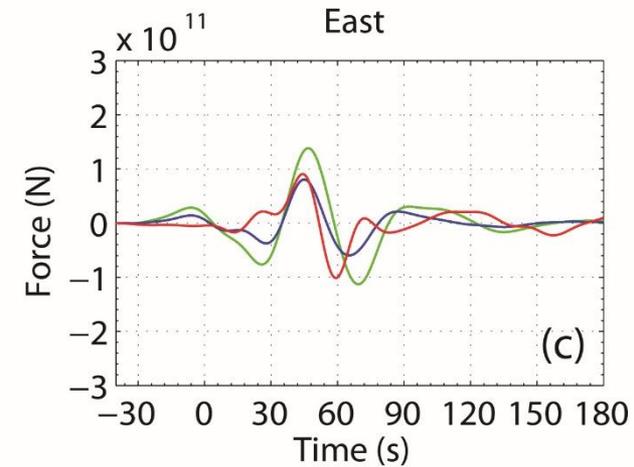
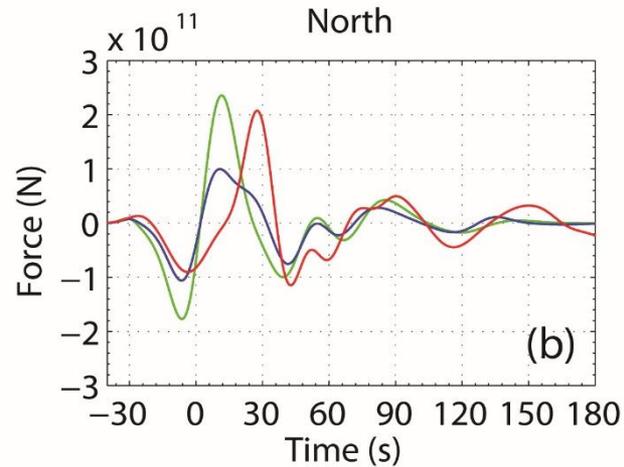
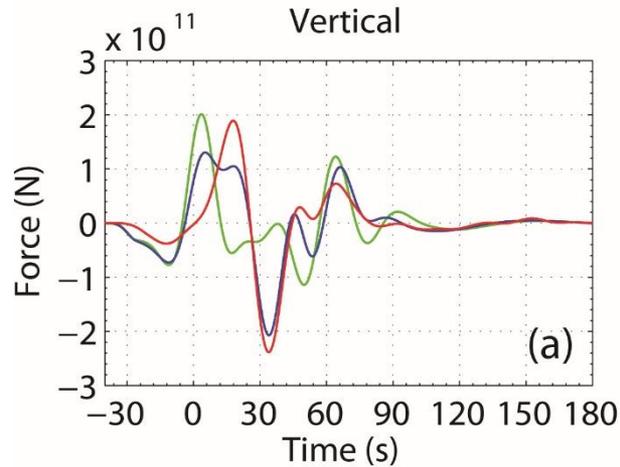


**with erosion**

**The deposit area is not enough to constrain landslide models !!**

# Low frequency: inverted and simulated force

Force filtered between 20-80s

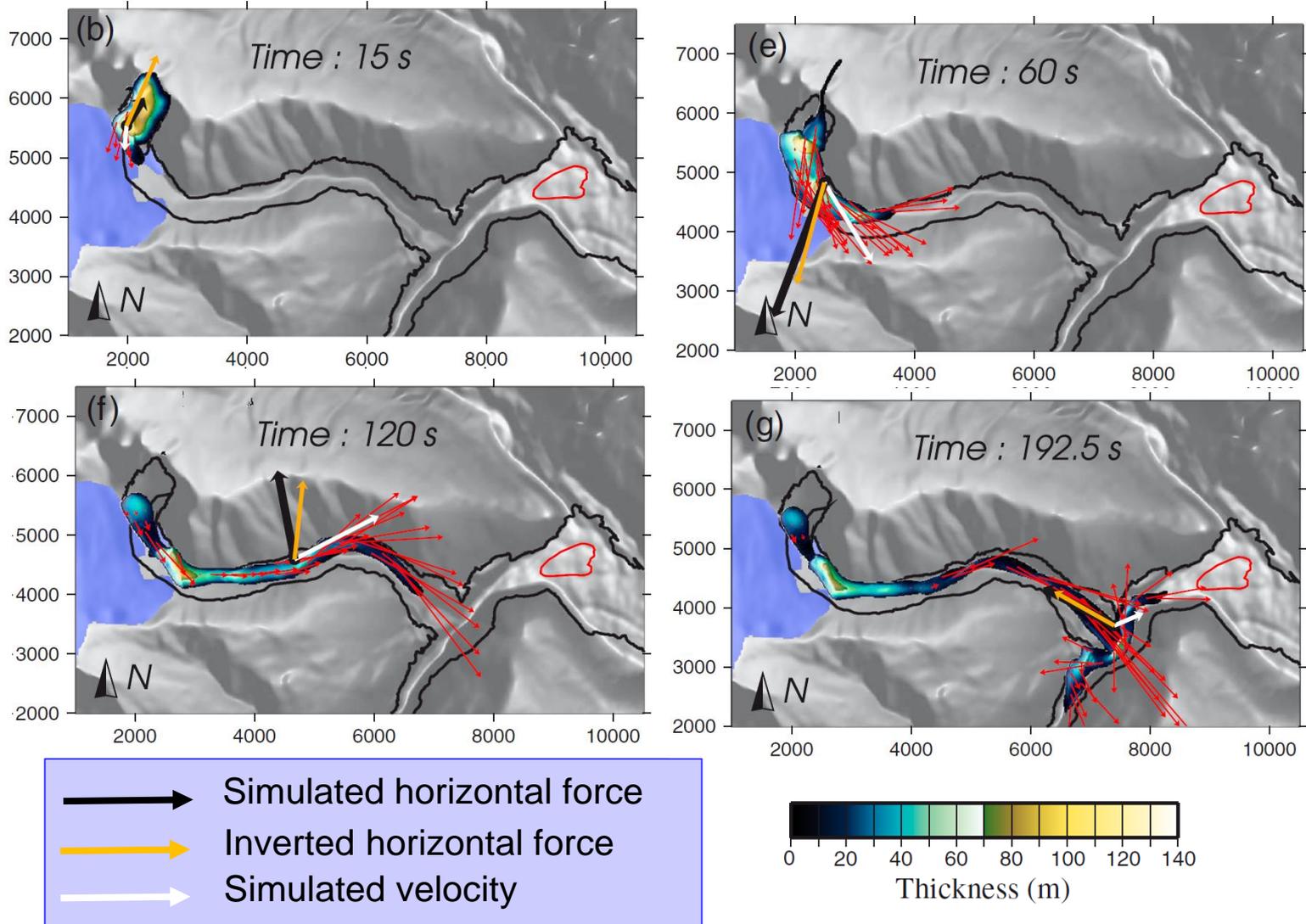


Taking into account **erosion is necessary to reproduce the dynamics**

*Moretti, Mangeney, Capdeville, Stutzmann, Huggel, Schneider, Bouchut 2012*

# Sensitivity to friction coefficient

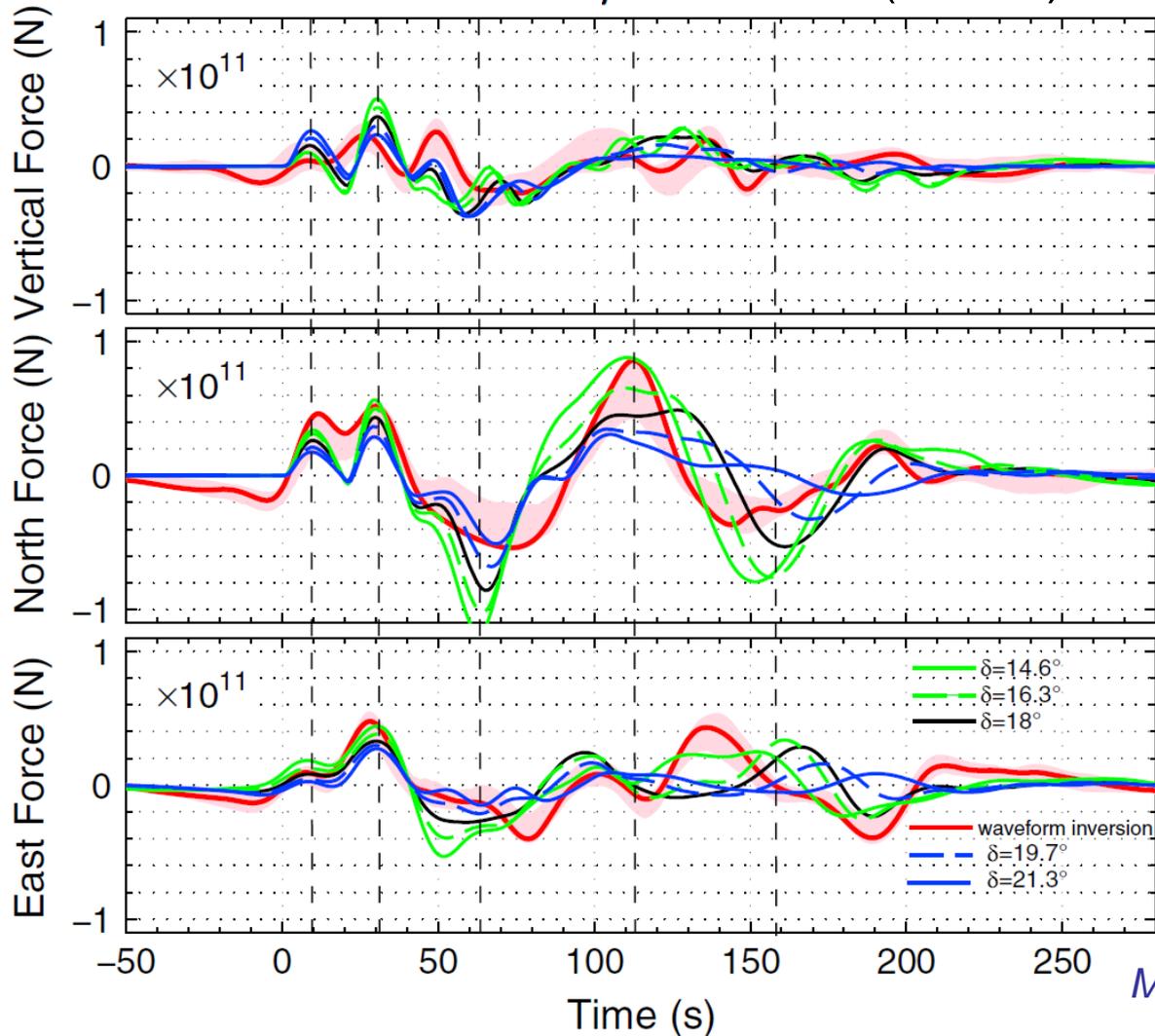
Simulation of Mount Meager landslide  $V = 50 \times 10^6 \text{ m}^3$



*Moretti, Allstadt, Mangeney, Capdeville, Stutzmann, Bouchut 2015*

# Sensitivity to friction coefficient

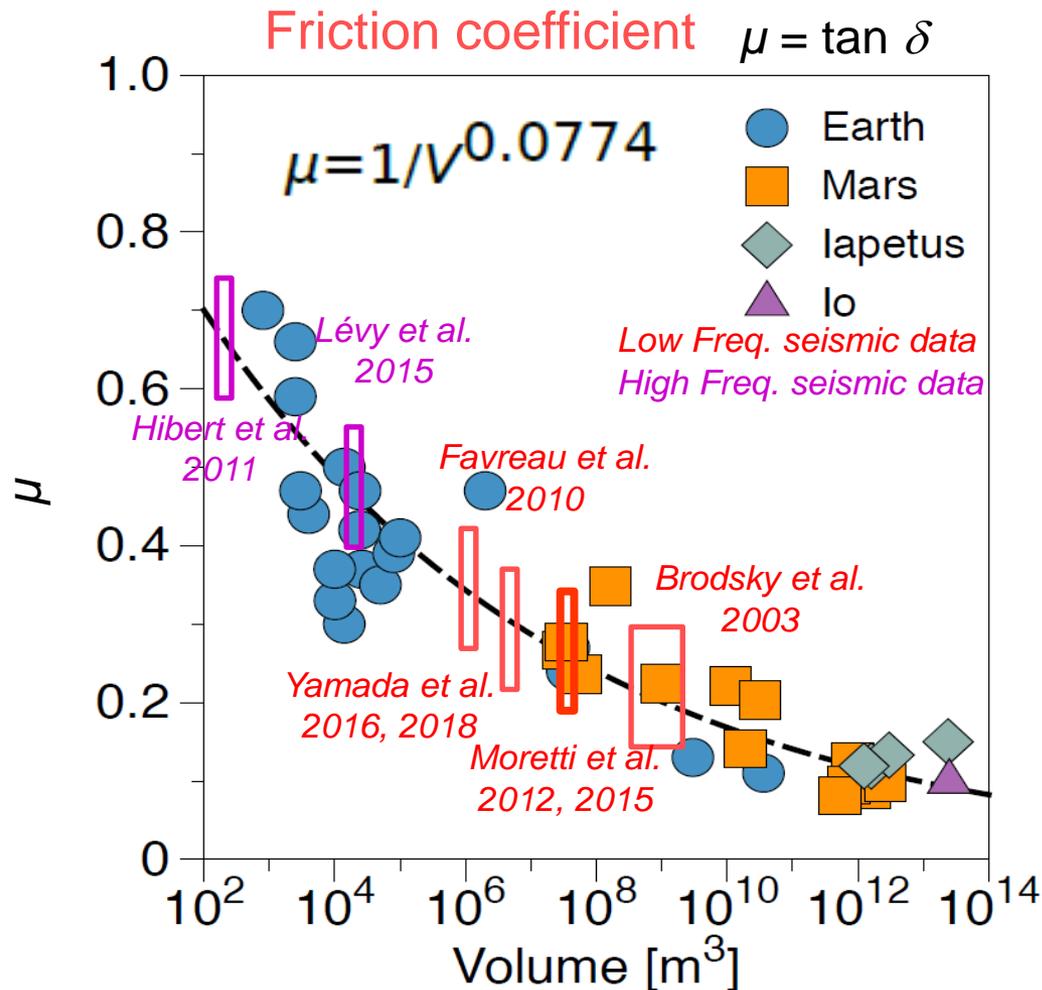
Best scenario  $\mu = 0.33$  ( $\delta = 18^\circ$ )



Moretti et al., 2016

Very small friction coefficient for this large landslide ( $V = 50 \times 10^6 \text{ m}^3$ )

# Empirical friction laws based on seismic data



Friction weakening with volume (or velocity, etc.)

Physical origin ?

*Lucas et al., 2014, Delannay et al., 2018, Yamada et al., 2018*

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**a – Laboratory experiments**

**b – Field measurements/numerical modelling**

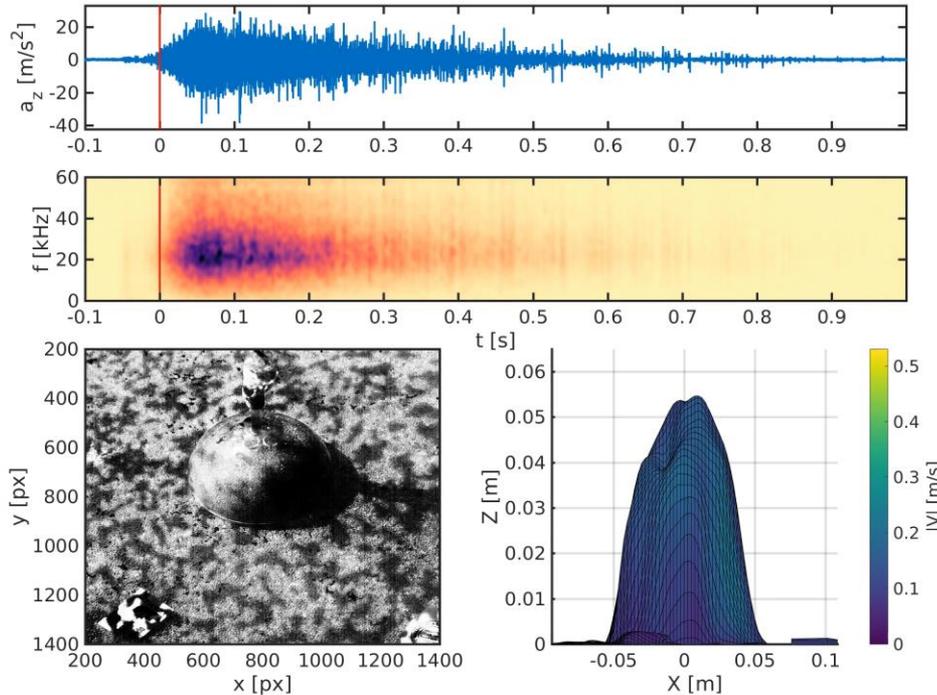
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# Physical origin of high frequency seismic waves ?

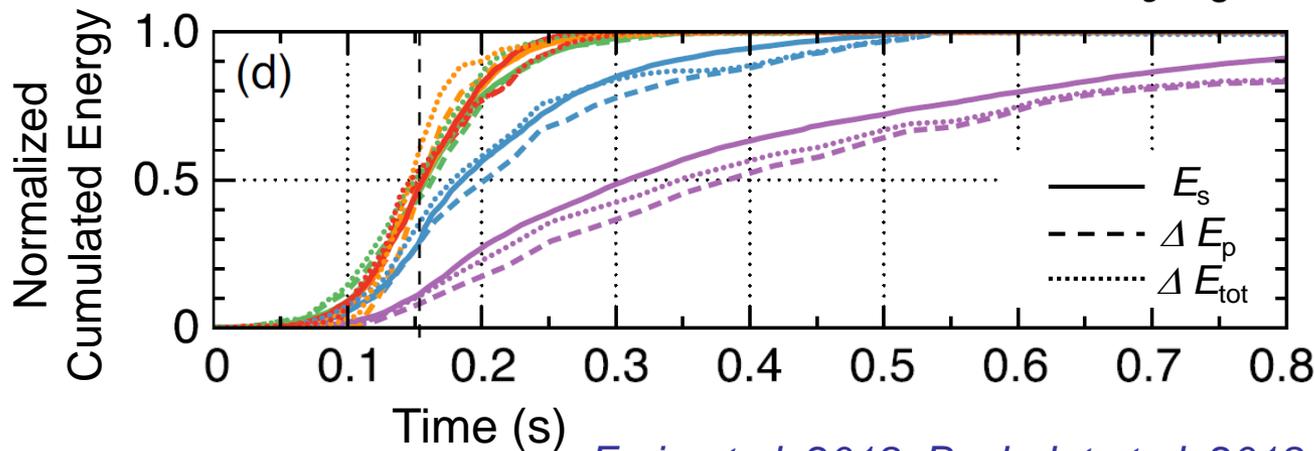
Laboratory experiments of granular flows and seismic emissions

Glass beads  $d = 2 \text{ mm}$



Optics :  
fast camera 5000 fps  
Acoustics :  
accelerometers [10Hz-54kHz]

—  $\theta = 0^\circ$     —  $\theta = 10^\circ$     —  $\theta = 20^\circ$   
—  $\theta = 5^\circ$     —  $\theta = 15^\circ$

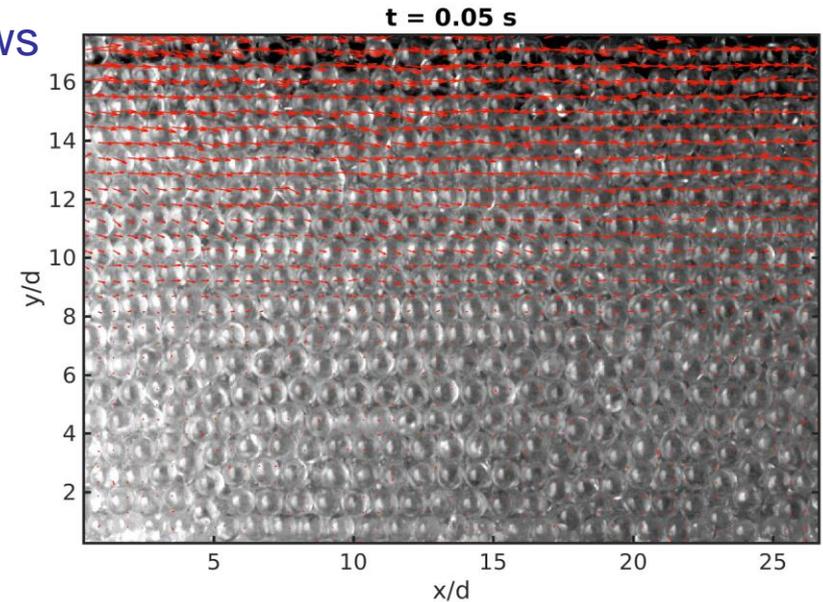
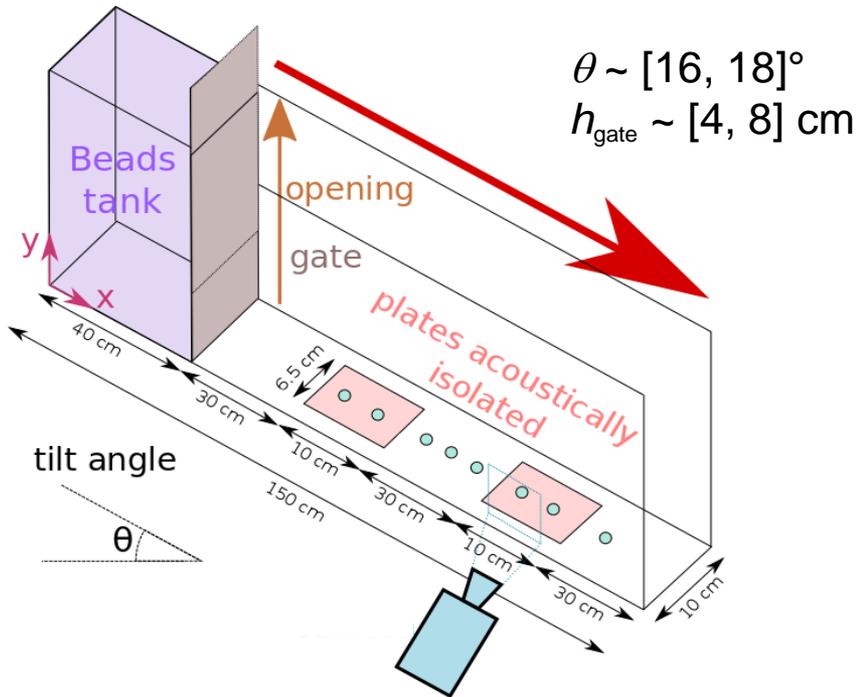


$$E_s \propto \Delta E_p$$

$$E_s / \Delta E_p \sim 2-5 \times 10^{-4}$$

# Physical origin of high frequency seismic waves ?

Almost steady and uniform free surface flows



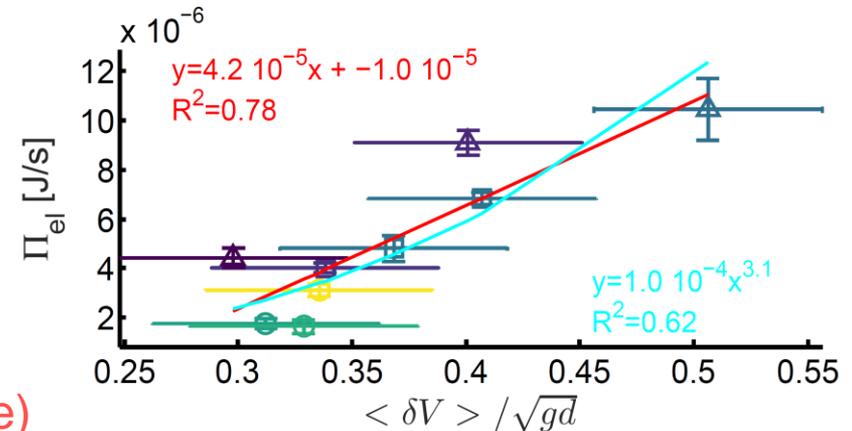
Seismic energy per second

$$\Pi_{\text{el}} \propto \delta V \propto V$$

$$f_{\text{mean}} \sim f_{\text{Hertz's contact theory}}$$



grain collisions (granular temperature)



Velocity fluctuations

# Seismic signal generated by rockfalls

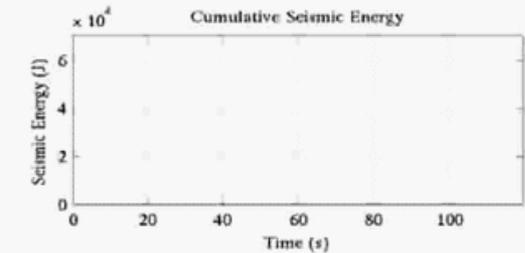
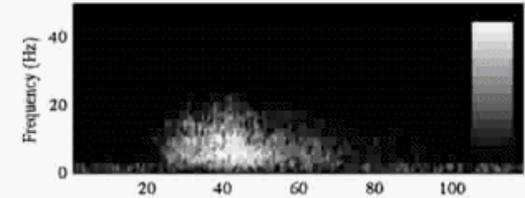
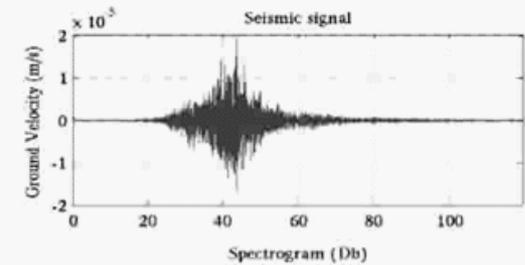


*Piton de la Fournaise volcano, La Réunion*

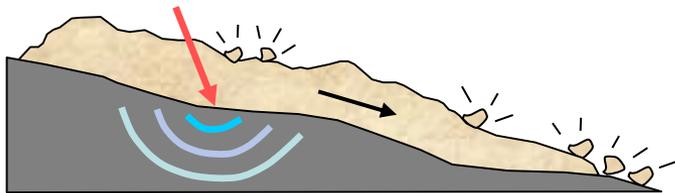
*Hibert, et al. 2011, 2014, 2017, Durand et al., 2018*



SFRC video - 23/09/2011 : 12-10-00-15



$F(x,t)$  : generated force



Seismic waves

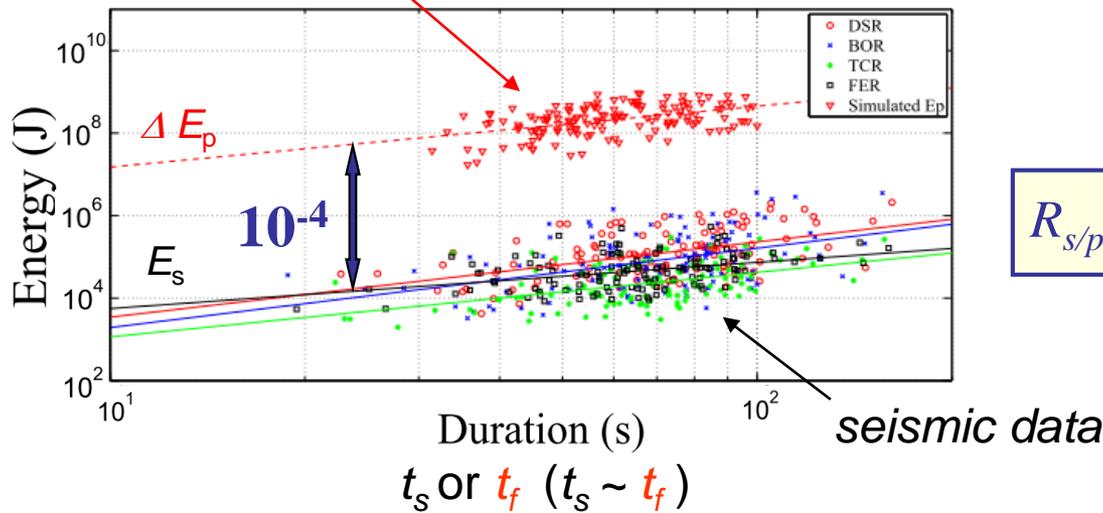
$$t_{\text{seismic}} \approx t_{\text{flow}}$$

# From seismic energy to rockfall volume

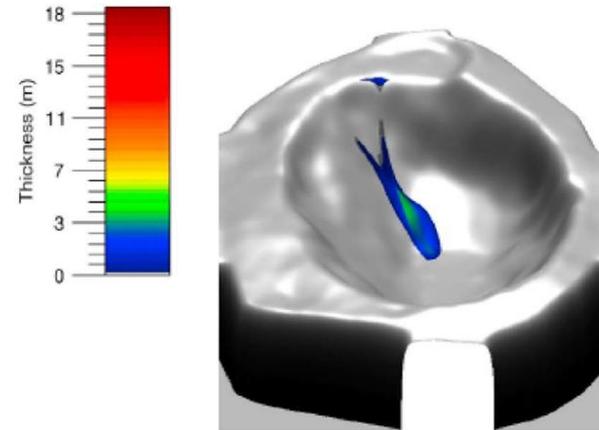
- Power law energy versus duration :

$$E_{\text{seismic}} \propto t_s^\beta \quad \text{and} \quad \Delta E_{\text{potential}} \propto t_f^\beta$$

simulation with friction angle  $\delta = 35^\circ$



Piton de la Fournaise



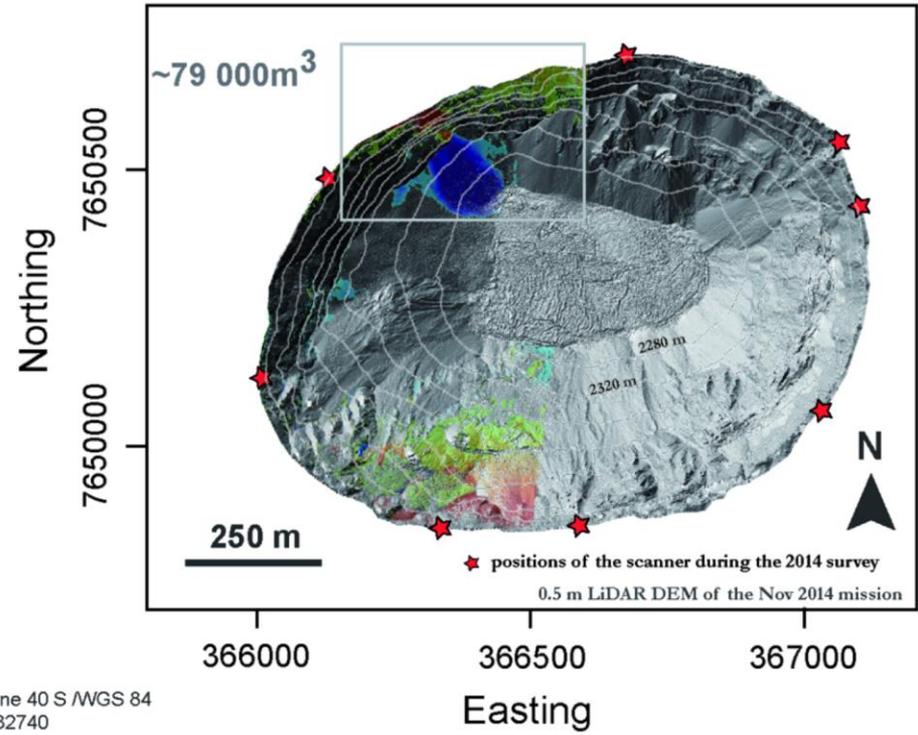
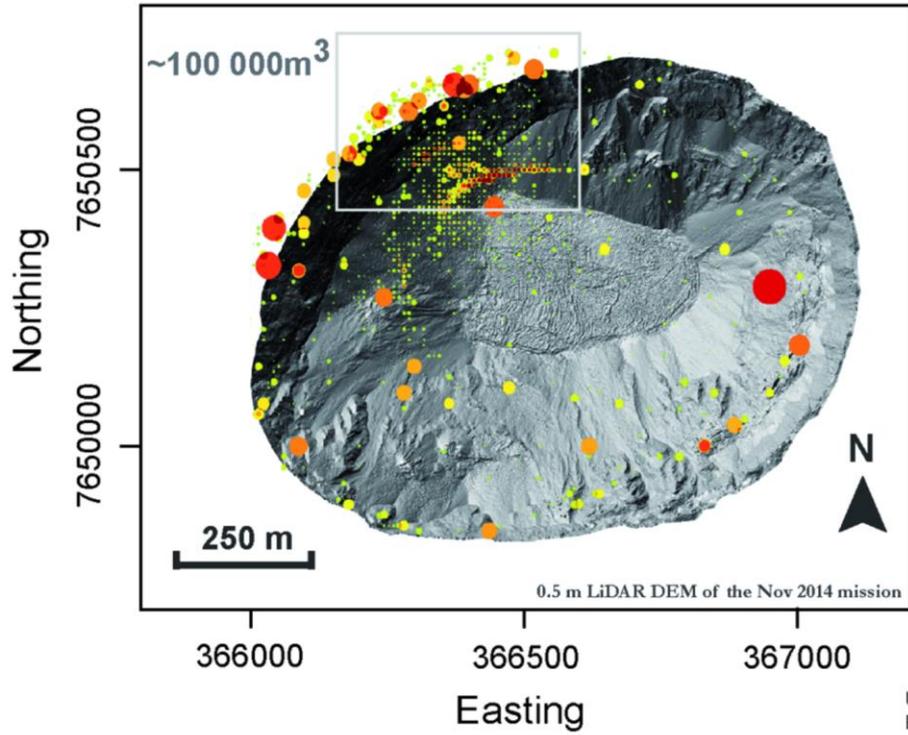
$$R_{s/p} = E_s / \Delta E_p \sim 10^{-4}$$

Hibert et al., 2011, 2014, 2017

➔ **Volume**  $V = \frac{3E_s}{R_{s/p} \cdot \rho g L (\tan \alpha \cos \theta - \sin \theta)}$

# Volumes: seismic and laser/photogrammetry

Durand et al., 2018



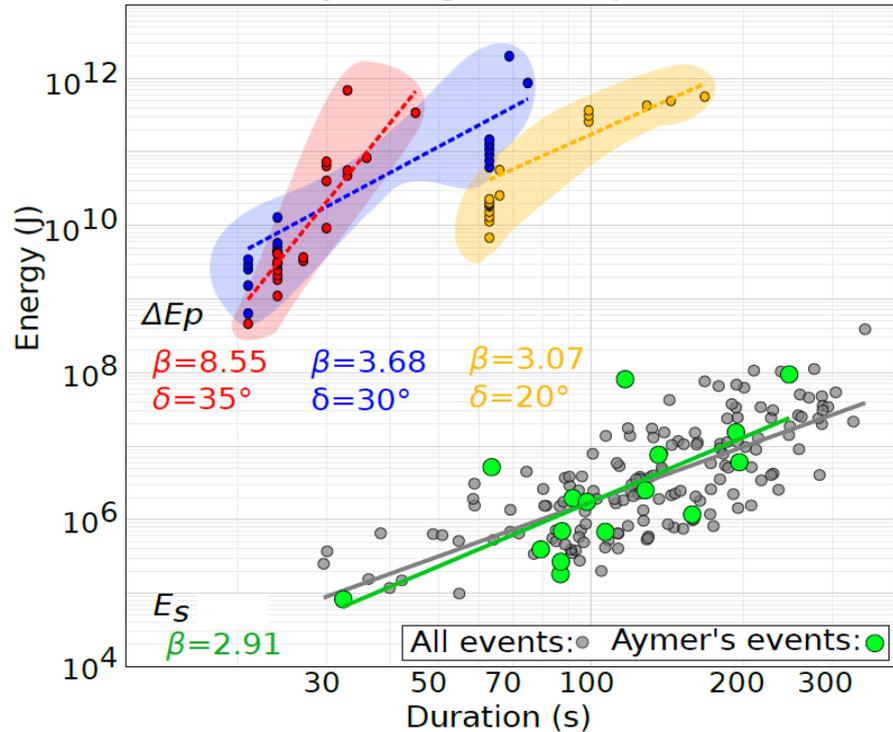
- Good agreement in the North-East part
- Seismic data: higher temporal resolution than photogrammetry

# Friction weakening signature on seismic data

Rockfalls and pyroclastic flows in Montserrat

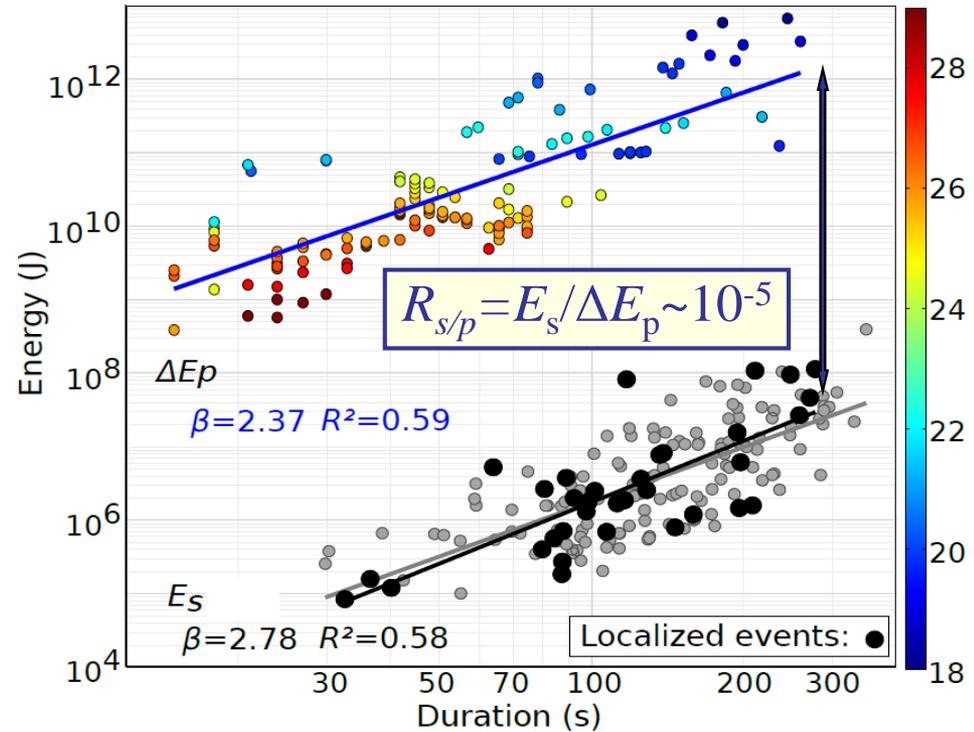
$$\mu = \tan \delta = 1/V^{0.0774}$$

Aymer's ghaut with  $\mu = C^{ste}$



White river, Gages & Aymer's ghauts

$\delta$  (°)



Friction weakening makes it possible to reproduce seismic data

Levy, Mangeney, Bonilla, Hibert, Calder, Smith, 2015

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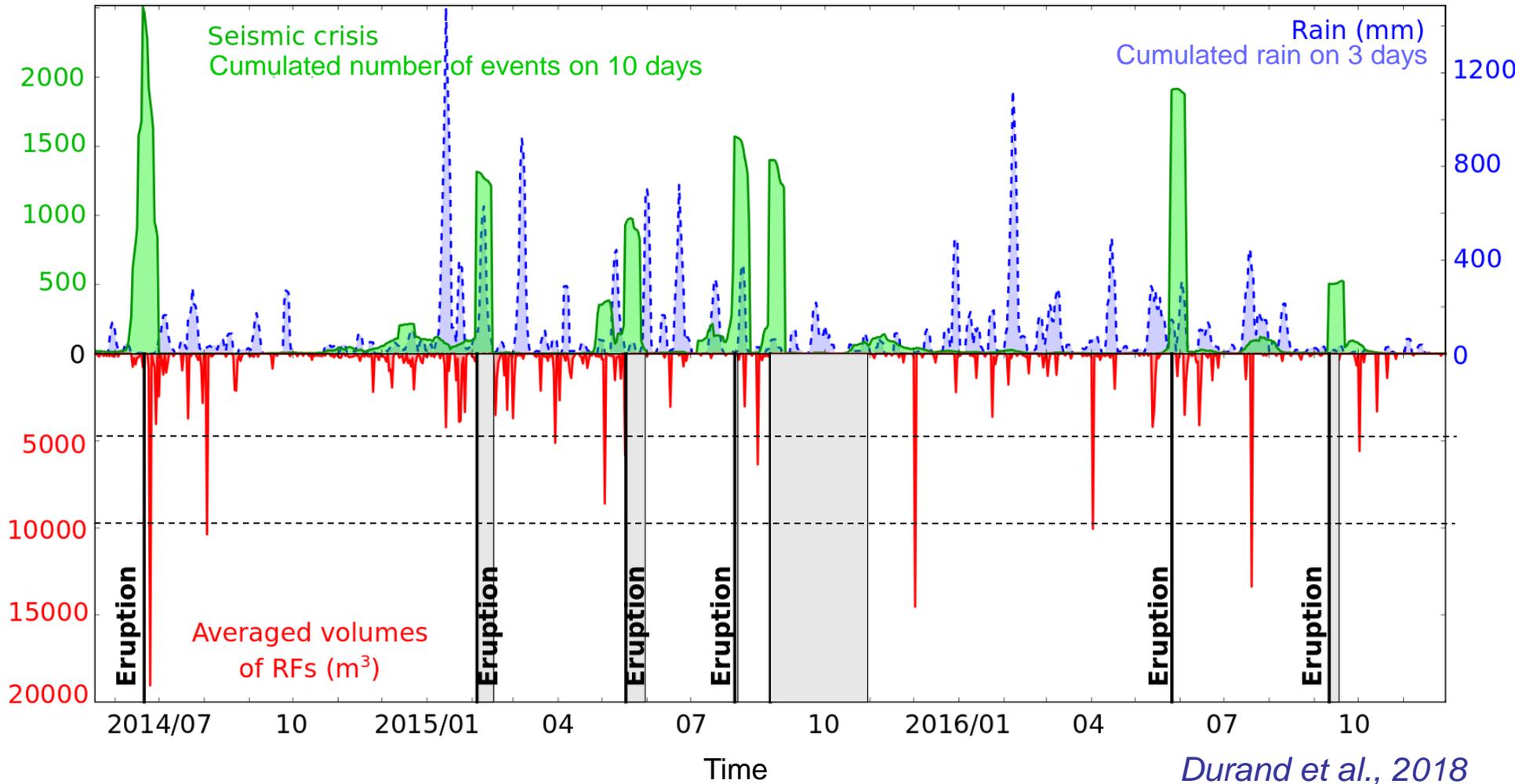
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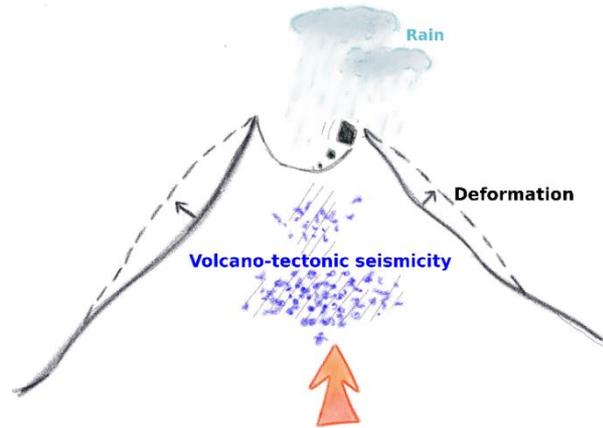
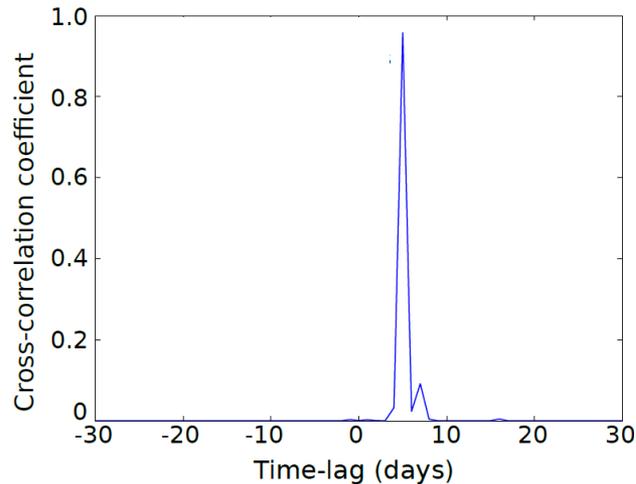
# Rockfall triggering



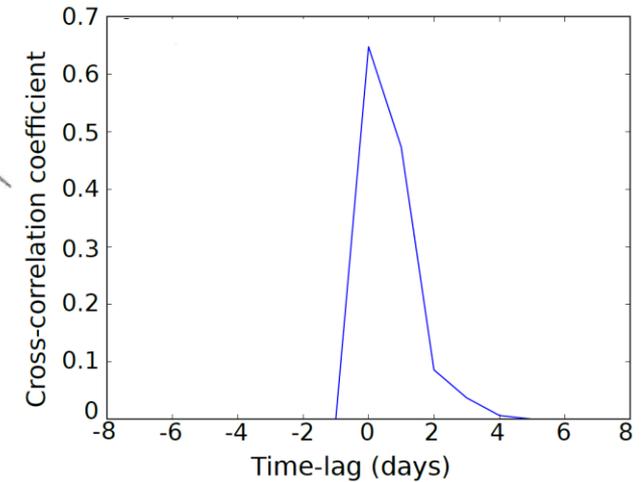
*Durand et al., 2018*

# Rockfall triggering

*Mean volume of rockfalls & earthquakes magnitude*



*Mean volume of rockfalls & rain*



- Strong correlation between rockfall volume and cumulative magnitude/number of Volcano-Tectonic events (time-lag 1-20 days)
- Weak correlation with maximum amplitude of volcano-tectonic seismicity
- Moderate correlation with rain with time-lag of 1-5 days

# Conclusion

- **Force history at low frequency** :  
constrain on landslide volume, rheology, physical processes involved (erosion, fluid content, etc.) when **compared to physical simulation**
- **Energy at high frequency** :  
constrain on landslide volume, rheology, localization
- **Monitoring rockfall activity**:  
link with seismic, volcanic, meteorological activity

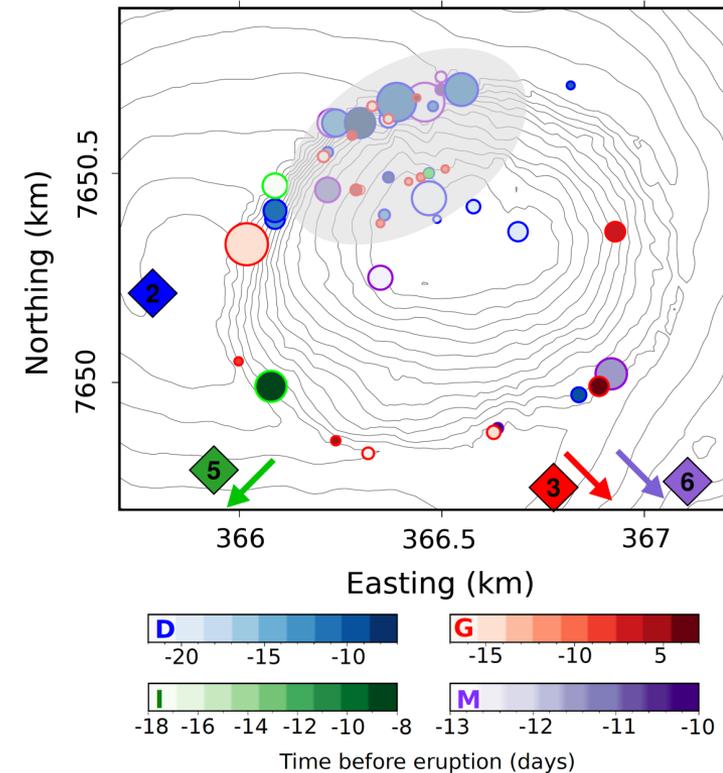
Large rockfalls seem to occur close to the next eruption ?



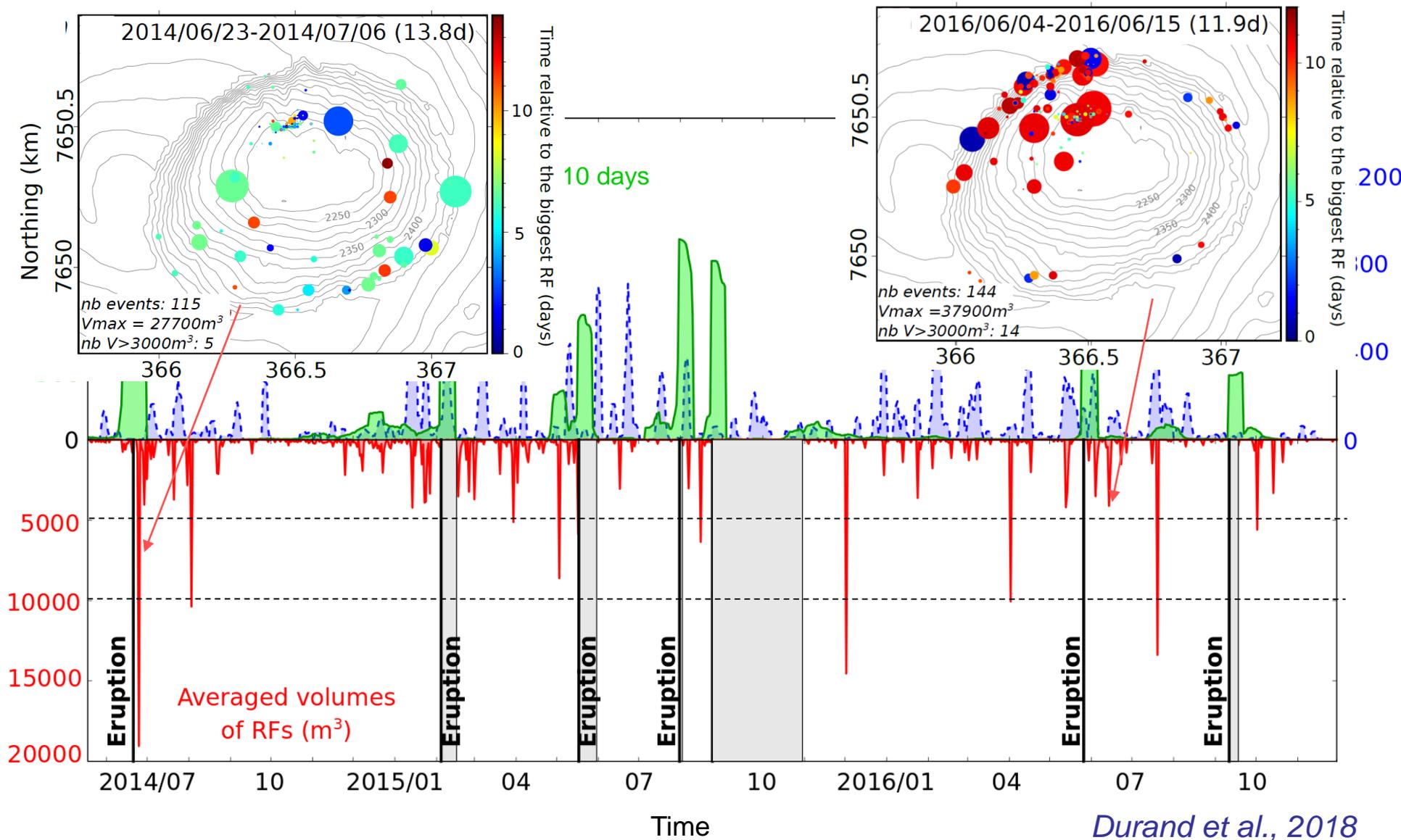
European Research Council  
Established by the European Commission

**SLIDEQUAKES**

*Durand et al., 2018*



# Rockfall triggering

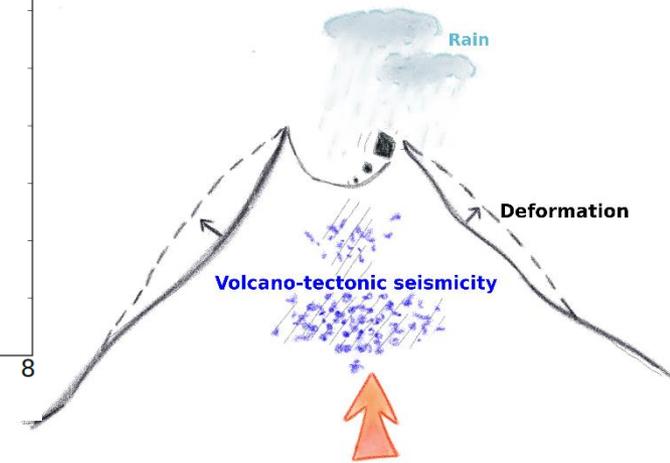
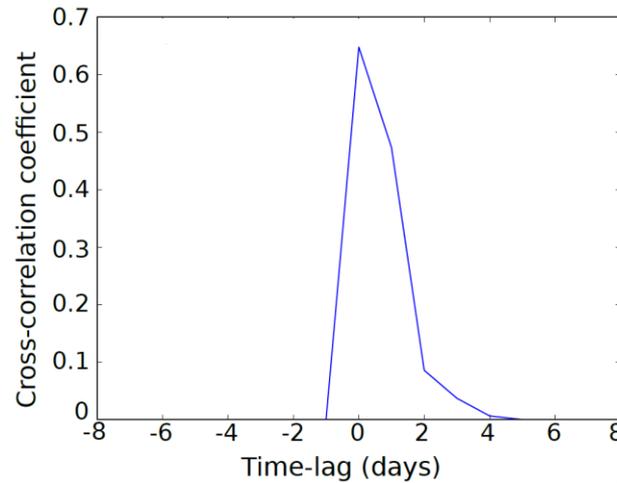
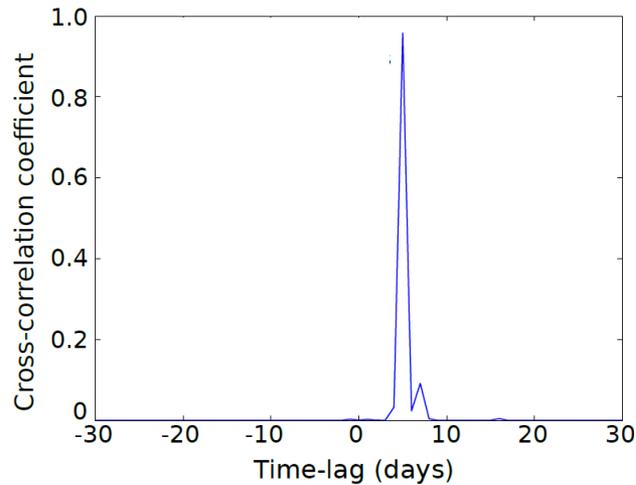


# Rockfall triggering

*Durand et al., 2018*

*Daily mean volume of rockfalls  
vs daily magnitude of Volcano-  
Tectonic earthquakes*

*Daily mean volume of rockfalls  
vs daily rain*

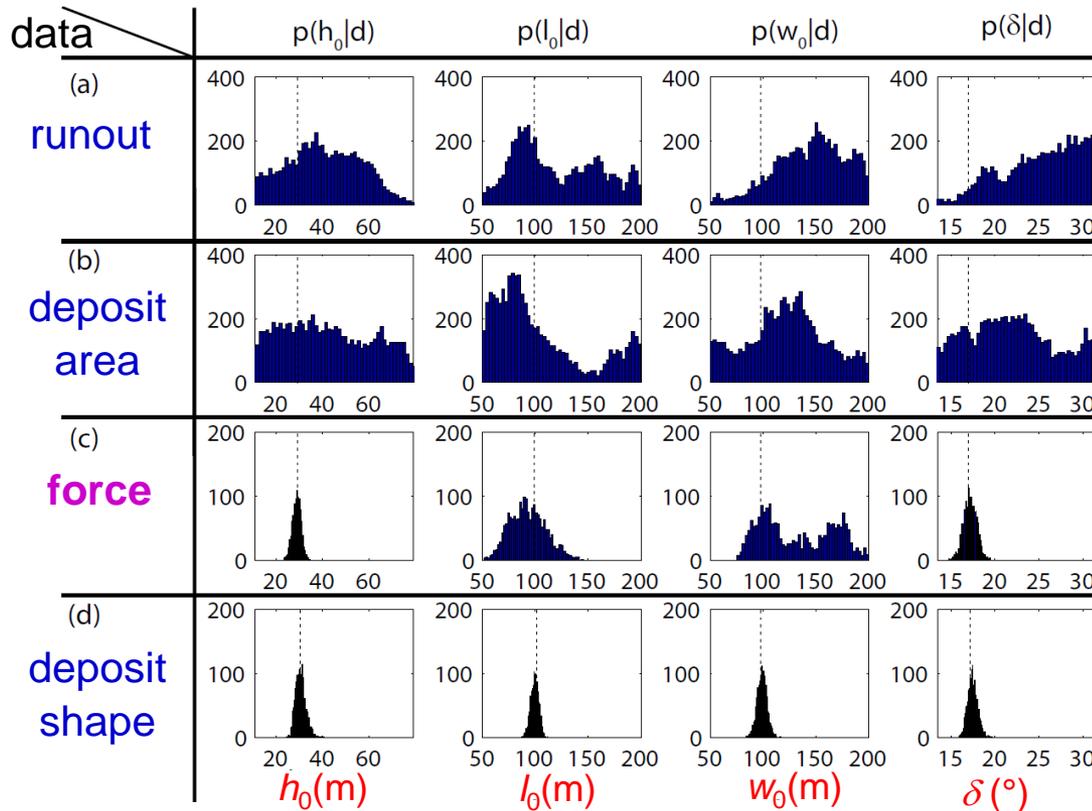
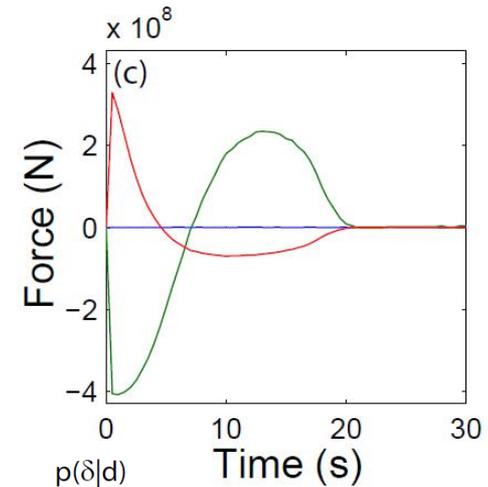
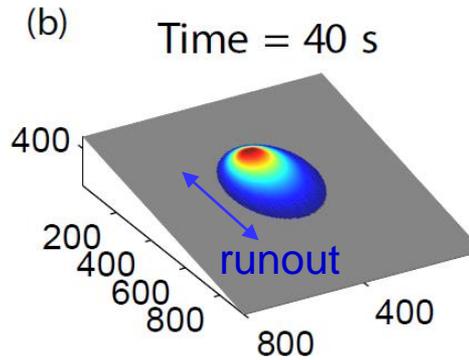
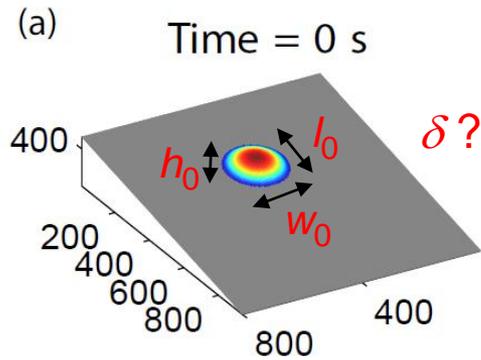


- **Strong correlation between rockfall volume and cumulative magnitude/number of Volcano-Tectonic events (time-lag 1-20 days)**
- Weak correlation with maximum amplitude of volcano-tectonic seismicity
- Moderate correlation with rain with time-lag of 1-8 days

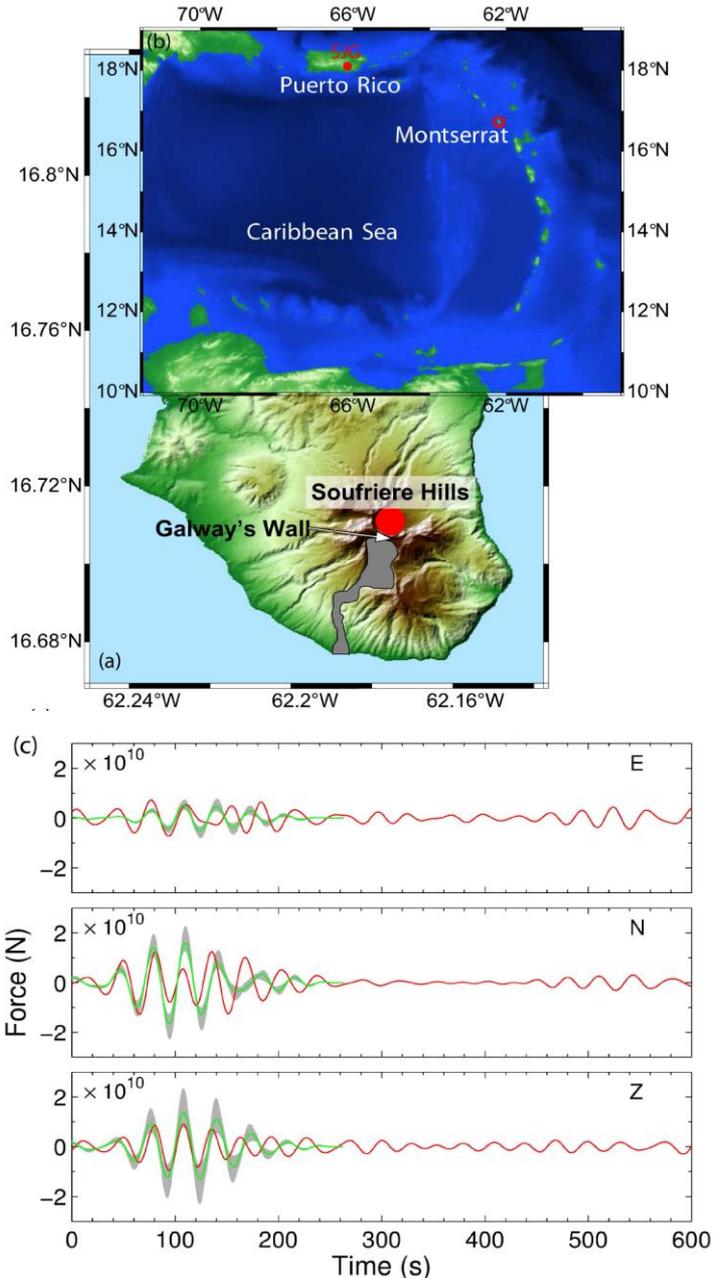
*Bontemps et al., 2018*

# Bayesian inversion of landslide characteristics

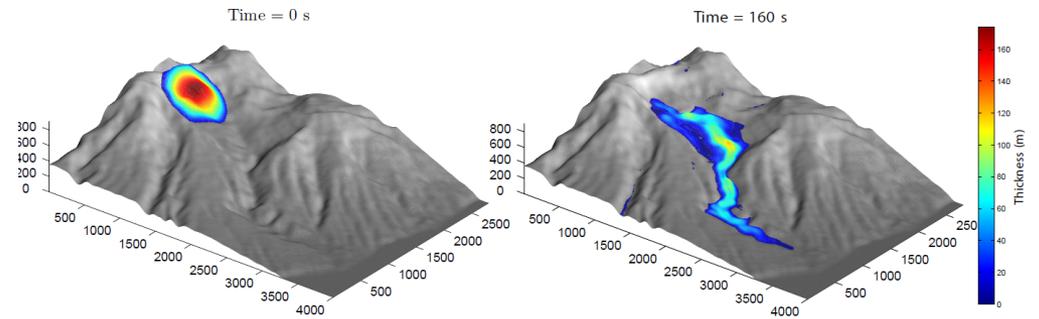
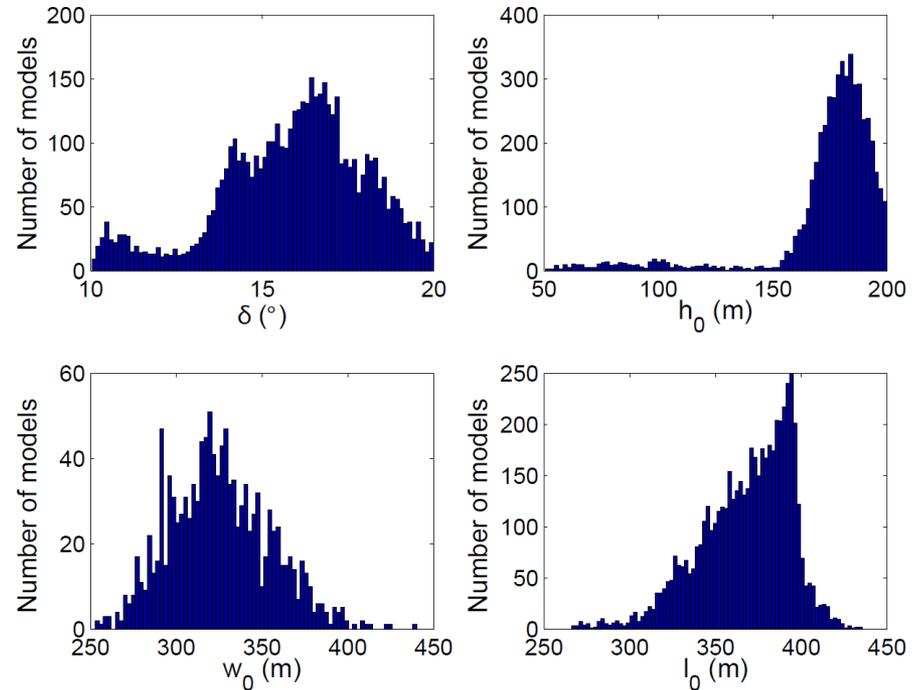
Synthetic data



# Bayesian inversion of landslide characteristics



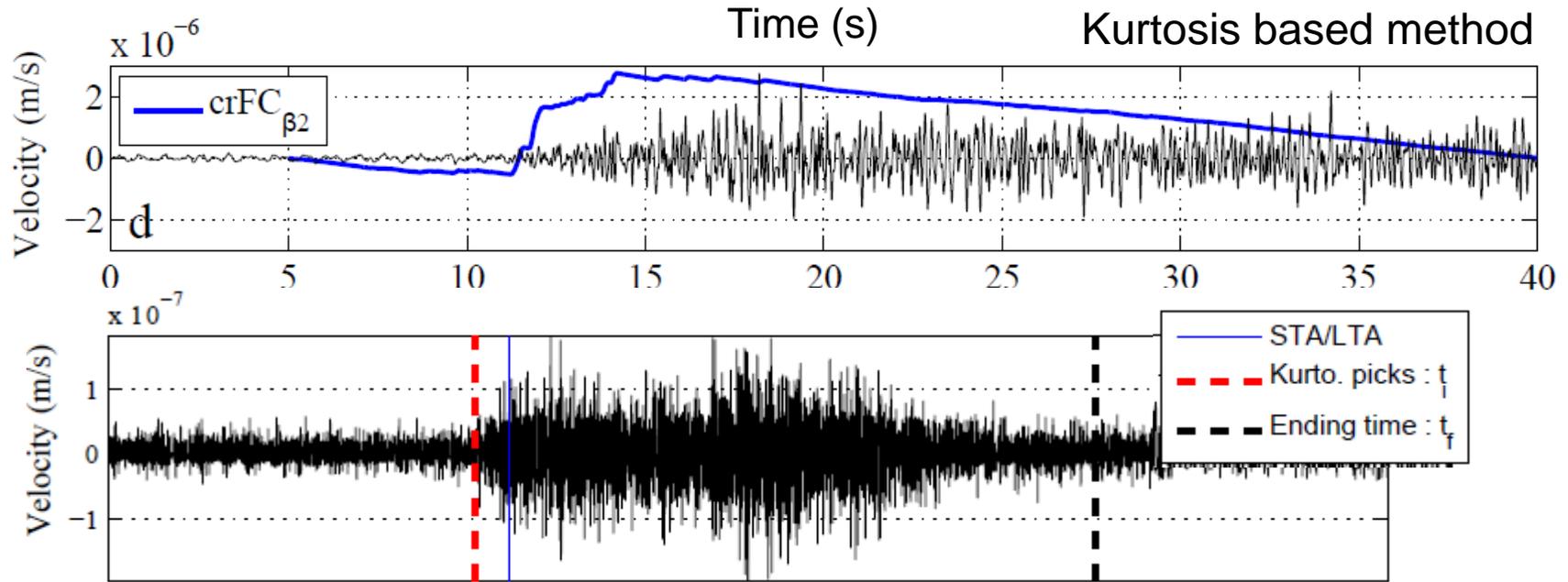
## Boxing Day debris avalanche, Montserrat



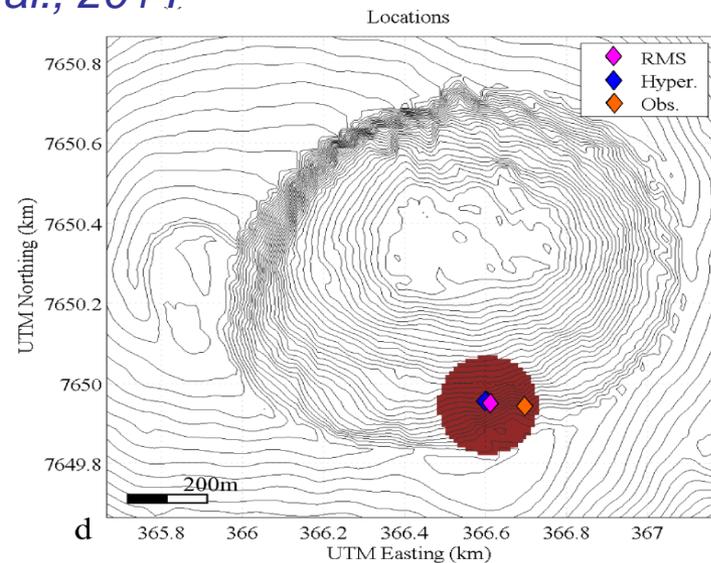
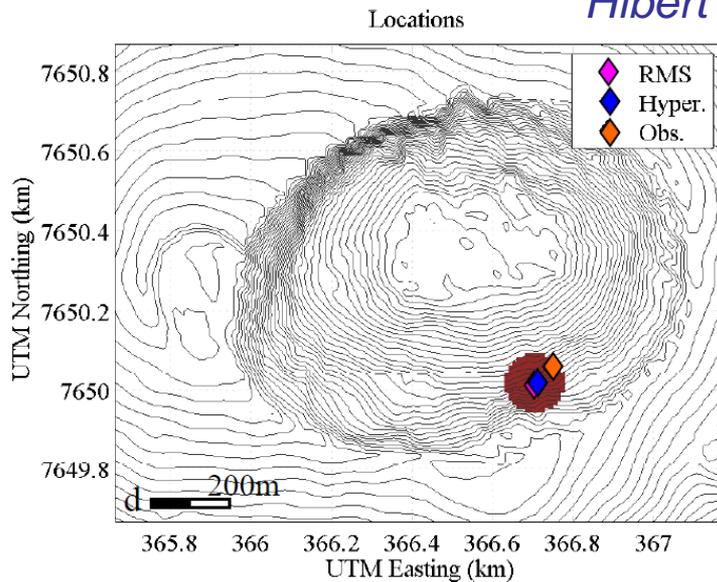
$V \in [32 \ 59] \text{ Mm}^3$  with a central value of  $V=45.8 \text{ Mm}^3$

*Moretti et al. 2017*

# High frequency Detection, localization, monitoring

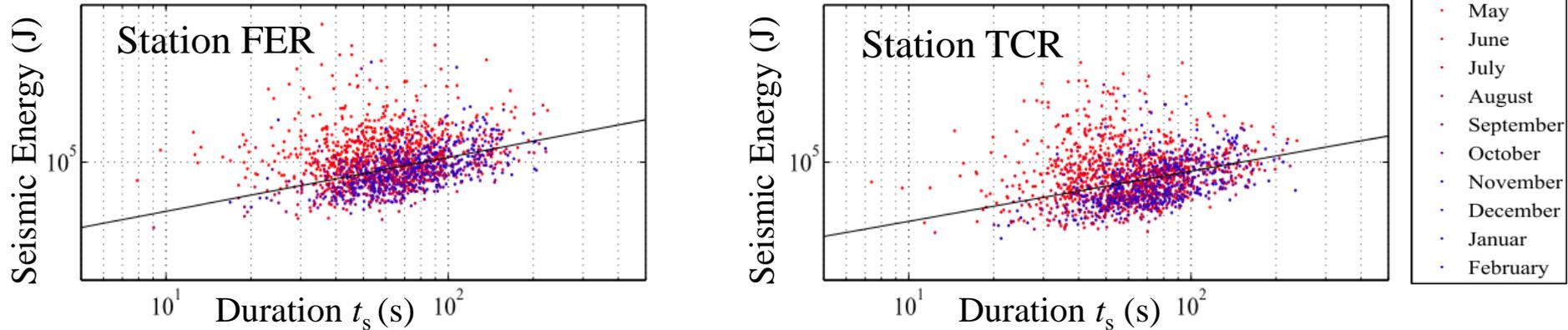


*Hibert et al., 2014*

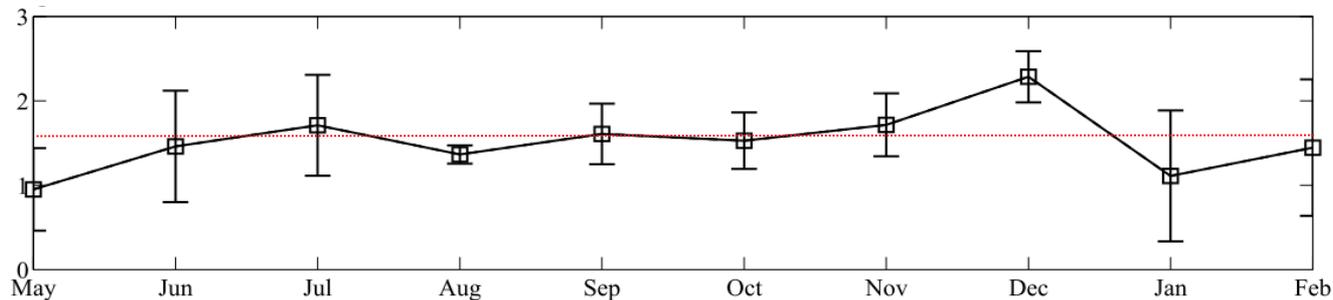


# Power law: seismic energy versus duration

**Seismic energy :** 
$$E_s = \int_{t_1}^{\infty} 2\pi r \rho h c u_{env}(t)^2 e^{\alpha r} dt$$
 *Vilajosana et al., 2008*



Regression lines and corresponding coefficients computed for each month



**Scaling law** between seismic energy and duration :

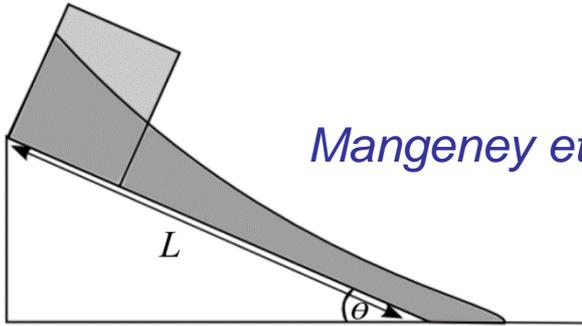
$$E_s \propto t_s^{\beta_s}$$

with

$$\beta_s \approx 1.56$$

# Power law: potential energy versus flow duration

- Analytical development for a rectangular mass on a flat slope



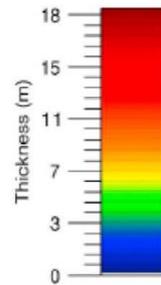
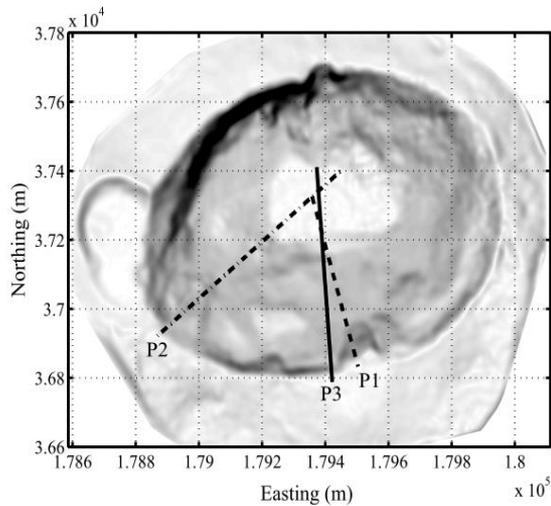
*Mangeney et al., 2010*

$$\Delta E_p \propto t_f^{\beta_a}$$

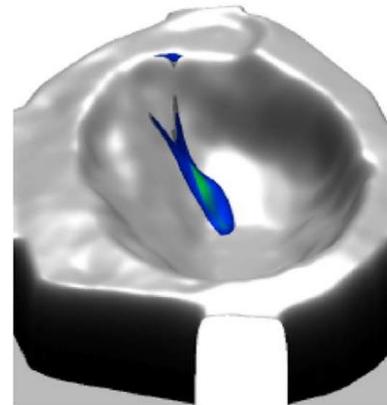
with

$$\beta_a = 2$$

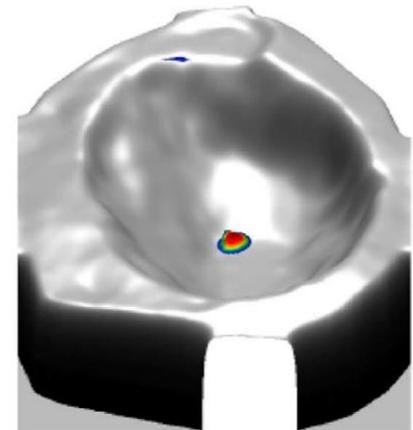
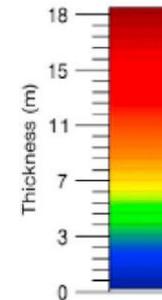
- Numerical simulation of granular flows over real topography using the code SHALTOP *Mangeney et al., 2007*



Time: 20 secs.



Time: 60 secs.



## Topography Effects

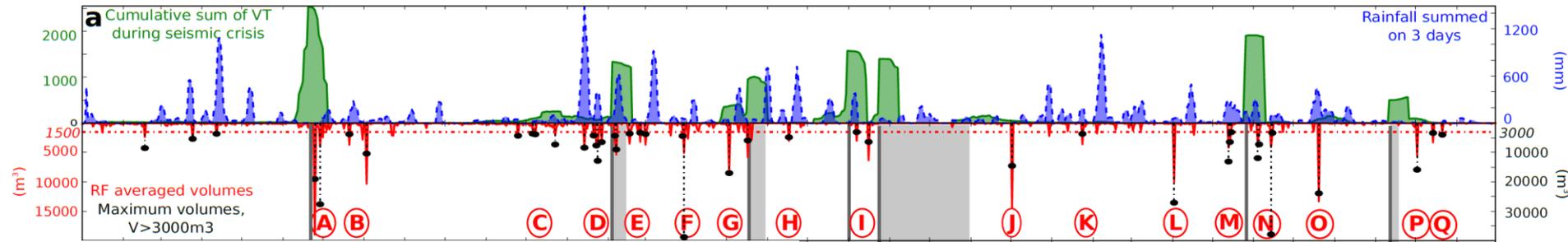
Rugosity  $\nearrow \Rightarrow \beta_p \searrow$

$$\Delta E_p \propto t_f^{\beta_p}$$

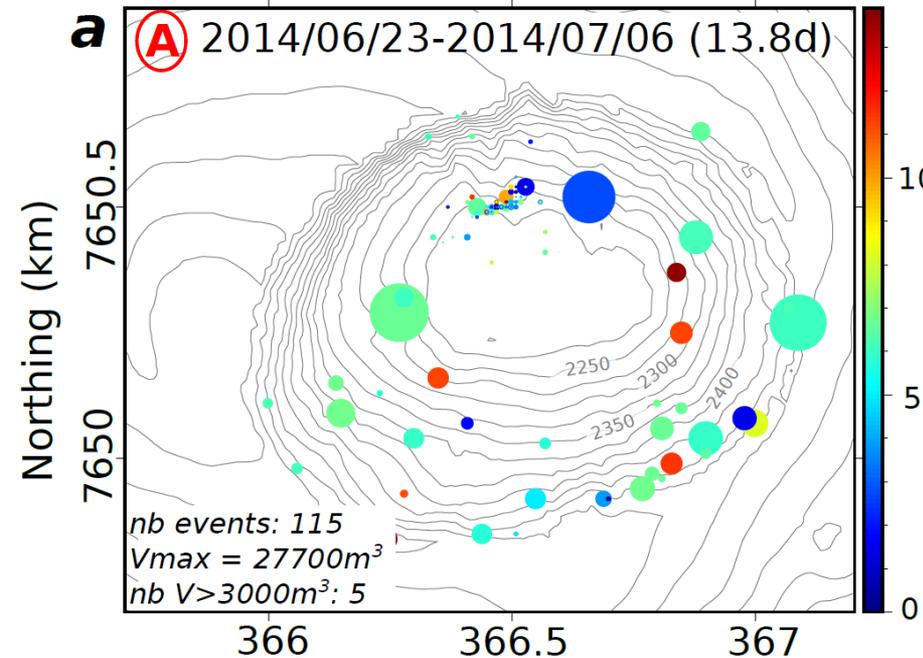
with

$$\beta_p = 1.65$$

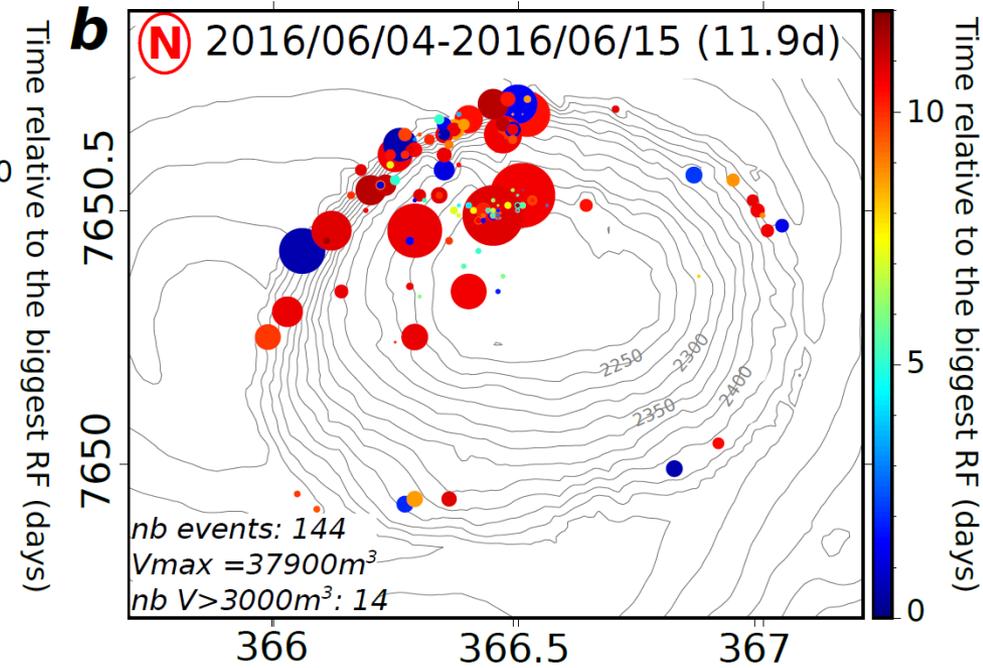
# Spatio-temporal distribution



**a** (A) 2014/06/23-2014/07/06 (13.8d)



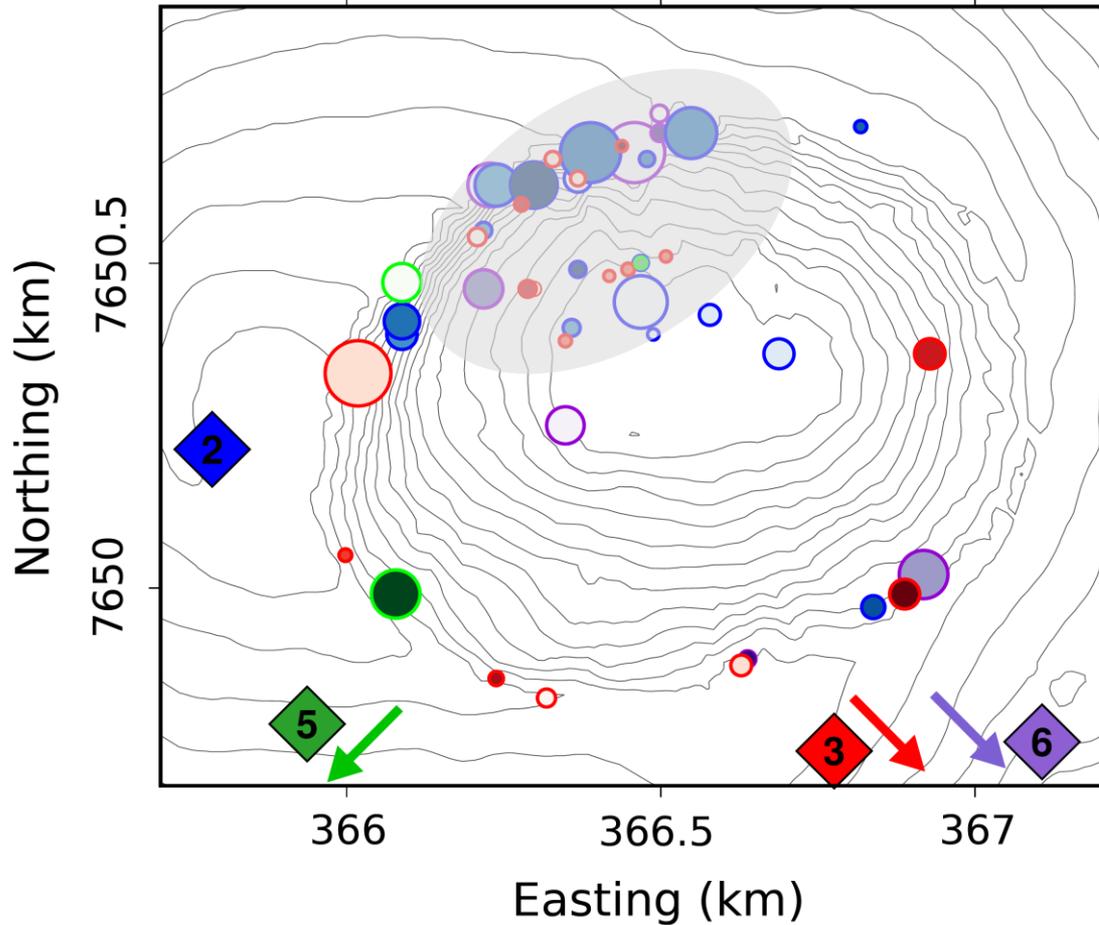
**b** (N) 2016/06/04-2016/06/15 (11.9d)



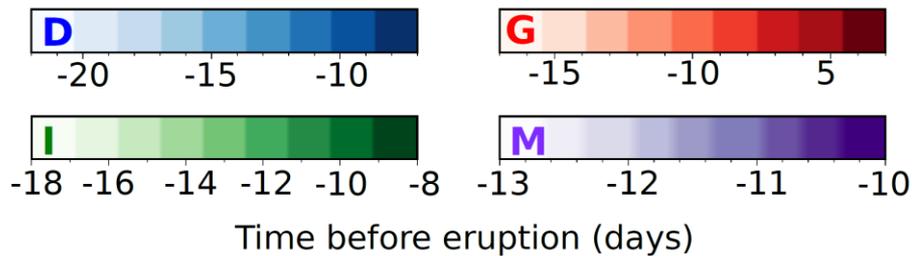
• Strong heterogeneity !

# Rockfall triggering

*Durand et al., 2018*



Large rockfalls seem to occur close to the next eruption ?

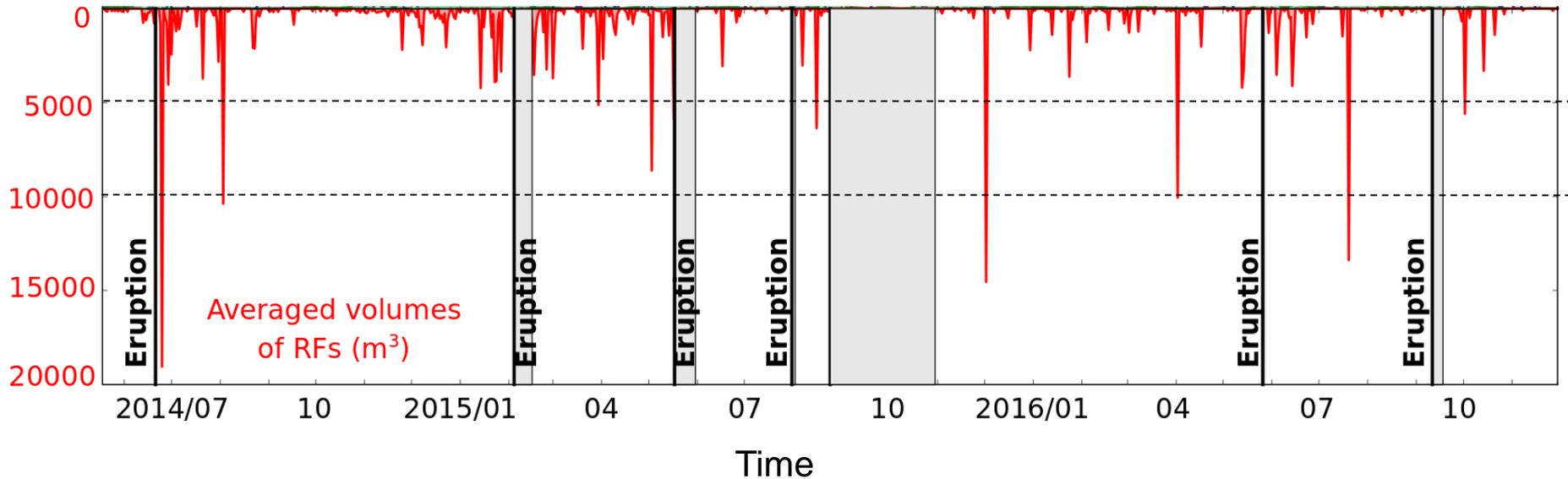


# Rockfall triggering

Rockfall crises with volume  $V > 3000 \text{ m}^3$

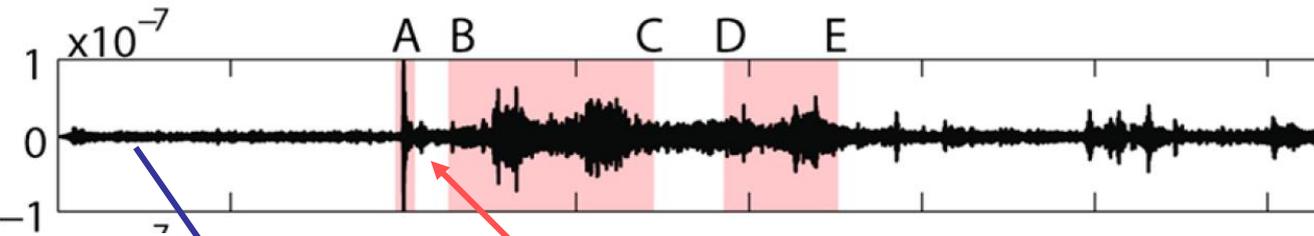
*Piton de la Fournaise*

*Durand et al., 2018*



Correlation between rockfall volume and external forcing  
(3-year time series)?

# Permanent seismic waves all around us

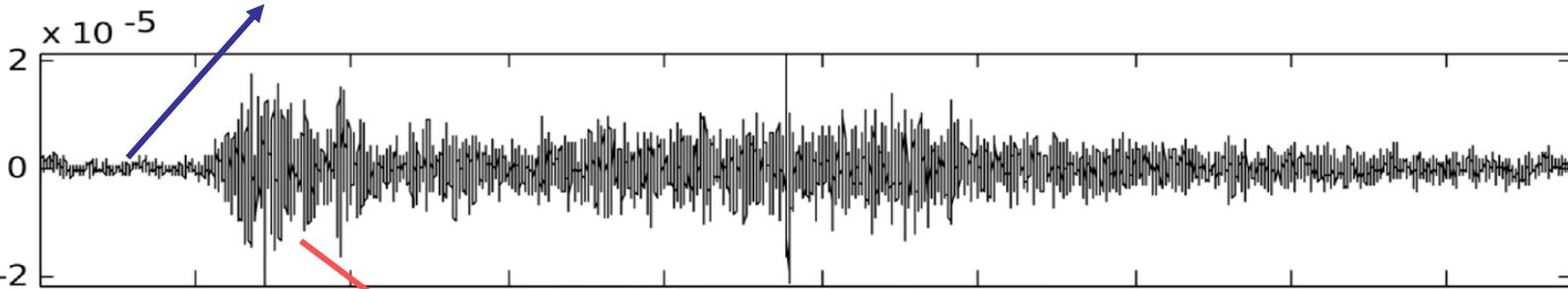
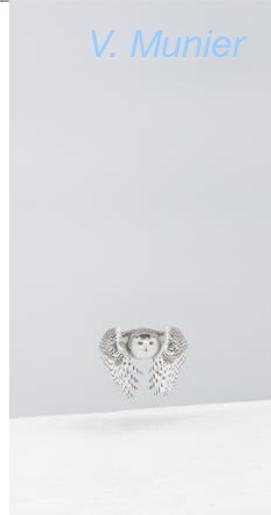


Ice avalanche and iceberg calving



Magnitudes up to 4-5  
Recorded at 10's to 100's km !  
Huge database since 10's years

V. Munier

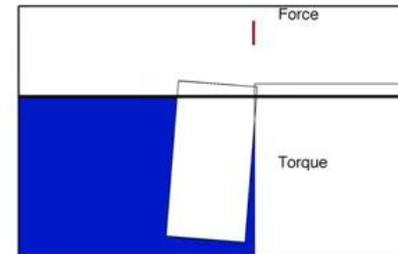
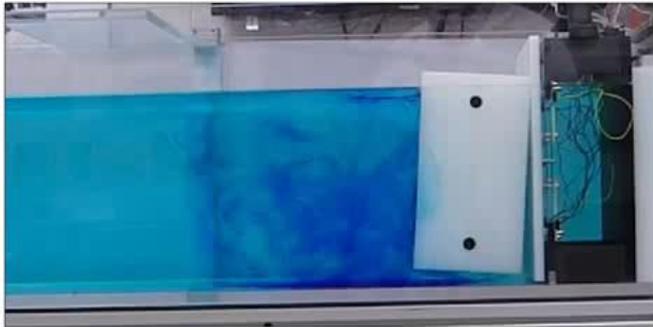
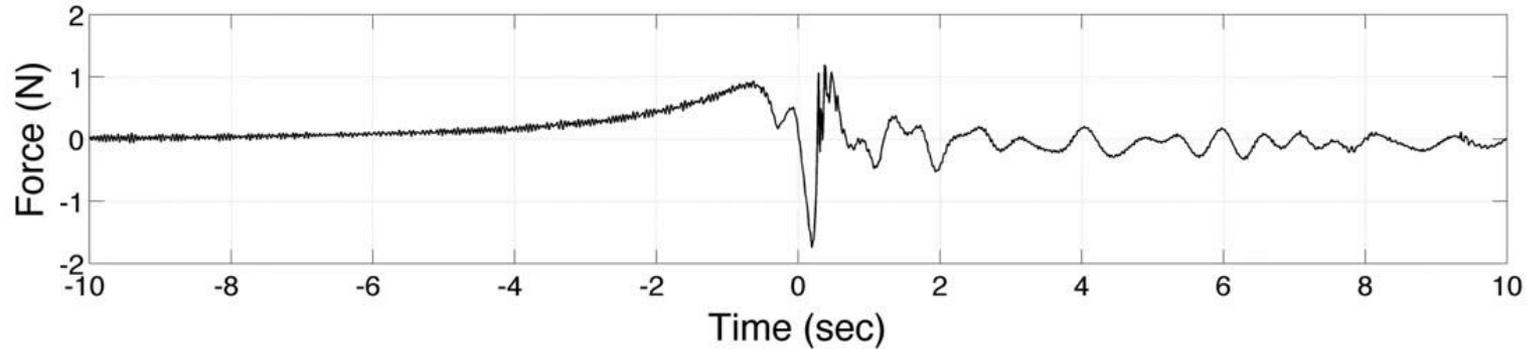


Landslide



# Physical processes at the origin of seismic waves ?

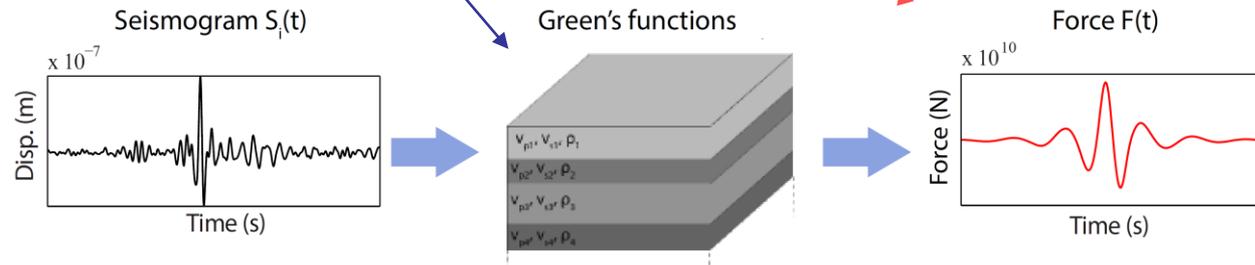
Laboratory experiments of iceberg calving



# Recovering the force due to iceberg calving

Force inversion

$$\mathbf{u}_i(x, t) = \mathbf{g}_{ij}(x, t) * \mathbf{F}_j(0, t)$$

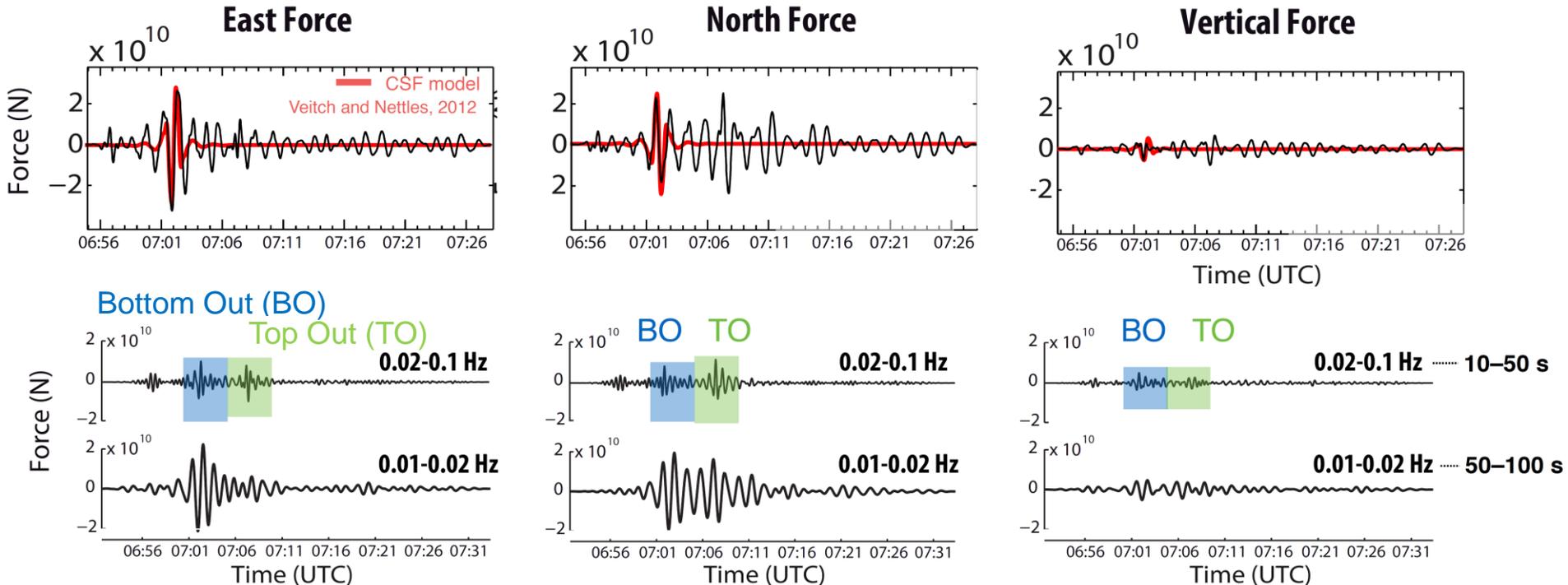


Low frequency

0.01-0.1 Hz -- 10-100 s

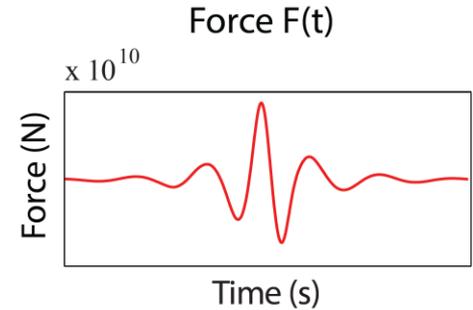
Jakobshavn Isbrae, Greenland

Sergeant et al. 2016



# What do we expect from calving modelling ?

- ▶ Can we reproduce the force inverted from seismic records ?
- ▶ Can we estimate iceberg volume from the force ?

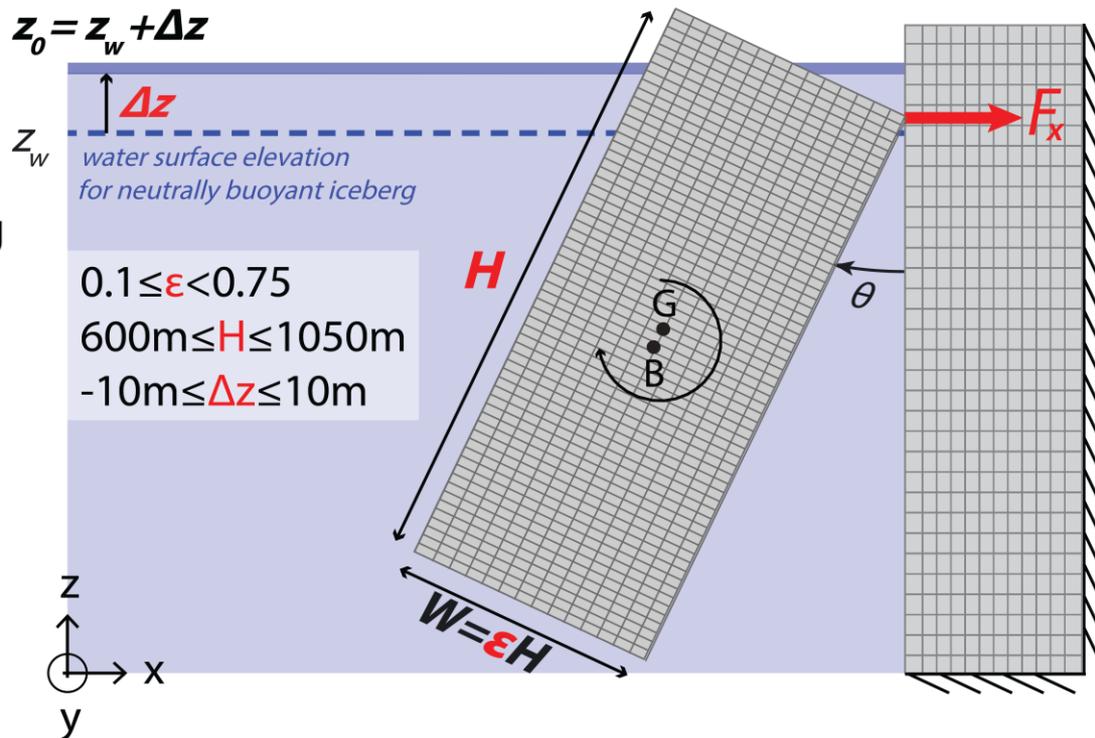
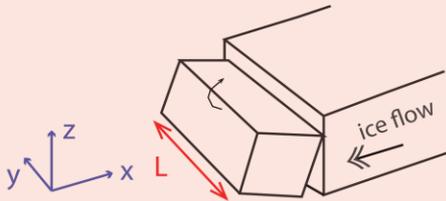


## Finite element model Z-set

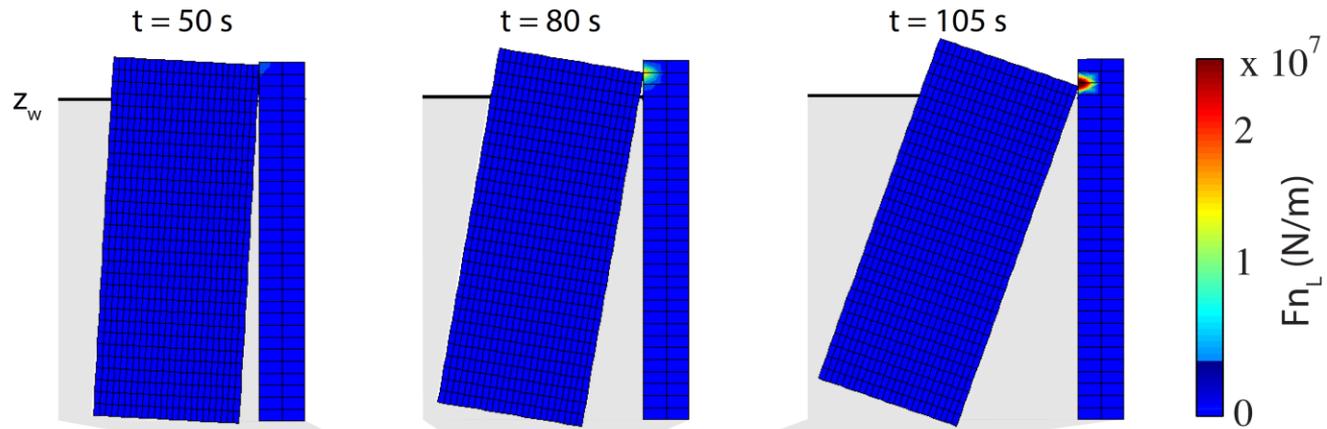
- ▶ Contact force computation with varying geometrical parameters:

### Total force from 2D modeling:

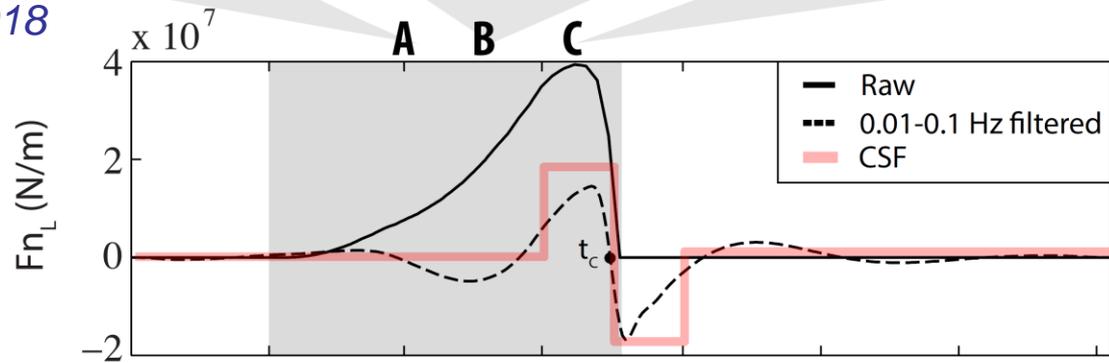
- ▶  $F^T(t) = LF(t)$
- ▶ Iceberg length  $500m \leq L \leq 5000m$



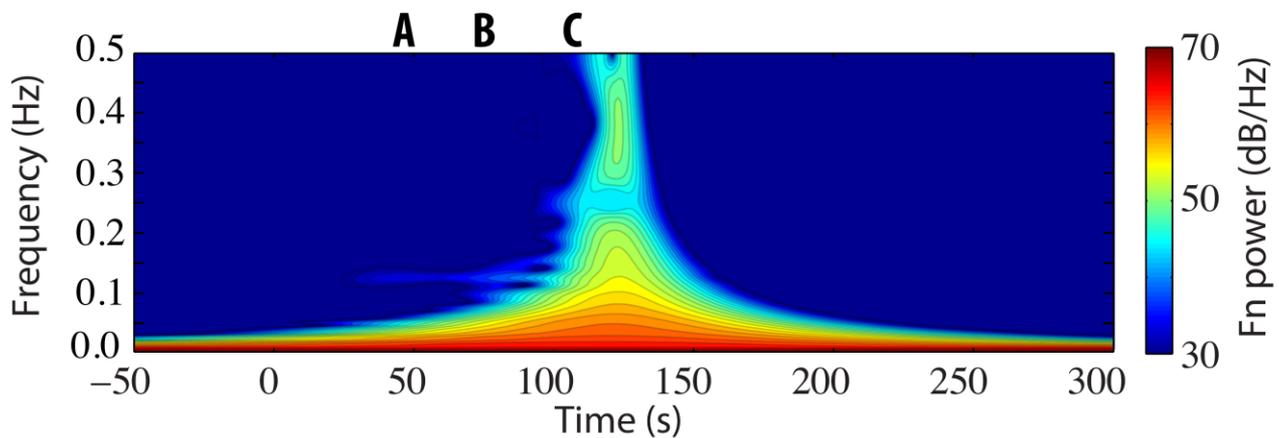
# Simulation of the contact force



*Sergeant et al. 2018*

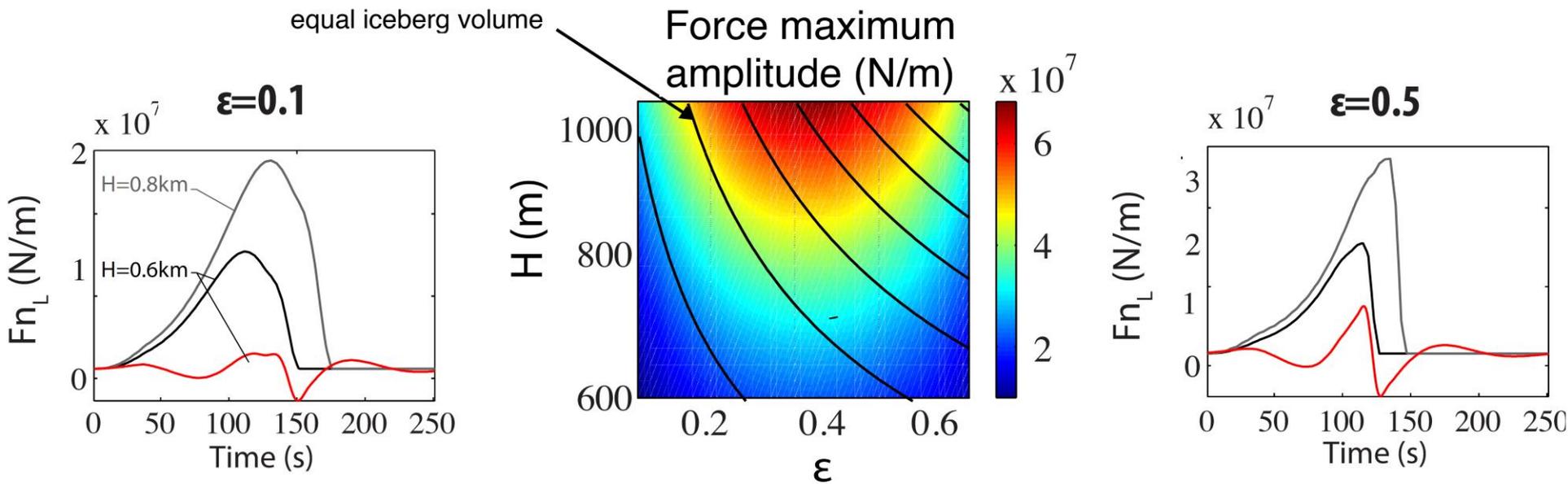


10-100s of periods



# Force dependency on iceberg characteristics

- ▶ Catalog of 1000 models for BO capsizes and varying values of  $H$  and  $\epsilon$ .
- ▶ Filtered force amplitudes match seismic observations (*Veitch an Nettles, 2012, Sergeant et al., 2016*)



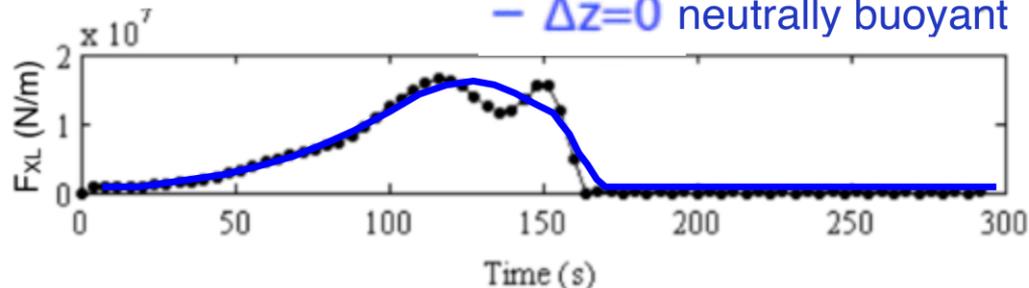
- ▶ Force amplitude, duration and history vary with  $H$  and  $\epsilon$ .

Impossible to recover the volume from  $F_{\max}$  :  
**need of full force history  $F(t)$  !**

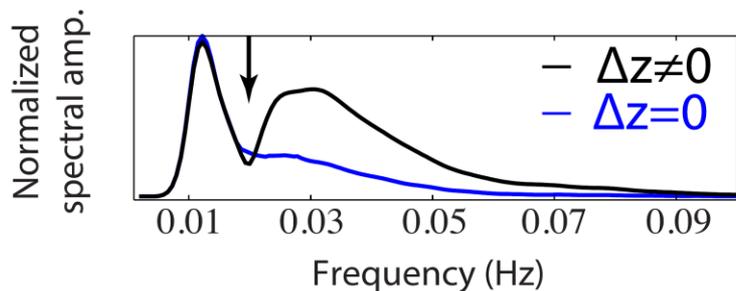
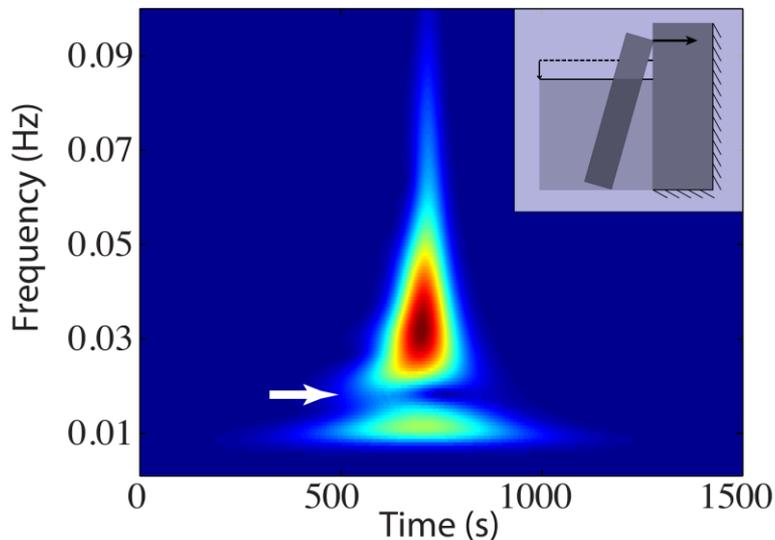
**Force Bottom-out**  
 $\neq$   
**Force Top-out**

# Influence of initial buoyant conditions

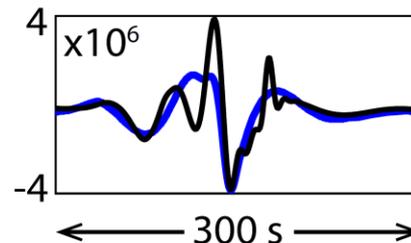
—  $\Delta z \neq 0$  out-of-equilibrium  
—  $\Delta z = 0$  neutrally buoyant



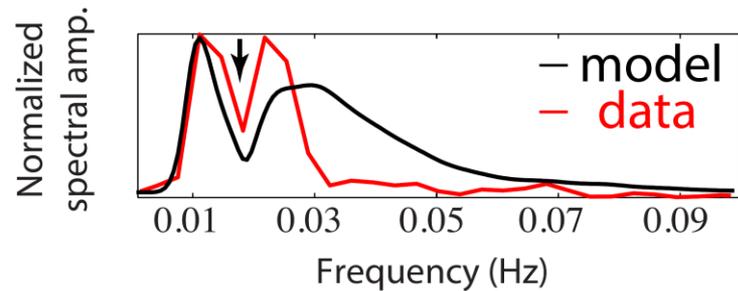
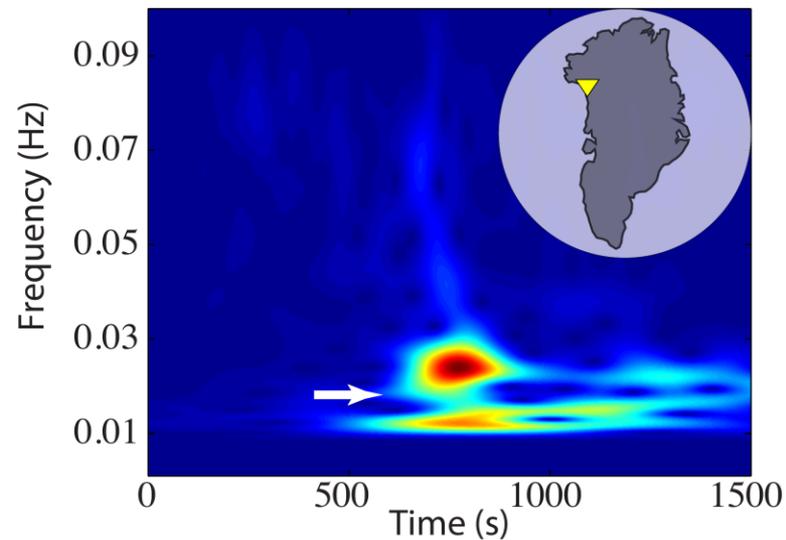
Modeled force



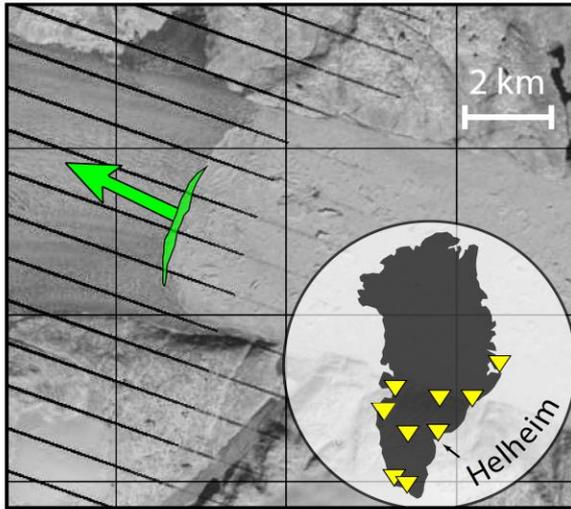
0.01-0.1 Hz force



Force inverted from seismic data



# From the force to the iceberg volume

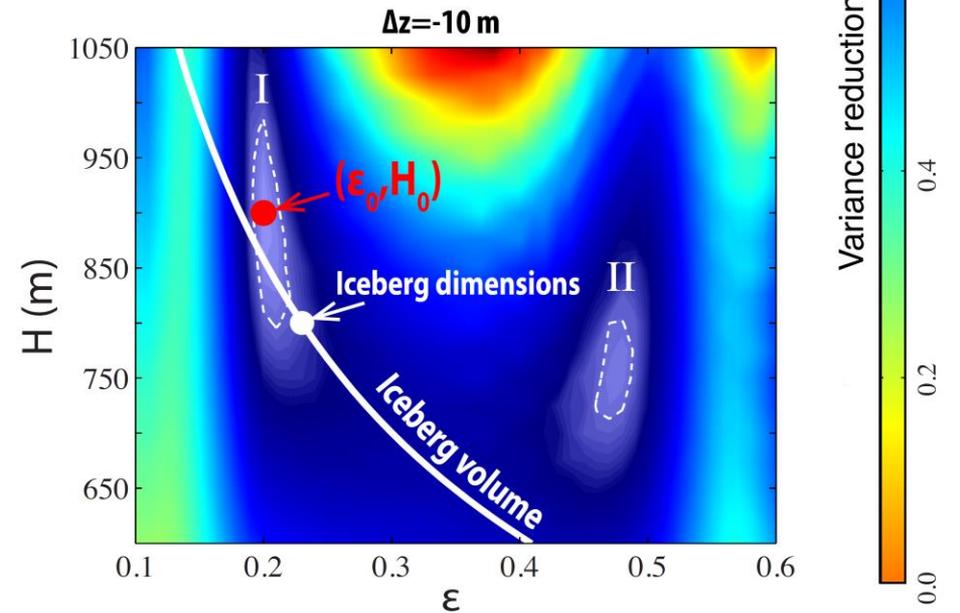
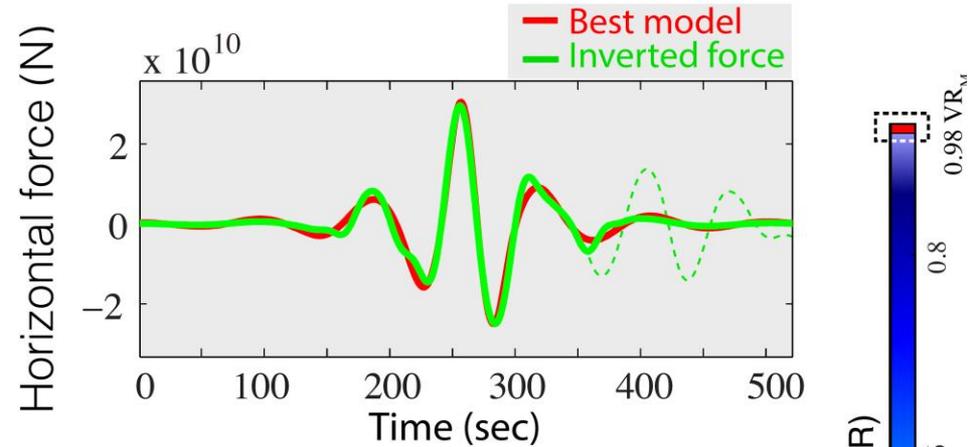


- ▶ BO capsizes captured on cameras  
(Murray et al., Science, 2015):

25/07/2013:  $H \approx 800$  m  
 $\epsilon \approx 0.23$   
 $L \approx 2500$  m  
 $\rightarrow \epsilon H^2 L \approx 0.37$  km<sup>3</sup>

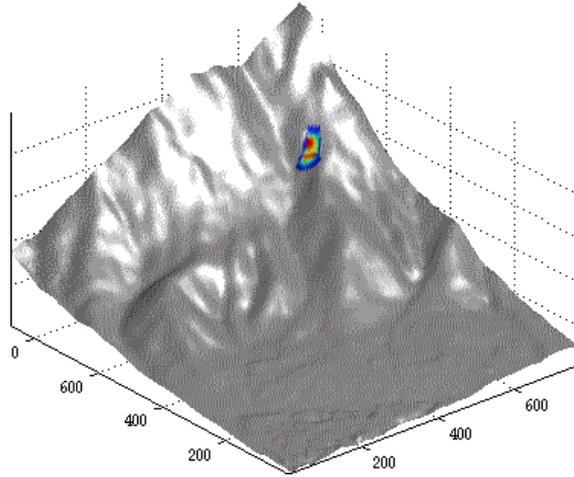
$H_0 = 900$  m  
 $\epsilon_0 = 0.2$   
 $L \approx 2500$  m  
 $\epsilon H^2 L = 0.4$  km<sup>3</sup>

- ▶ Glacial earthquake (force  $\sim 3 \times 10^{10}$  N)



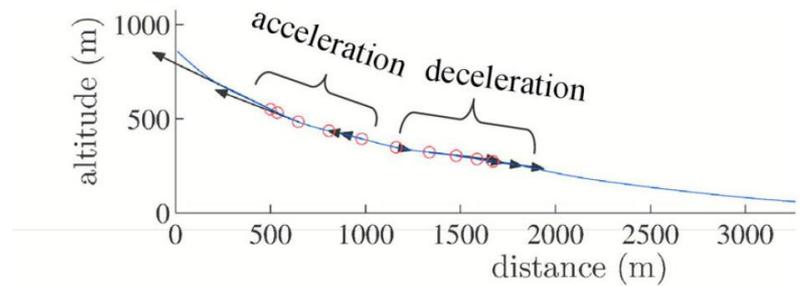
Sergeant et al. 2018

# Modelling of the basal force

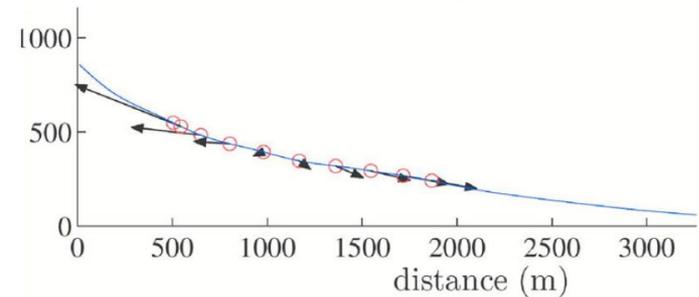


*Favreau, Mangeney, Lucas, Crosta, Bouchut 2010*  
*Moretti, Mangeney, Capdeville, Stutzmann, et al. 2012*

$\Delta F$  for a block sliding in the White River valley

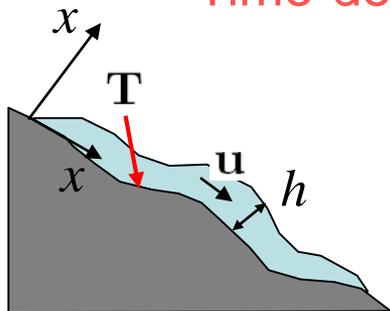


(c)  $\Delta F$  for a 2D deformable granular mass



*Zhao, Moretti, Mangeney, Stutzmann, Kanamori, Capdeville, Calder, Hibert et al., 2015*

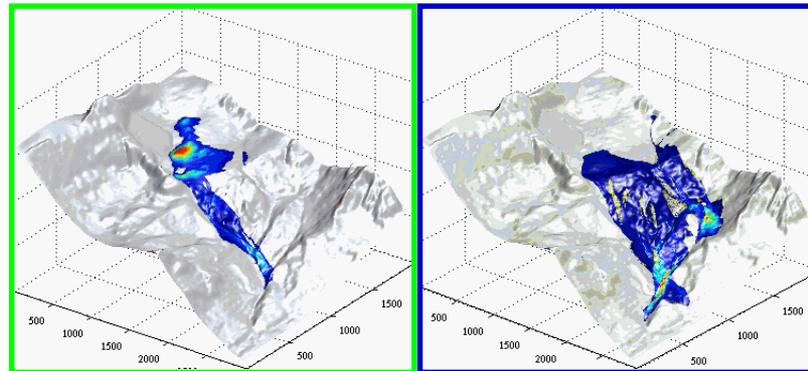
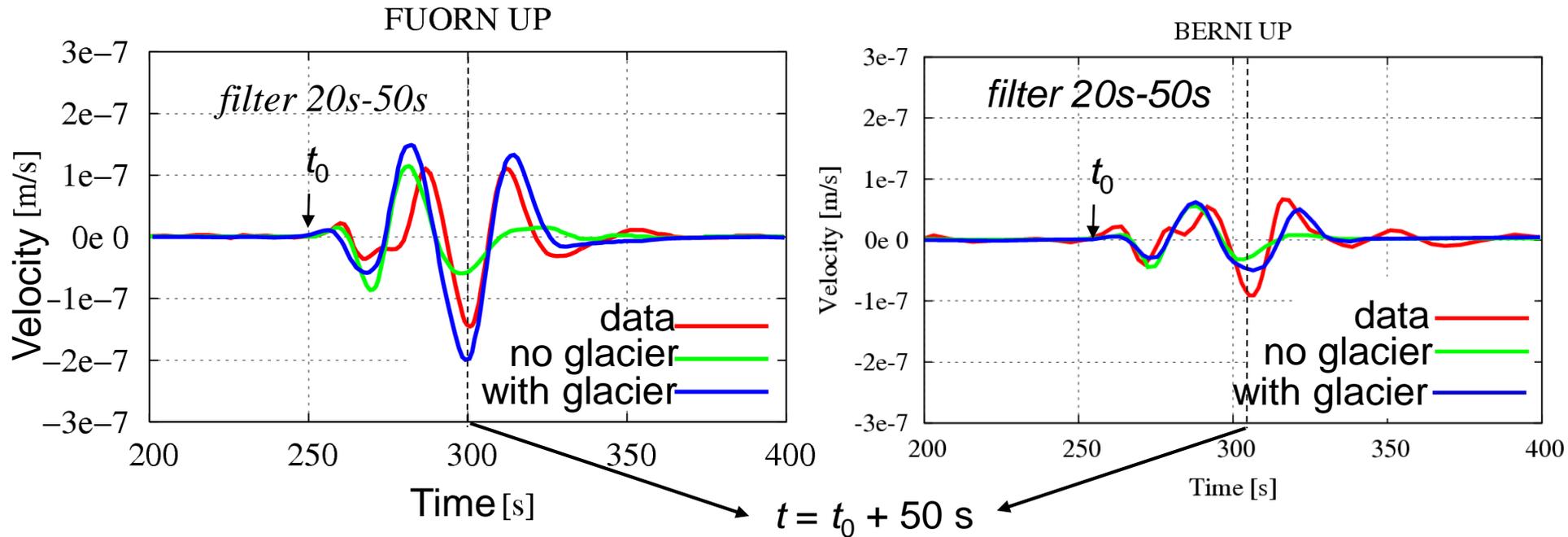
Time-dependent basal stress field applied on top of the terrain



$$\mathbf{T} = \rho g h \left( \cos \theta + \frac{\mathbf{u}_h^t \mathcal{H} \mathbf{u}_h}{g \cos^2 \theta} \right) \left( \mu \frac{u_X}{\|\mathbf{u}\|}, \mu \frac{u_Y}{\|\mathbf{u}\|}, -1 \right)$$

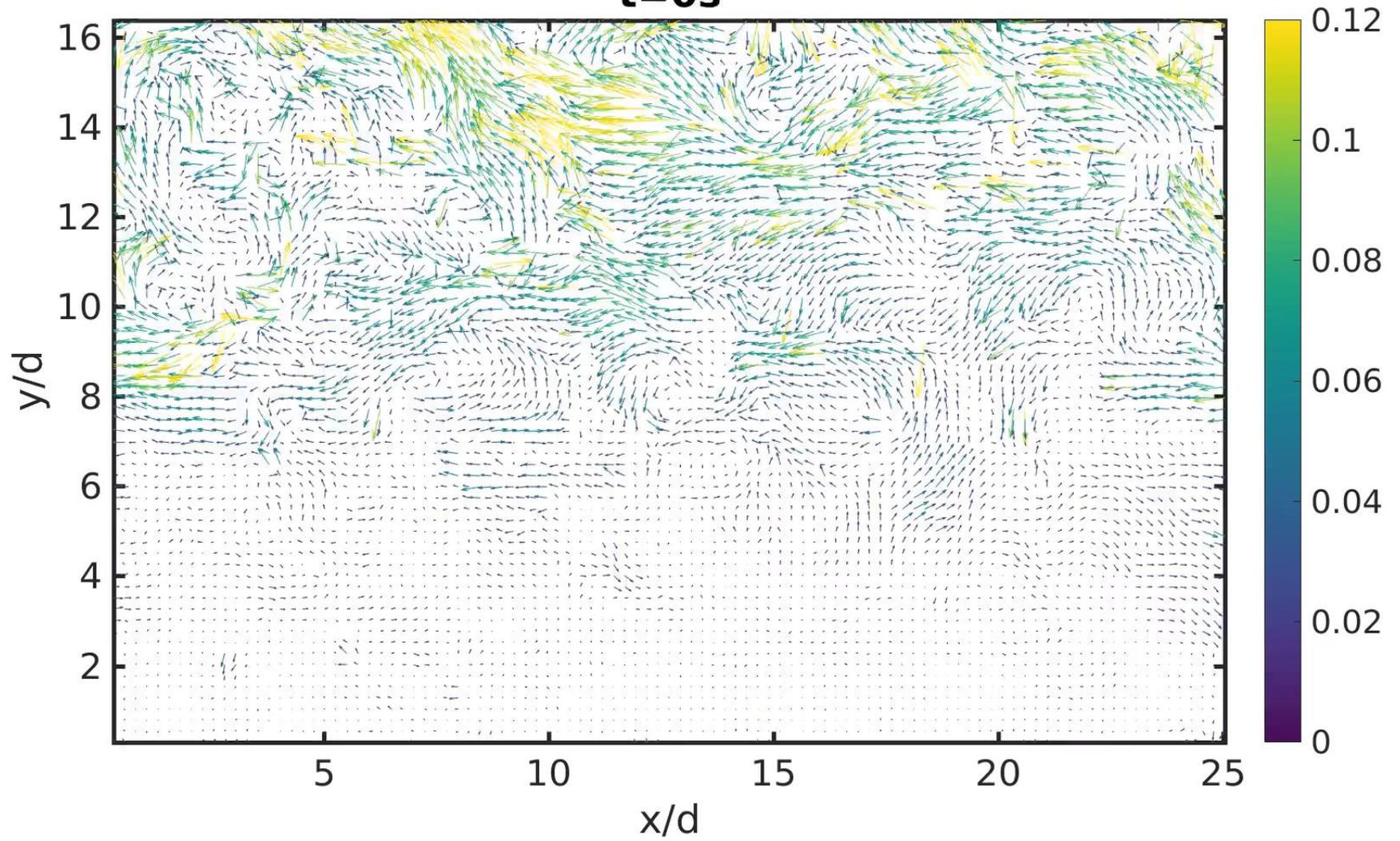
Curvature effects

# Thurweiser landslide seismic waves



- The scenario **with glacier** better reproduces the vertical waveform

**t=0s**



# Acoustic waves in laboratory experiments

Grain impact

Seismic signal



Impactor's characteristics



*On thin plates*

mass  $m \propto W_{el}^{3/16} f_{mean}^{-33/16}$

speed  $V_z \propto W_{el}^{5/16} f_{mean}^{25/16}$

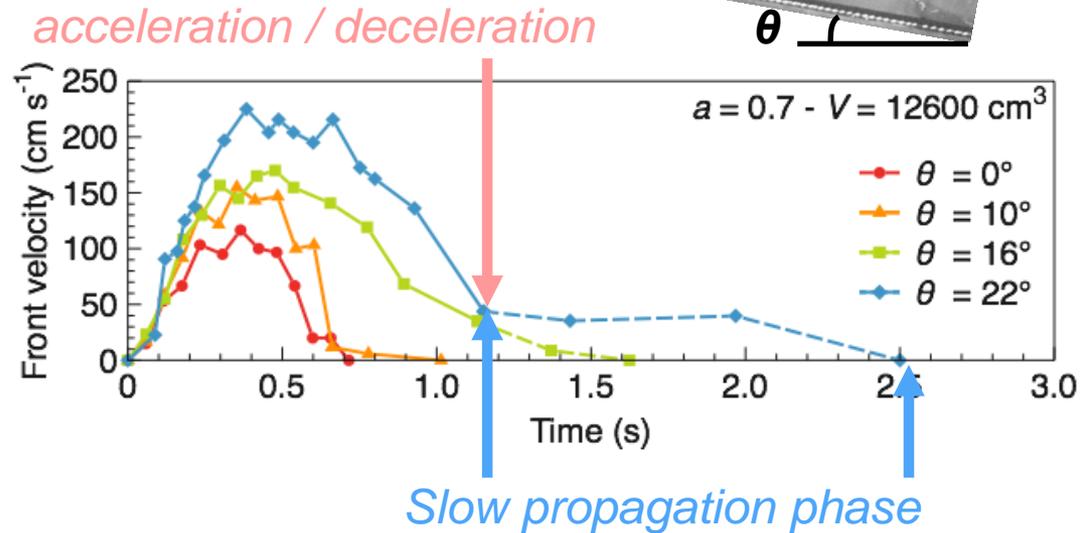
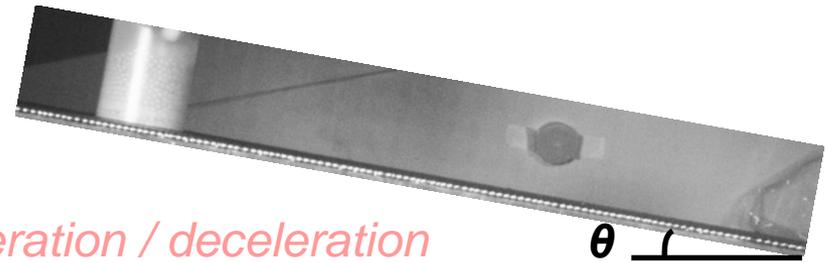
$W_{el}$  = radiated seismic energy

$f_{mean}$  = mean frequency

**Validated experimentally**

Farin, Mangeney, Toussaint, De Rosny, Shapiro, Dewez, Hibert et al. 2015

Granular flows on sloping beds



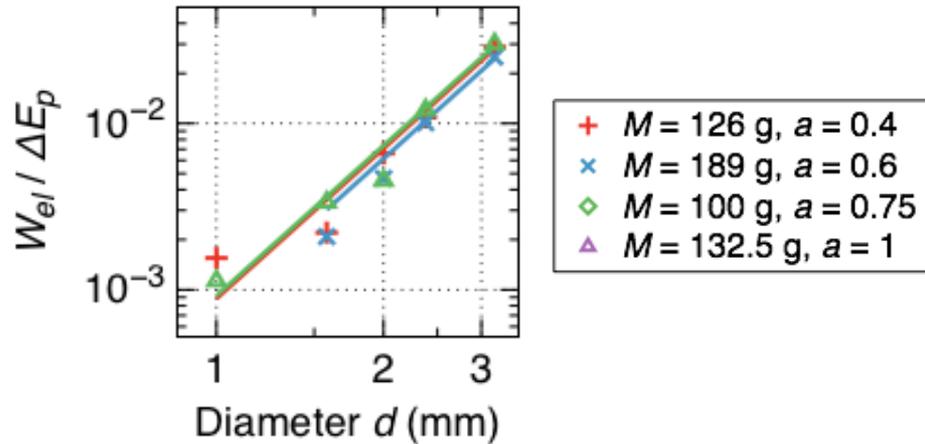
The dynamic regime of granular flows changes at high slopes  $\theta > 15^\circ$

**Seismic signal ?**

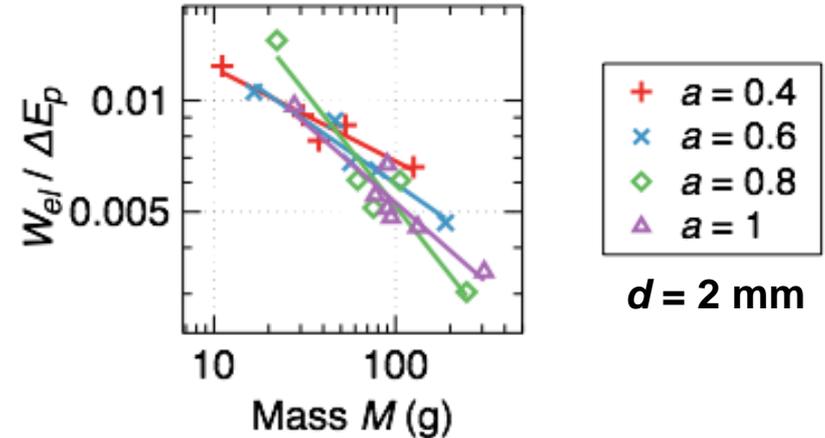
Mangeney et al. 2010, Farin et al. 2014, 2015

# Seismic efficiency

Diameter



Mass



$$W_{el} / \Delta E_p \propto d^3$$

$$W_{el} / \Delta E_p \propto M^{-0.4}$$

$W_{el} / \Delta E_p$  depends essentially on particle diameter, flow mass and slope angle

May explain dispersion observed on the field  $W_{el} / \Delta E_p = 10^{-5} - 10^{-3}$

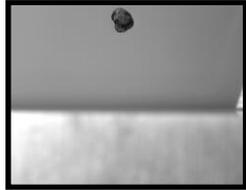
*e.g. Hibert et al. 2011, 2014, Lévy et al. 2015*

*Farin, Mangeney, De Rosny, Toussaint, Trinh, 2017*

# Laboratory/field for impacts/flows

## Single impacts

### Laboratory



*Farin et al., 2014*

### Field

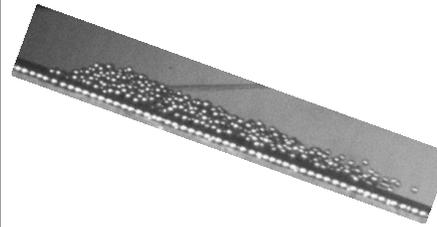


*Farin et al., 2014*

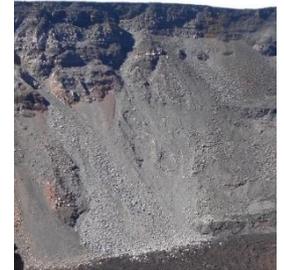
**/ 5 - 20**

## Granular flows

### Laboratory



### Field



Characteristic  
Frequency

**~ 100 kHz**

**~ 100 Hz**

**/ 1000**

**/ 10-100**

**~ 5 - 20 kHz**

**/ 1000**

**~ 5 - 20 Hz**

**deduced  
values**

Seismic  
efficiency  
 $W_{el} / \Delta E_p$

**~ 0.2**

**$10^{-3} - 10^{-2}$**

**/ 20 - 200**

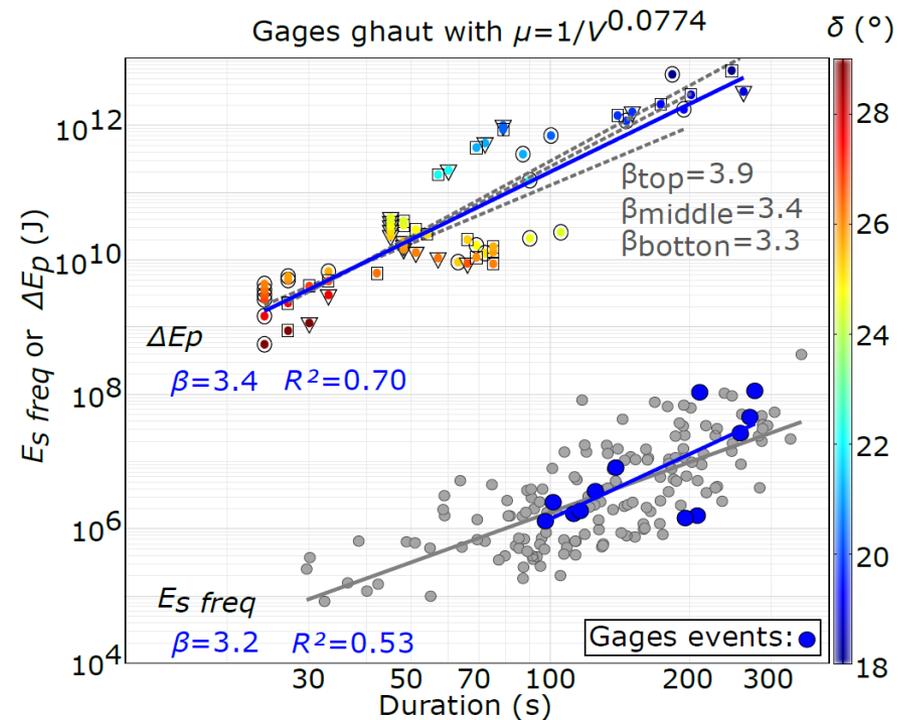
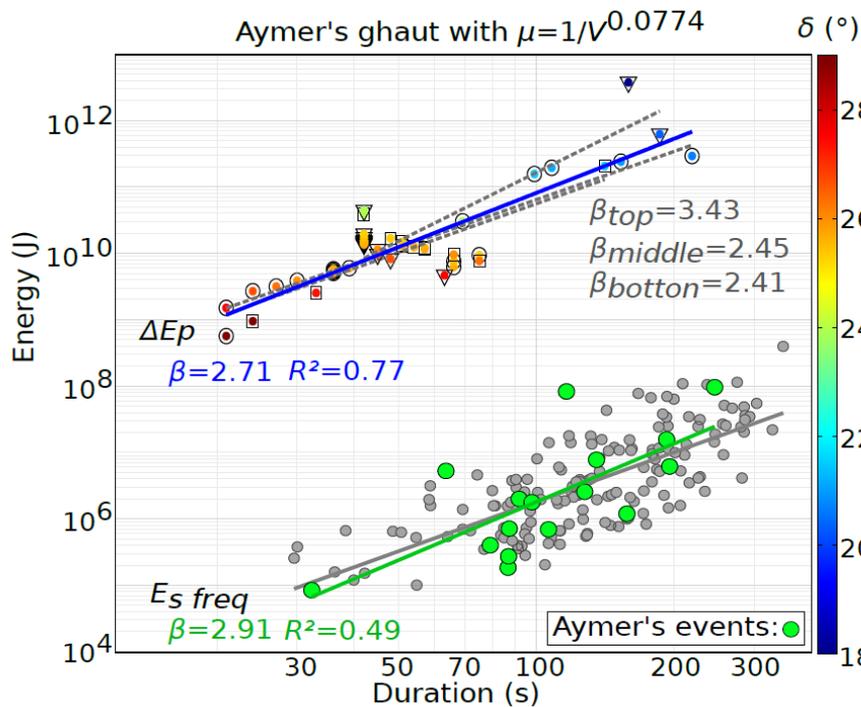
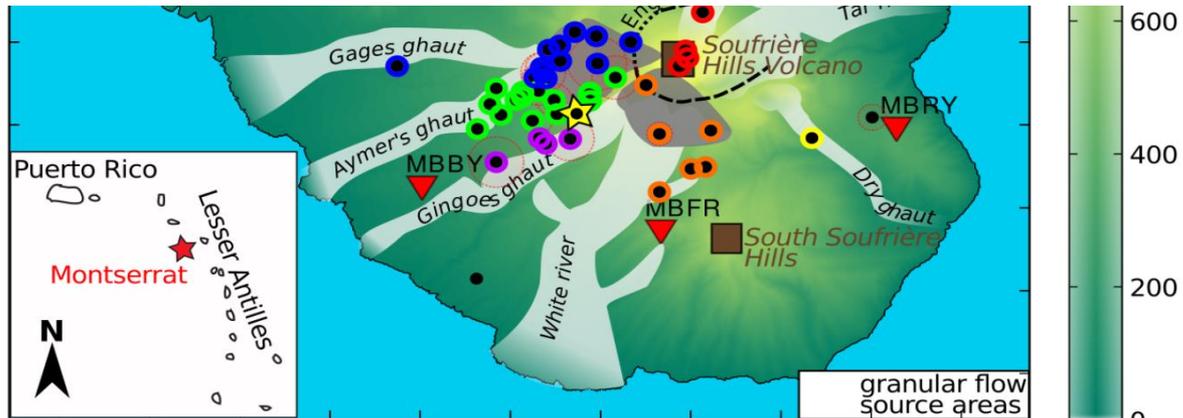
**$2 \cdot 10^{-3} - 2 \cdot 10^{-2}$**

**/ 20 - 200**

**$10^{-5} - 10^{-3}$**

**in agreement with  
field studies**

# Friction weakening signature on seismic data



The parameters of the power law depend on the valley !

# Granular flows over complex topography

$$\partial_t(h/c) + \nabla_x \cdot (hu') = 0$$

$$\partial_t \mathbf{u}' + c\mathbf{u}' \cdot \nabla_x \mathbf{u}' + \frac{1}{c}(\text{Id} - \mathbf{s}\mathbf{s}^t)\nabla_x(g(hc + b)) =$$

$$\frac{-1}{c}(\mathbf{u}'^t H \mathbf{u}')\mathbf{s} + \frac{1}{c}(\mathbf{s}^t H \mathbf{u}')\mathbf{u}' - \frac{g\mu c\mathbf{u}'}{\sqrt{c^2\|\mathbf{u}'\|^2 + (\mathbf{s} \cdot \mathbf{u}')^2}} \left(1 + \frac{\mathbf{u}'^t H \mathbf{u}'}{gc}\right) +$$

$$\vec{n} = \left( -\frac{\nabla_x b}{\sqrt{1 + \|\nabla_x b\|^2}}, \frac{1}{\sqrt{1 + \|\nabla_x b\|^2}} \right) \equiv (-\mathbf{s}, c) \in \mathbb{R}^2 \times \mathbb{R}$$

$$\mu = \mu_s$$

or

$$\mu(\text{Fr}, h) = \mu_s + \frac{\mu_2 - \mu_s}{\frac{\beta h}{\mathcal{L}\text{Fr}} + 1}$$

Other empirical terms can be added .... with more unconstrained parameters...

# Long period observed and simulated seismograms

Filtered between 20 and 80s

