News from magnetohydrodynamics and anomalous transport

Adrien Florio

with:

Arpit Das

Nabil Iqbal

Napat Poovuttikul

Work in progress

Le Studium, 5^{th} of July 2023

Context

- Improve understanding of hydrodynamics as universal dynamics
- Improve understanding of anomalous transport
- Reevaluate phenomenology

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- Improve understanding of anomalous transport
- Reevaluate phenomenology
  - Chiral magnetic effect (heavy ions)
  - Magnetogenesis, baryogenesis (cosmology)
  - Chiral Q-bits
  - ...

Plan

Hydrodynamics

Magneto Hydrodynamics (MHD)

Microscopic realization: Classical EM on a lattice

Application: Chiral decay rate Γ_5

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Microscopic realization: Classical EM on a lattice

Application: Chiral decay rate Γ_5

Main result: verified from micro. $\Gamma_5 \propto$ resistivity $\propto \lim_{t\to\infty} \langle E(t)E(0)\rangle$

Hydro from my UG

Euler,

Navier-Stokes, . . .

$$\partial_t(\rho v_i) + \partial_j(\cdots ?) = 0$$

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Poiseuille

Fig. 20. Chien curarisé par ingestion. — La branche horizontale d'un manomètre est engagée dans l'artère carotide [1].

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Hydro as an EFT

Symmetries $\longleftrightarrow$ Conservation laws

Equilibrium (static) state $\hat{\rho}$

Hydro: Systematic expansion of cons. laws around $\hat{\rho}$

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Relativistic hydro

$\hat{\rho}$: $\quad \mathrm{Tr}\left(e^{\frac{1}{T}u_\mu P^\mu}\right)$

$$u^\mu = (1-\mathbf{v}^2)^{-1/2}(1,\mathbf{v})$$

↑ Medium velocity

Typically $\mathbf{v} = 0$

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$\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$

Generically:

$$T_{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + (q^\mu u^\nu + q^\nu u^\mu) + t^{\mu\nu}$$

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Hydro:

$\mathcal{E}$

$\mathcal{P}$

q^μ

$t^{\mu\nu}$

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↑ 0^{th} order ("ideal")

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For example ("Landau frame")

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$$\sigma^{\mu\nu} = \Delta^{\alpha\nu}\Delta^{\beta\nu}\left(\partial_{\alpha} u_{\beta} + \partial_{\beta} u_{\alpha} - \frac{2}{d}\partial_{\alpha} u^{\alpha}\eta_{\mu\nu}\right)$$

ζ = bulk viscosity

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Recap #1

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- Built from conservation laws

$T_{\mu\nu} +$ other internal symmetries

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MHD =Magneto HydroDynamics

Goal: electrogmagnetic system

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Ohm's law: $J^\perp_\mu = \sigma \underbrace{F_{\mu\rho} u^\rho}_{E_\mu}$

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Power counting:

• $O(\partial^0)$: $u^\mu, T, \vec{B}$
• $O(\partial^1)$: $\partial u^\mu, \partial T, \vec{E}, \ldots$

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• Extra assumptions to hydro

• Ohm's law at weak coupling only

More "hydro EFT" approach?

Higher form MHD

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[Grozdanov, Hofman, Iqbal, 16]

$$J_{\mu\nu} = \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$$

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conserved "2-current"

Conservation # of magnetic lines!

See also [Landry, Liu, 22] for equivalent Schwinger-Keldysh derivation

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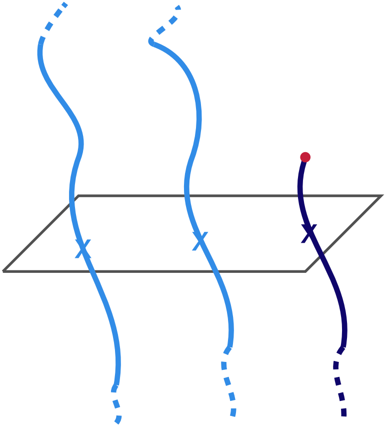

magnetic lines: 2

Particle

electric lines: 1

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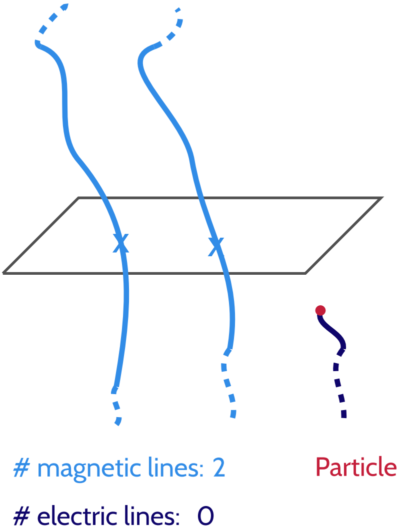
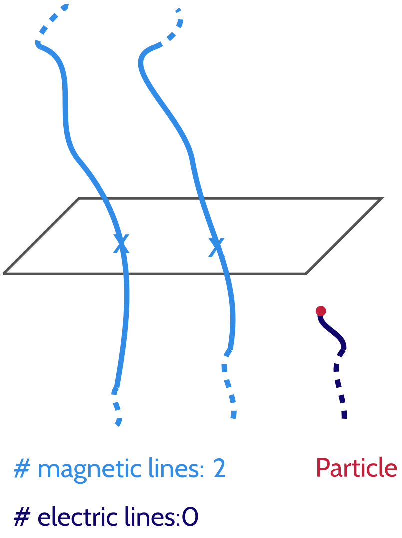


Hydro, again



$\hat{\rho}$: $\quad \mathrm{Tr}\left(e^{\frac{1}{T}\left(u_\mu P^\mu - \mu h_i \Phi^i\right)}\right)$

$\Phi^k = \frac{1}{2}\int B^k \epsilon_{kij} \mathrm{d}x^i \mathrm{d}x^j$ (no k-sum)

$$u^\mu = (1-\mathbf{v}^2)^{-1/2}(1, \mathbf{v})$$

↑ Medium velocity

$$h^\mu = (1-\mathbf{h}^2)^{-1/2}(0, \mathbf{h})$$

↑ Direction along field lines

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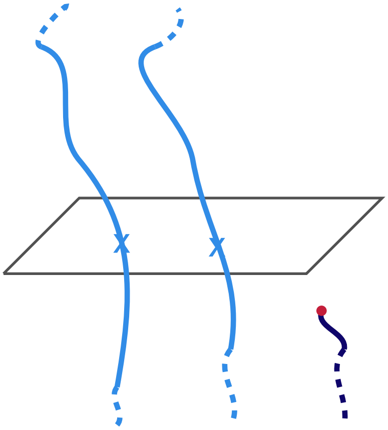

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For instance:

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r vs σ

Retarded propagator:

$$G_R^O(x,t) = \theta(t)\langle[O(x,t)O(0,0)]\rangle$$

Kubo formula:

$$r = \lim_{\omega\to 0} \frac{1}{\omega}\frac{1}{3}\sum_{i=1}^{3} \mathrm{Im}\left(G_R^{E_i}(k=0,\omega)\right)$$

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- Can formulate from conservation of magnetic fluxes
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Does it matter in practice?

↓

Study microscopic realization!

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Model

Scalar QED at finite T

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E.o.M.:

$$D_\mu D^\mu \phi + \frac{\partial V}{\partial \phi^*} = 0$$

$$\dot{E}_i + \sum_{j,k} \epsilon_{ijk}\partial_j B_k = 2e^2 \mathrm{Im}\left(\phi^* D_i \phi\right)$$

$$\uparrow \; j_i^{el}$$

Gauss law: $\vec{\nabla} \cdot \vec{E} = j_0^{el}$

$$V(\phi) = \lambda|\phi|^4 + m^2|\phi|^2$$

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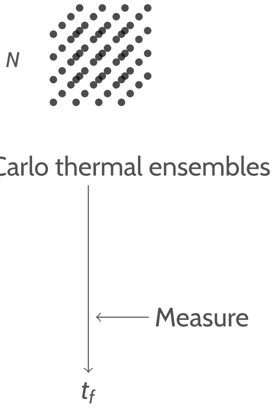
Gauss law: $\vec{\nabla} \cdot \vec{E} = j_0^{el}$

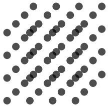
$$V(\phi) = \lambda|\phi|^4 + m^2|\phi|^2$$



Solve classical E.o.M. numerically!

Method

N



Monte-Carlo thermal ensembles

$\downarrow$ ← Measure

t_f

Solve classical E.o.M. numerically!

Classical limit

$$G^O_{cl}(x,t) = \langle O(x,t) O(0,0) \rangle^{cl.}_T$$

$$\frac{2T}{\omega} \mathrm{Im}\left(G^O_R\right) \approx G^O_{cl}$$

↑ Thermal equilibrium (KMS)

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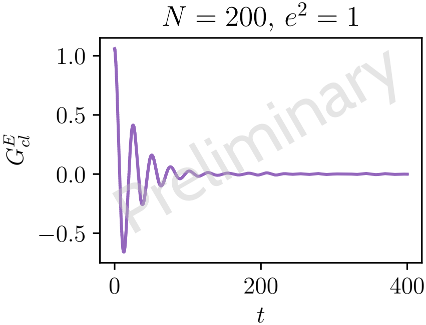
Strategy

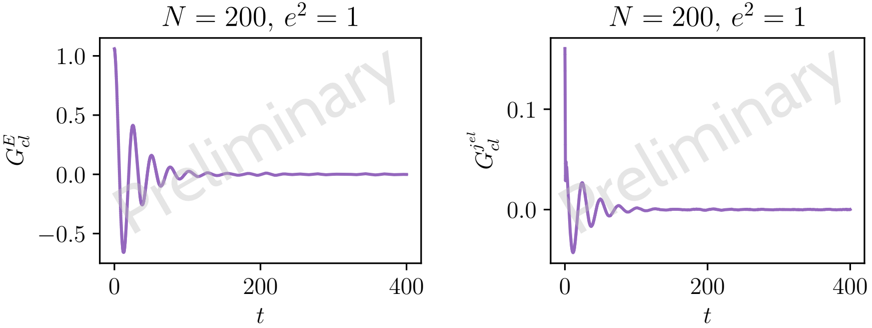
Strategy:

Compute:
- $G^{E}_{cl}(t) = \int \mathrm{d}x^3 G^{E}_{cl}(x,t)$
- $G^{j^{el}}_{cl}(t) = \int \mathrm{d}x^3 G^{j^{el}}_{cl}(x,t)$
- $r = \int_0^\infty \mathrm{d}t G^{E}_{cl}(t)$
- $\sigma = \int_0^\infty \mathrm{d}t G^{j^{el}}_{cl}(t)$

Compare!



$r = 1.059 \pm 0.032$



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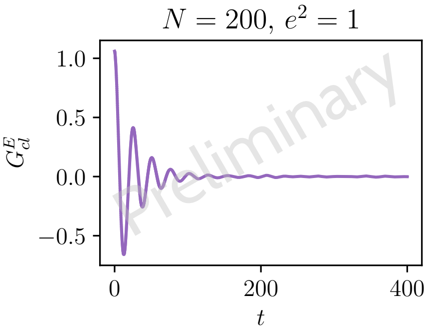

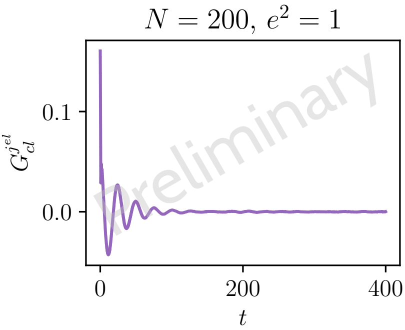

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$\sigma^{Kubo} = 0$ **in dynamical EM**

Reason:

$$\partial_t E_i(k=0,t) = j_i^{el}(k=0,t)$$
$$\downarrow$$
$$-\omega^2 G_{cl}^{E}(k=0,\omega) = G_{cl}^{j^{el}}(k=0,\omega)$$

Recap #3

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- Study MHD of micro. model
- r finite, $\sigma^{Kubo} = 0$

Recap #3

- Study MHD of micro. model
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Application: chiral physics

Chiral Magnetic Effect (CME):

Constant $\vec{B}$ background

$+$

Chiral imbalance μ_5

$=$

Magnetic current

$$\vec{j}^{CME} = \frac{1}{4\pi^2}\mu_5\vec{B}$$

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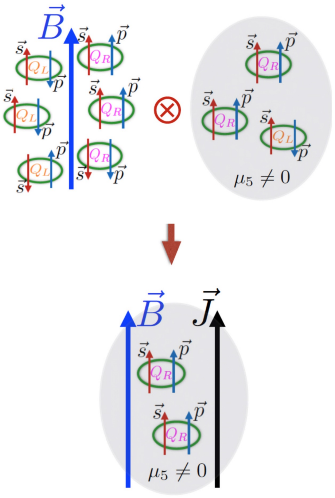

Credit: Kharzeev, Liao, Voloshin, Wang, arXiv: 1511.04050

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[Das, Iqbal, Poovuttikul, 22]
[Landry, Liu, 22]

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What happens in our micro.?

Scalar QED + massless fermions

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Open questions

- Recover σ from micro.?
- Relevance to pheno.?

Take home

- Fundamental transport of MHD is resistivity not conductivity
- Exemplified in a micro. description
- Affects chiral phyiscs

[Grozdanov, Hofman, Iqbal, 16]

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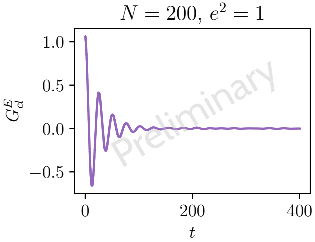

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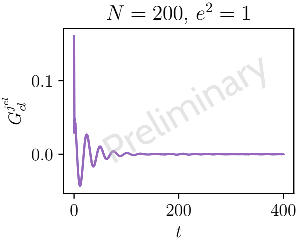

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Thanks!