Chern-Simons, Black Holes, Topological Matter

Instituto de Física Teórica UAM-CSIC

EXCELENCIA SEVERO OCHOA

Campus Internacional excelencia UAM CSIC+

Karl Landsteiner

Chiral Matters, Tours, 2023

Outline

- What is an anomaly?
  - Quantum Mechanics
  - Quantum Field Theory
- Anomalies in matter
- Topological quantum matter = Chern Simons on Black holes
  - CME
  - CVE
  - Superfluids

Anomalies

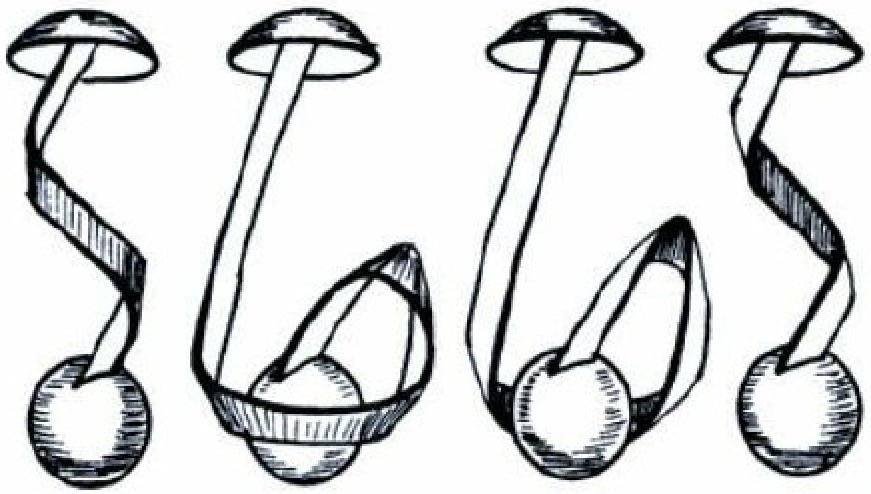
Ingredients to an anomaly:

- Classical symmetry broken by quantization
- Topology

Anomalies

Is there a classical model? YES!

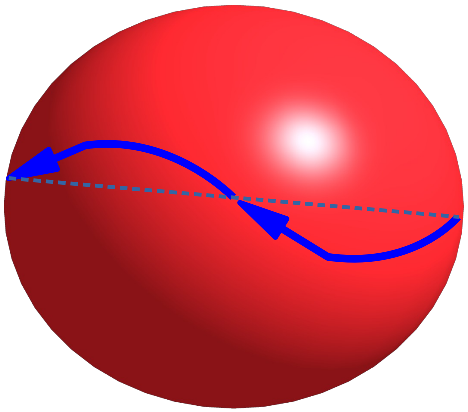
“Dirac’s Belt Trick”



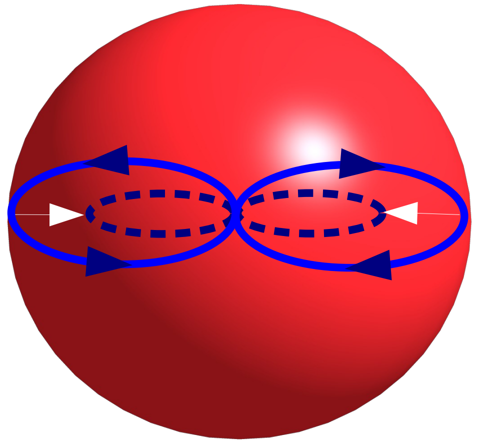
[G. K. Francis, A Topological Picturebook. Springer, New York. 1987]

Non-local object!

Anomalies



Rotation by 2π = non-contractible path



Rotation by 4π = contractible to identity

“Simple connected double cover” of SO(3) = SU(2)

$$U(\varphi) = \exp\left(i\frac{\varphi}{2}\hat{n}\vec{\sigma}\right)$$

$$U(2\pi) = -1$$ ← Anomaly !

[Uhlenbeck, Goudsmith], [Pauli] 1925

Anomalies

Can one see this anomaly in experiment?

20 October 1975

Volume 54A, number 6 PHYSICS LETTERS 20 October 1975

VERIFICATION OF COHERENT SPINOR ROTATION OF FERMIONS ☆

H. RAUCH, A. ZEILINGER, G. BADUREK, A. WILFING
Atominstitut der Oesterreichischen Hochschulen, A-1020 Wien, Austria

W. BAUSPIESS
Institut für Physik, Universität, D-46 Dortmund 50, Germany
and Institut Laue-Langevin, F-38042 Grenoble, France

U. BONSE
Institut für Physik, Universität, D-46 Dortmund 50, Germany

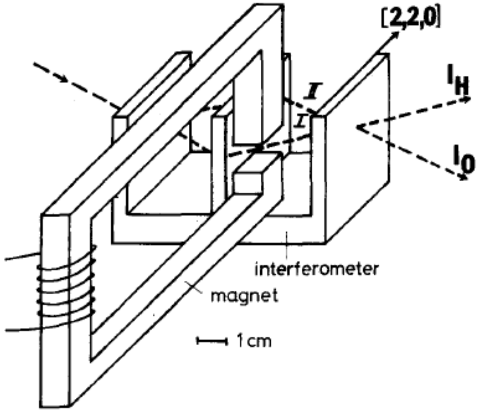


Fig. 1. Sketch of the experimental setup.

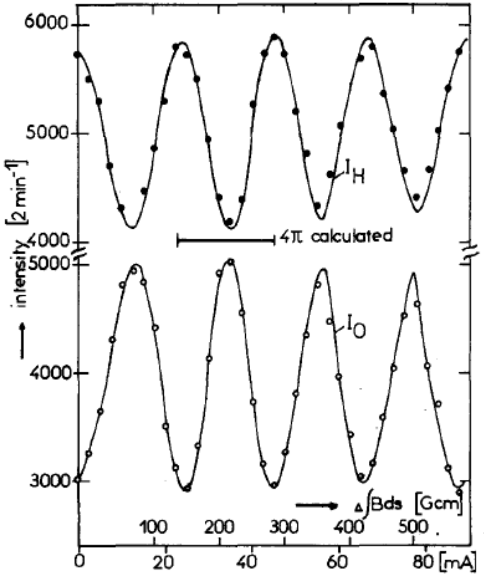


Fig. 2. Observed intensity oscillations of the 0- and H-beam as a function of the difference of the magnetic field action on beam I and II ($\Delta\int B\,ds = \int B_z\,ds$ (path I) $- \int B_z\,ds$ (path II).

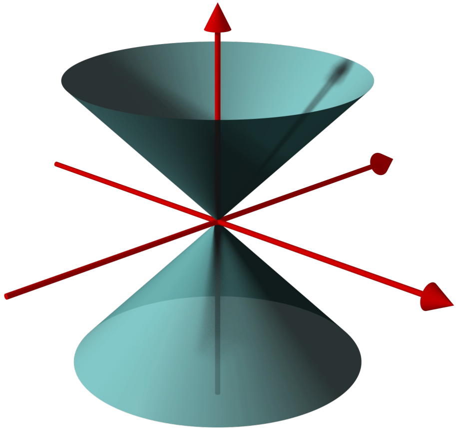
The oscillation period clearly shows that the identical wave function is reproduced after a spinor rotation of 4π and a -1 occurs for a 2π rotation.

Chiral Fermions

Massless Dirac equation $i\gamma^\mu \partial_\mu \Psi = 0$ $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

Weyl: independent right- left-handed components

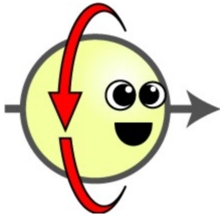
$$\gamma_5 \psi_\pm = \pm \psi_\pm$$



Hamiltonian

$$\boxed{H_\pm = \pm \vec{\sigma}.\vec{p}}$$

Spin-momentum locking: helicity = chirality

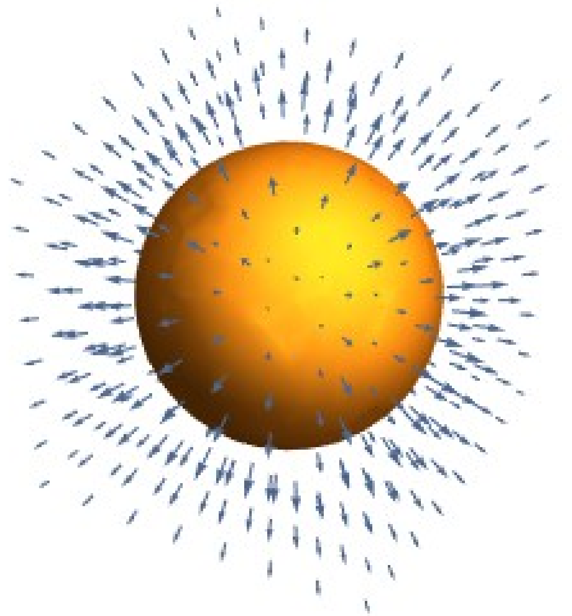


Chiral Anomaly - I

Topology-I: Chiral fermions as monopoles in momentum space

"The universe in a Helium droplet" [Volovik]

$$H_{\pm} = \pm\vec{\sigma}.\vec{p}$$



Berry connection: $$\mathcal{A}_i = \psi(p)^\dagger \frac{\partial \psi(p)}{\partial p_i}$$

Berry curvature: $$\mathcal{F} = \nabla_p \times \mathcal{A}$$

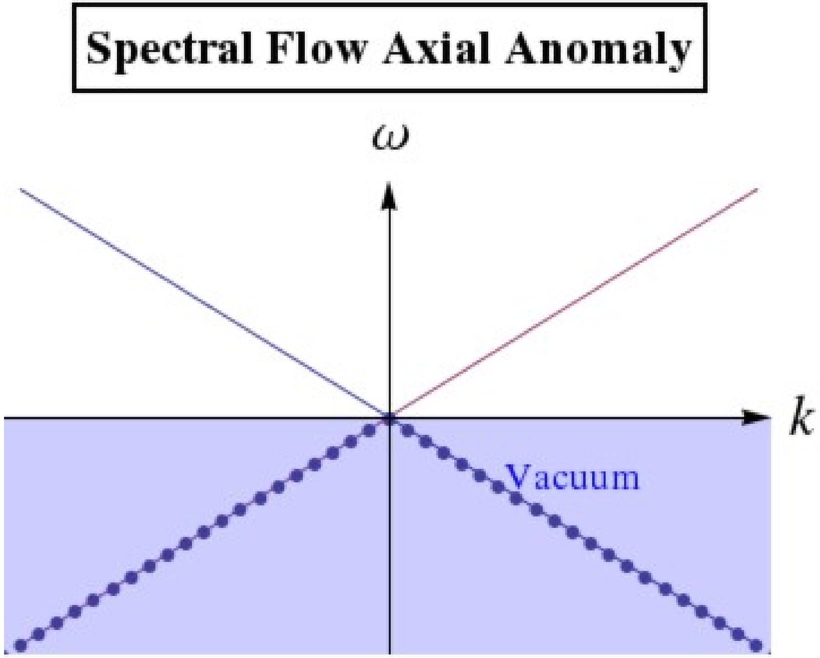
$$\int_{S^2} \mathcal{F} = \pm 2\pi$$

Topological invariant!

[Son, Yamamoto]

Axial Anomaly

Dirac fermion = 1 left handed + 1 right handed



Particle creation

Holes = Anti-Particles

$$\frac{dn_5}{dt} = \frac{1}{2\pi^2}\vec{E}.\vec{B} \qquad \frac{d}{dt}(n_R + n_L) = 0$$

Anomalies

"Chiral Magnetic Effect" $$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$ $$\mu_5 = \frac{1}{2}(\mu_r - \mu_L)$$

"Chiral Vortical Effect" $$\vec{J}_5 = \frac{T^2}{12}\vec{\Omega}$$ $$\frac{(2\pi T)^2}{48\pi^2}\vec{\Omega}$$

Anomalies - II

Anomaly comes from non-local term in effective action !

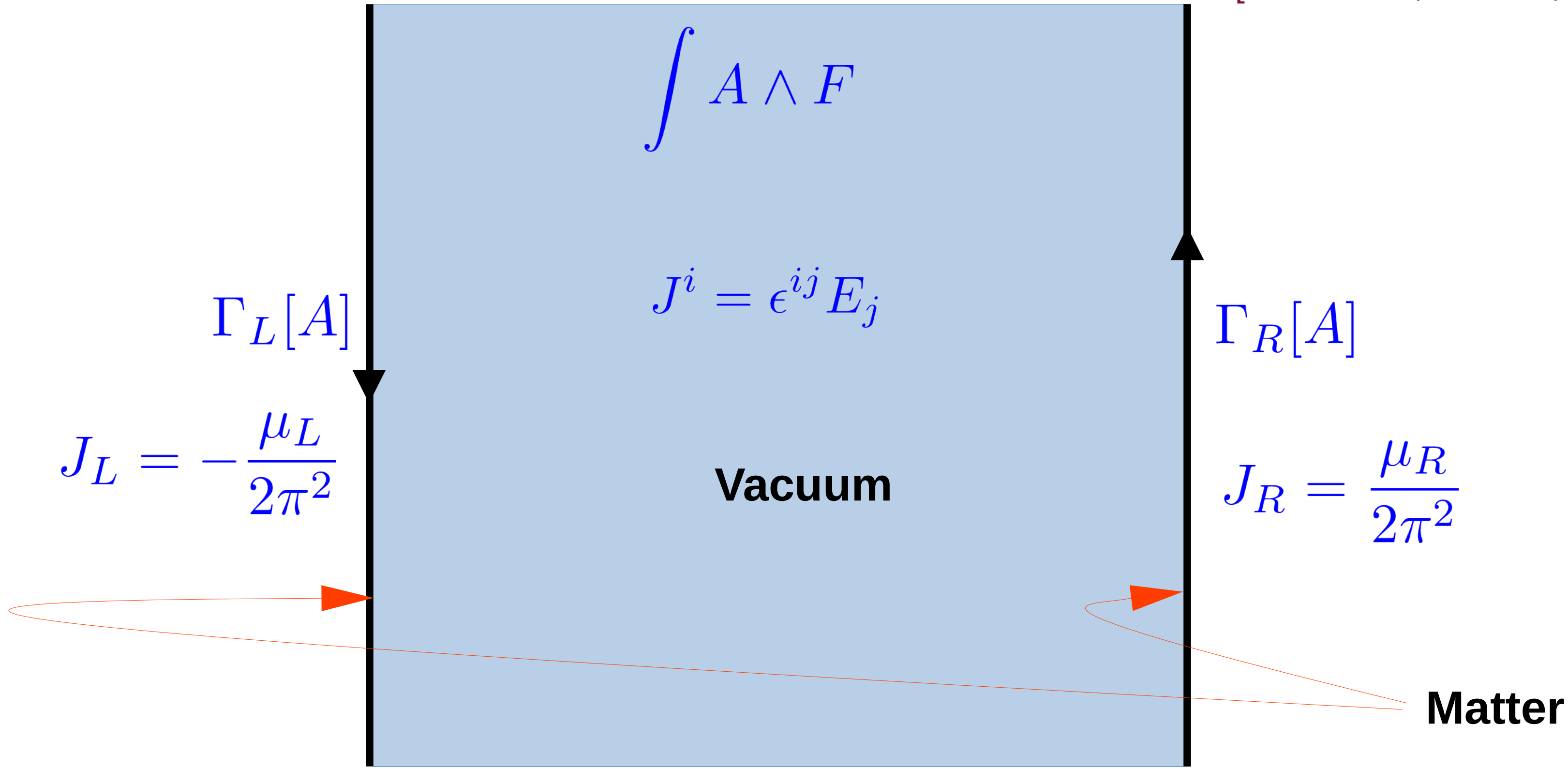
Example: single chiral fermion $$\mathcal{A} = \frac{1}{24\pi^2}\int d^4x\, c\, F \wedge F$$

- Non-local term $$\int d^4x\, d^4y \frac{\partial A(x) F(y) \wedge F(y)}{\Box_{xy}}$$
- Local term in 5D $$\Gamma_{\text{CS}} = \int_{\text{M}} (cA \wedge F \wedge F + c_g A \wedge \text{tr}(R \wedge R))$$
- Gauge Trafo: $$\delta\Gamma_{\text{CS}} = \int_{\partial M} \lambda c F \wedge F + c_g (R \wedge R)$$ (QHE)

Application: QHE

What is M?

[TKNN] 1982
[Callan, Harvey] 1985
[Golterman, Jansen, Kaplan] 1992

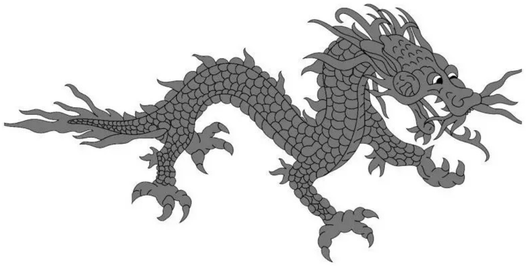


Edge currents are also determined by anomaly

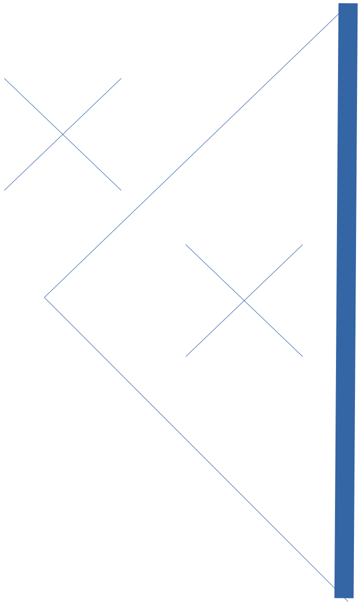
Matter = μ (and T)

Anomalous currents from black holes

What is M?
Answer: a Black Hole   At least if we want to encode “matter” = T,μ



Híc sunt dracones



$$ds^2 = dr^2 - f(r)^2 dt^2 + g(r)^2 d\vec{x}^2$$

$$2\pi T = f'(r_h)$$

Anomalous currents from black holes

3 dimensions: $$S = \int_{BH} A \wedge F$$

$$\delta S = 2\int_{BH} \delta A \wedge F + (\delta A \wedge A)|_{\partial BH}$$

$$F = \partial_r A_t dr \wedge dt$$

$$\mu = A_t(b) - A_t(r_h)$$

Encodes chemical potential

$$\delta S = \delta A.J$$

$$J = 2\mu - A_t(b)$$

Current of 2D Chiral Fermionl

Anomalous currents from black holes

3 gravity:

$$S = \int_{BH} (\Gamma d\Gamma + \frac{2}{3}\Gamma^3)$$

$$x^\mu \to x^\mu + \xi^\mu$$

$$\delta\Gamma = (i_\xi d + d i_\xi)\Gamma - D\Lambda_\xi$$

$$(\Lambda_\xi)^\alpha_\beta = \frac{\partial \xi^\alpha}{\partial x^\beta} \qquad D\cdot \ = d\cdot \ + [\Gamma, \cdot]$$

$$\delta S = \int_\partial \Lambda_\xi R$$

2D Gravitational anomaly on boundary

Important detail: vanishing extrinsic curvature on boundary!

Anomalous currents from black holes

Calculate current as before:

$$ds^2 = dr^2 - f(r)^2(dt - \delta A_g dx)^2 + g(r)^2 dx^2)$$

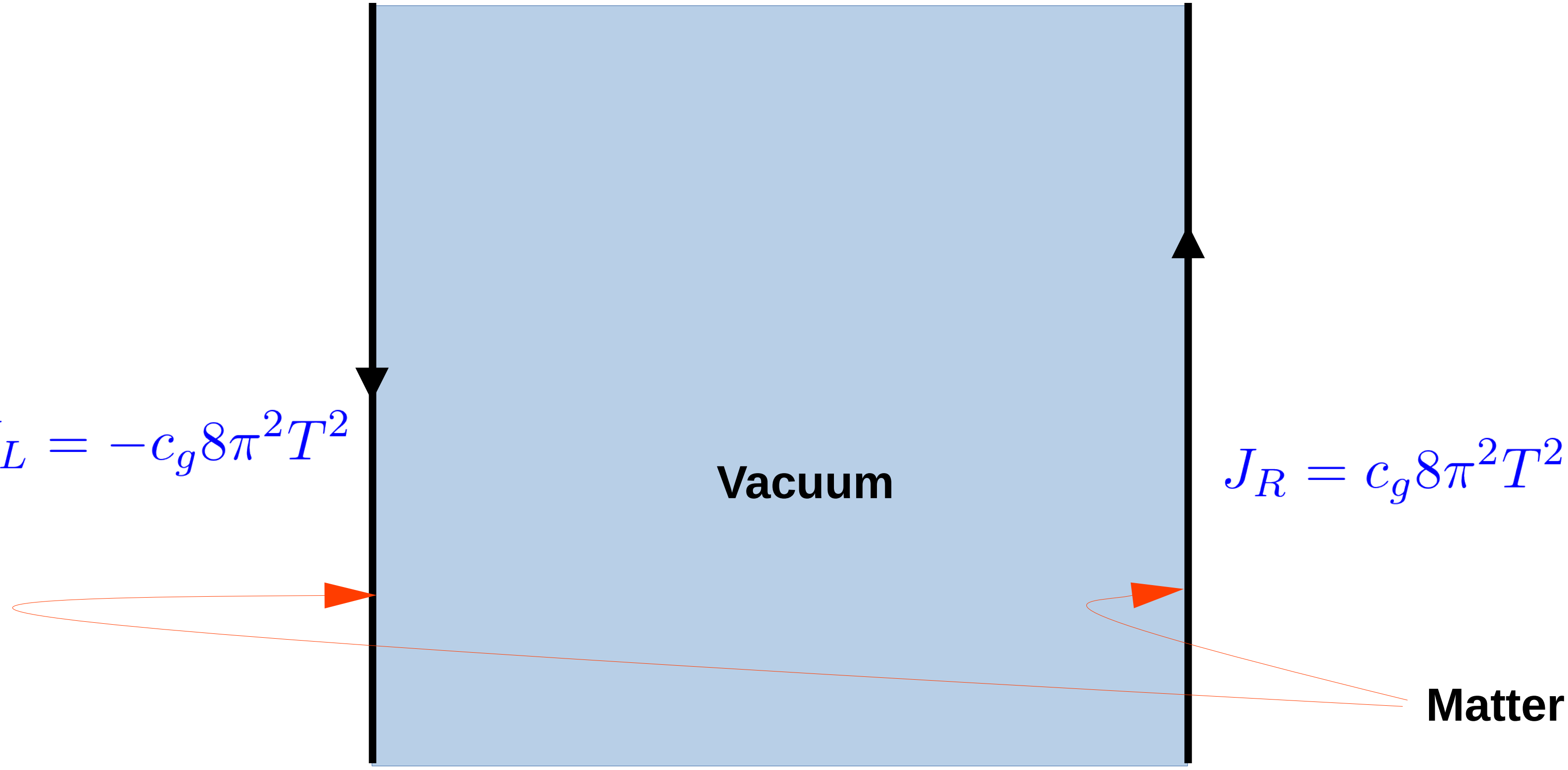
$$\delta S = 2A_g \int_{r_H}^{b} [\frac{f f' g'}{g} - (f')^2]' dr$$

$$T^{tx} = J_E = 8\pi T^2$$

Energy current of chiral fermion

Application: thermal QHE

$$S = c_g \int_M (\Gamma d\Gamma + \frac{2}{3}\Gamma^3) - c_g \int_{BH} (\Gamma d\Gamma + \frac{2}{3}\Gamma^2)$$



$$J_L = -c_g 8\pi^2 T^2$$

$$J_R = c_g 8\pi^2 T^2$$

Application: CME

Chiral Magnetic Effect $\Gamma[A] = C \int_{\mathcal{M}} A \wedge F_V \wedge F_V$

Calculate current: $\delta\Gamma = \int_{\partial\mathcal{M}} \delta V \wedge J = 2\int_{\mathcal{M}} \delta V \wedge F_A \wedge F_V + 2\int_{\partial\mathcal{M}} \delta V \wedge A \wedge F_V$

Field configuration represents physical state: $A_t(\partial\mathcal{M}) = \mu_5$

Regular field configuration at origin: $A_t(0) = 0$

Static solution = equilibrium: $F_A = \partial_r A_t dr dt$

Magnetic field obeys Bianchi identity: $\partial_r B = 0$

CME vanishes in equilibrium $J = 2C(\mu_5 - A_t)B$

Application: CSE

<u>Chiral Separation Effect</u> $\Gamma[A] = C \int_{\mathcal{M}} A \wedge F_V \wedge F_V$

Calculate current: $\delta\Gamma = \int_{\partial\mathcal{M}} \delta V \wedge J = \int_{\mathcal{M}} \delta A \wedge F_V \wedge F_V$

Field configuration represents physical state: $V_t(\partial\mathcal{M}) = \mu$

Regular field configuration at origin: $V_t(0) = 0$

Static solution = equilibrium: $F_V = \partial_r V_t dr dt + B dx dy$

Magnetic field obeys Bianchi identity: $\partial_r B = 0$

CSE does not vanishes in equilibrium $J_5 = 2C\mu B$

Application: CVE

Chiral Vortical Effect: $\Gamma[A] = C_g \int_{\mathcal{M}} A \wedge \mathrm{tr}(R \wedge R)$

$$ds^2 = dr^2 - f(r)^2(dt - \vec{A}_g.d\vec{x})^2 + g(r)d\vec{x}^2$$

- SO(3) symmetric state
- Rotation = gravito-magnetic field $B_g = 2\Omega = \nabla \times A_g$
- Non-rotating regular Euclidean section (equilibrium) $f(0) = 0 \quad , \quad f'(0) = 2\pi T$
- No contribution from extrinsic curvature at boundary $f'(\partial\mathcal{M}) = g'(\partial\mathcal{M}) = 0$

$$J = C_g B_g \int_0^{r_b} \frac{1}{2} \left(-f^2 \frac{g'^2}{g} - f'^2 + 2f \frac{f'g'}{g} \right)'$$

CVE given by gravitational anomaly $J = 8\pi^2 C_g T^2 B_g$

Application: Superfluid

Condensate: $\langle \Phi \rangle = \Phi_0 e^{i\theta}$

$$\partial_\mu \Phi_0 = 0$$

Superflow: $J^\mu = \partial^\mu \theta - A^\mu$

Gauge invariant: $\theta \to \theta + \lambda \quad , \quad A_\mu \to A_\mu + \partial \lambda$

Hodge dual current: $K_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\lambda} J^\lambda$ [Delacratz, Hofman, Mathys]

Higher form symmetry
Anomaly
Vortex creation

$$\partial_\mu K^\mu{}_{\rho\lambda} = -F_{\rho\lambda}$$

Usual charge: $\int_{\text{space}} *J = Q$ Vortex number: $\oint *K = N_v$

Application: Superfluid

Chern-Simons (BF) theory: $$S_{BF} = \int B \wedge F$$

Source for current K $$B = B_{\mu\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho$$

Gauge transformation: $$B \to B + d\lambda$$

BF theory on Black Hole: $$\delta S_{BF} = \delta B_{xyz} \int_{r_h}^{b} \partial_r A_t = \delta B_{xyz} \mu$$

$$K^{xyz} = \mu$$

What does this mean? $$K^{xyz} = J_t = \partial_t \theta = \mu$$

Application: Superfluid

Variation with A_z: $J^z = B_{txy}(b) - B_{txy}(r_h) - B_{txy}(b) = \mu_K - B_{txy}(b)$

Exactly like in CME: superflow vanishes in thermal equilibrium!
Here due to *consistent* anomaly (sorry, just couldn't resist...)

Mixed anomaly just like CME and CSE

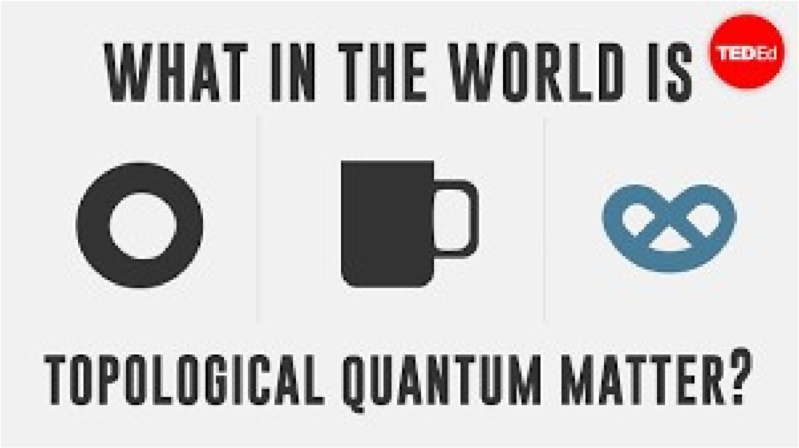
$$J^z = \mu_K \qquad \partial_t J^t + \partial_z J^z = 0 \qquad J^t = \chi\mu$$

$$K^{zxy} = \mu \qquad \partial_t K^{txy} + \partial_z K^{zxy} = 0 \qquad K^{txy} = \chi_K \mu_K$$

$$\omega^2 - \frac{1}{\chi\chi_K} k^2 = 0$$

Chiral Magnetic Wave = 2nd sound!

Summary



[Fan Zhang]

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Thank you!