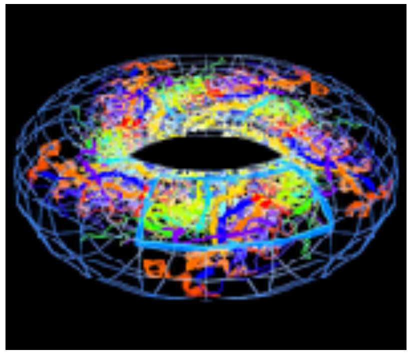
Casimir effect: entanglement of quantum and classical physics

Alexander Molochkov
Pacific Quantum Center, Vladivostok, Russia

In collaboration with
Maxim Chernodub, Vladimir Goy and Alexey Tanashkin



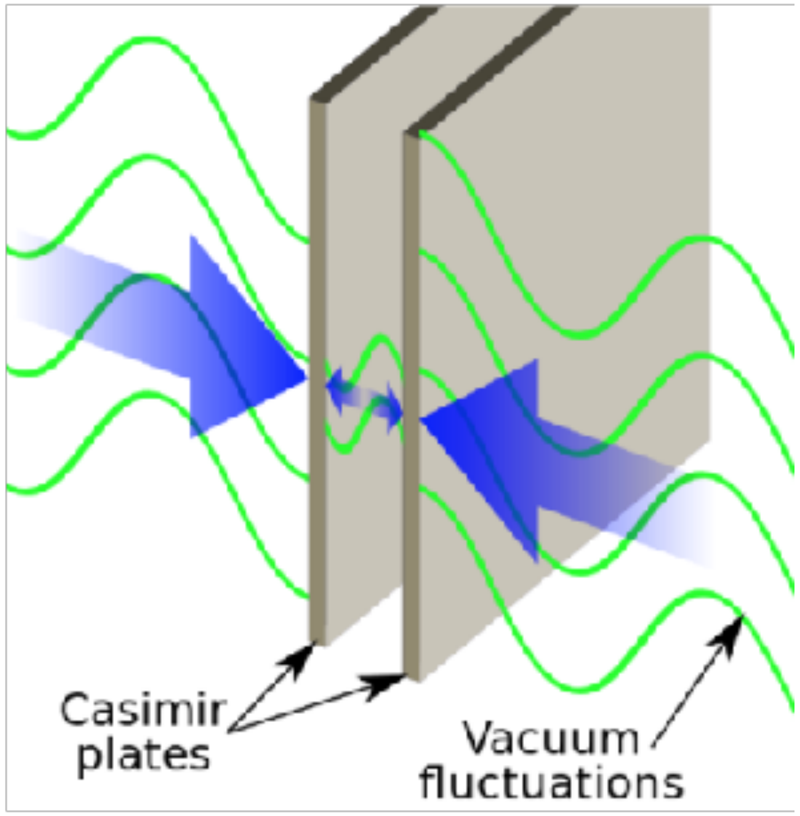
Le Studium Conference Chiral Matter
July 5 - 7, 2023, Tours, France

FEFU
FAR EASTERN FEDERAL UNIVERSITY

Casimir effect: occurrence of mechanical forces between classical objects due to quantum vacuum fluctuations

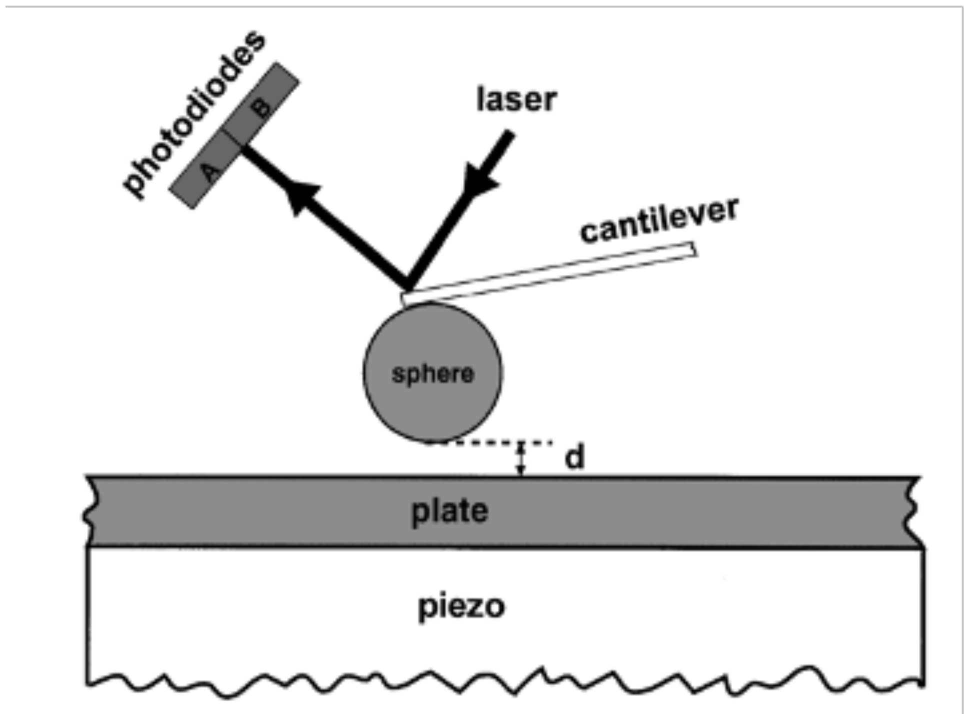
[H . B.G. Casimir, Proc. K. Ned . Acad . Wet. 51 , 793 (1948)]

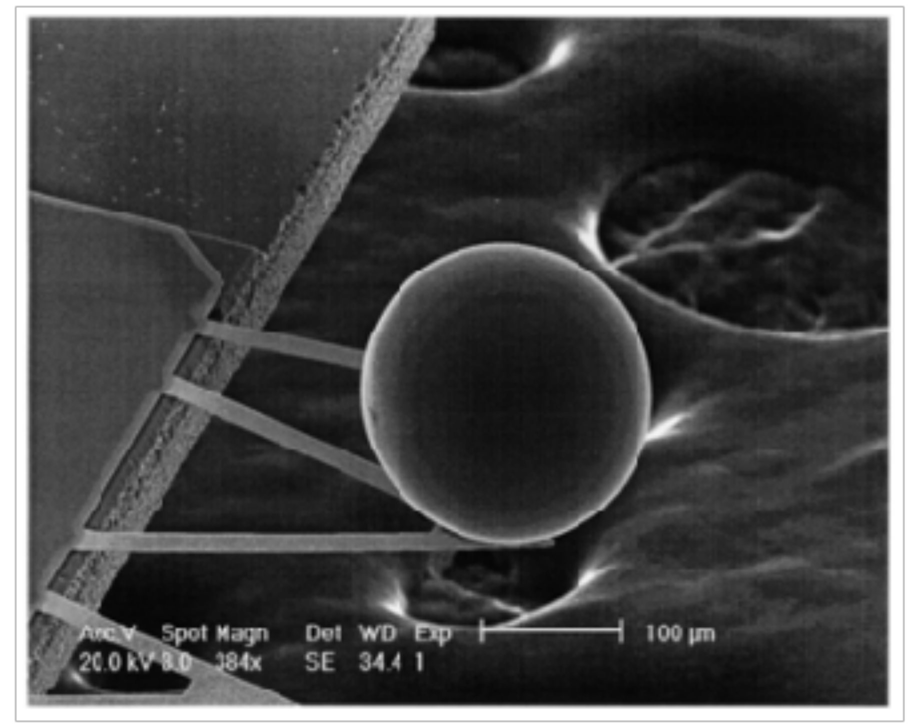
Simplest setup: Two parallel perfectly conducting plates at finite distance R



The Casimir force is considered as a possible proof of the reality of quantum vacuum fluctuations.

Experimentally CE confirmed in plate-sphere geometries.
(S. K. Lamoreaux, Phys. Rev. Lett. 78, 5 (1997)):





A very small force at human scales. However, at R=10 nm the pressure is about 1 atmosphere. (from U. Mohideen and A. Roy, Phys. Rev. Lett. 81, 4549 (1998), down to 100 nm scale.)

Can vacuum fluctuations affect classical physics parameters?

Important example: Cosmological constant

Vacuum fluctuations energy density

$$\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

Vacuum energy density contribution to the cosmological constant

$$\lambda = 8\pi G \mathcal{E}$$

The question of negative mass and negative energy

Qualitative effect:

– the Casimir energy between two perfectly conducting plates is negative

→ the gravitational mass and the inertial mass associated with the Casimir energy are equal and are negative as well:

$$M^{\text{Cas}}_{\text{Inertial}} = M^{\text{Cas}}_{\text{Gravitational}} = \frac{E}{c^2} < 0$$

[K. A. Milton et al, "How Does Casimir Energy Fall? I-IV" (2004-2007); J.Phys.A41, 164052 (2008); G. Bimonte et al., Phys.Rev. D76, 025008 (2007); V. Shevchenko, E. Shevrin, Mod.Phys.Lett. A31 (2016) no.29, 1650166]

→ A pure Casimir energy would levitate in a gravitational field due to existence of an upward "buoyant" force exerted by the outside vacuum on a "Casimir apparatus" following a quantum "Archimedes' principle".

However:

1. The buoyant force will be extremely small;
2. "... the mass energy of the cavity structure necessary to enforce the boundary conditions must exceed the magnitude of the negative vacuum energy, so that all systems of the type envisaged necessarily have positive mass energy." [J.D. Bekenstein, PRD 88, 125005 (2013)]

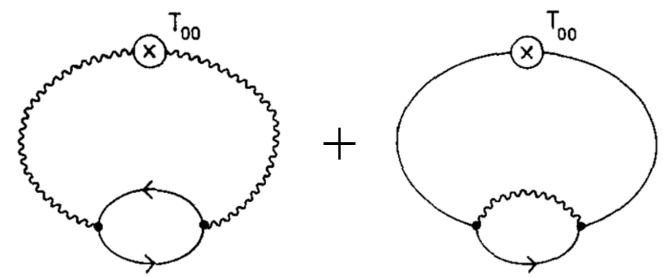
→ The Casimir apparatus will anyway be drown in the gravitational field.

Perturbative Casimir effect

The case of QED:
→ perturbative corrections are very small:

$$\langle \mathcal{E} \rangle = -\frac{\pi^2}{720}\frac{\hbar c}{R^3}\left(1 - \frac{9\alpha\hbar}{32 m_e c}\frac{1}{R}\right)$$

For the ideal plates separated at optimistic R =10 nm the radiative correction is 10^{-7} of the leading term.



$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

[M. Bordag, D. Robaschik, E. Wieczorek, Ann. Phys. 165, 192 (1985)]

→ Unexpected qualitative phenomenon: between the plates, light travels faster than light outside the plates (the Scharnhorst effect).

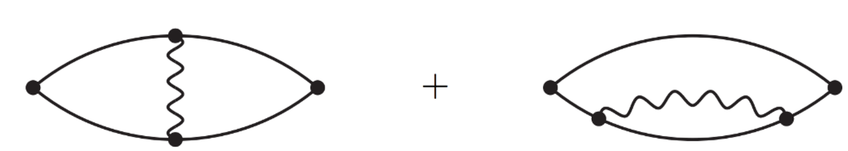
[G. Barton, K. Scharnhorst, J. Phys. A 26, 2037 (1993); K. Scharnhorst, Annalen Phys. 7, 700 (1998)]

- Despite the Scharnhorst effect formally implies "faster-than-light travel" it cannot be used to create causal paradoxes.

[S. Liberati; S. Sonego, M. Visser, Ann. Phys. 298, 167 (2002); J.-P. Bruneton, Phys. Rev. D, 75, 085013 (2007)]

- The excess of c_{cavity} over the usual c is tremendously small, given by a two-loop radiative contribution to a refractive index in between plates.

$$\delta c = +\frac{11\pi^2}{90^2}\alpha^2\left(\frac{\hbar}{m_e c}\frac{1}{R}\right)^4$$

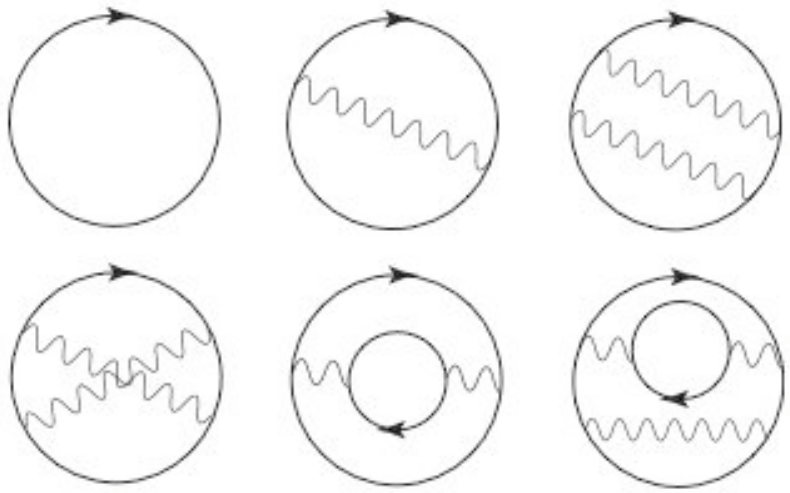


The Scharnhorst effect gives a 10^{-24} correction to c at optimistic R =10 nm.

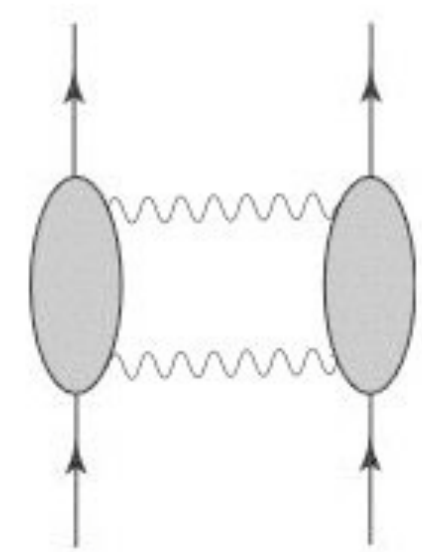
Does the Casimir force prove the reality of quantum vacuum zero-point fluctuations?

R. L. Jaffe, Phys. Rev. D **72**, 021301(R)

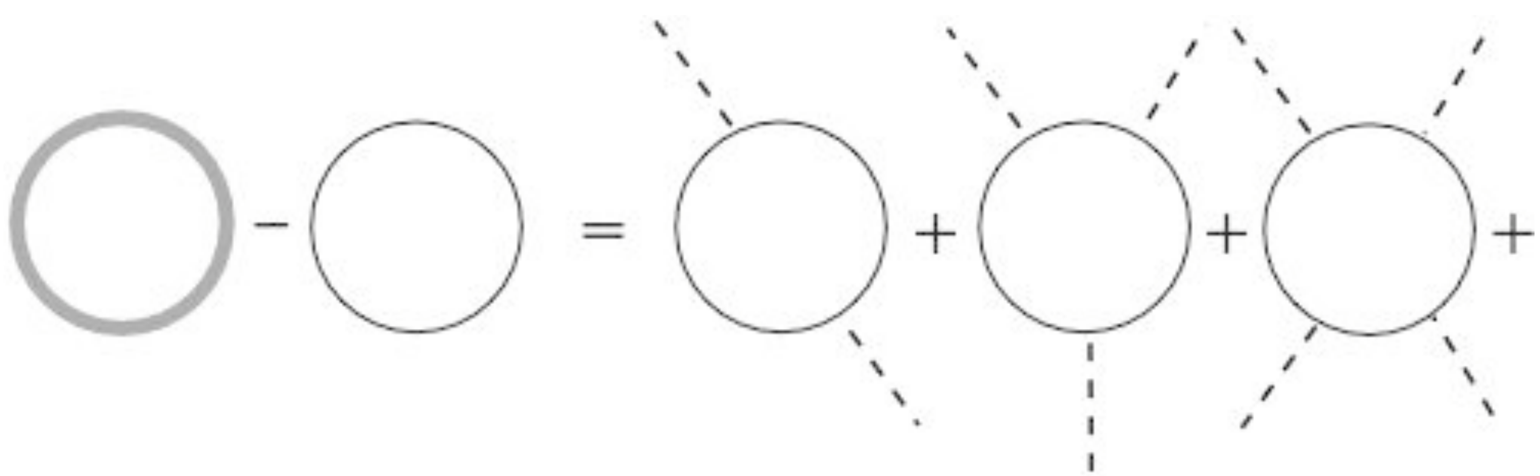
Field theory pure vacuum fluctuations:



Casimir-Polder force:



Field theory vacuum fluctuations in the presence of physical bodies:



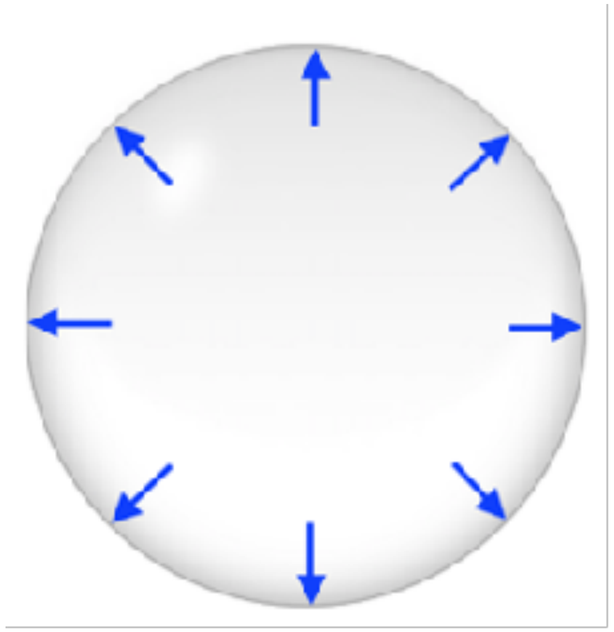
The presence of physical bodies changes the structure of the quantum vacuum

- Does the quantum vacuum contribute to the cosmological constant?
- Quantum vacuum non-perturbative interaction with boundaries
- Role of the quantum vacuum's condensates, topology and symmetry
- Phase transitions
- Finite volume effects (example: simple Casimir picture)
- Casimir effect: Is it a quantum vacuum effect or just classical forces in a finite volume?

A popular and simple explanation of the effect: *boundaries restrict the number of virtual photons inside a cavity so that the pressure of the virtual photons from outside prevails* is, actually, incorrect.

Boyer: A spherical geometry the Casimir force is acting outwards:

$$\langle E \rangle_{\mathrm{sphere}} = +\frac{0.0461765}{R}$$



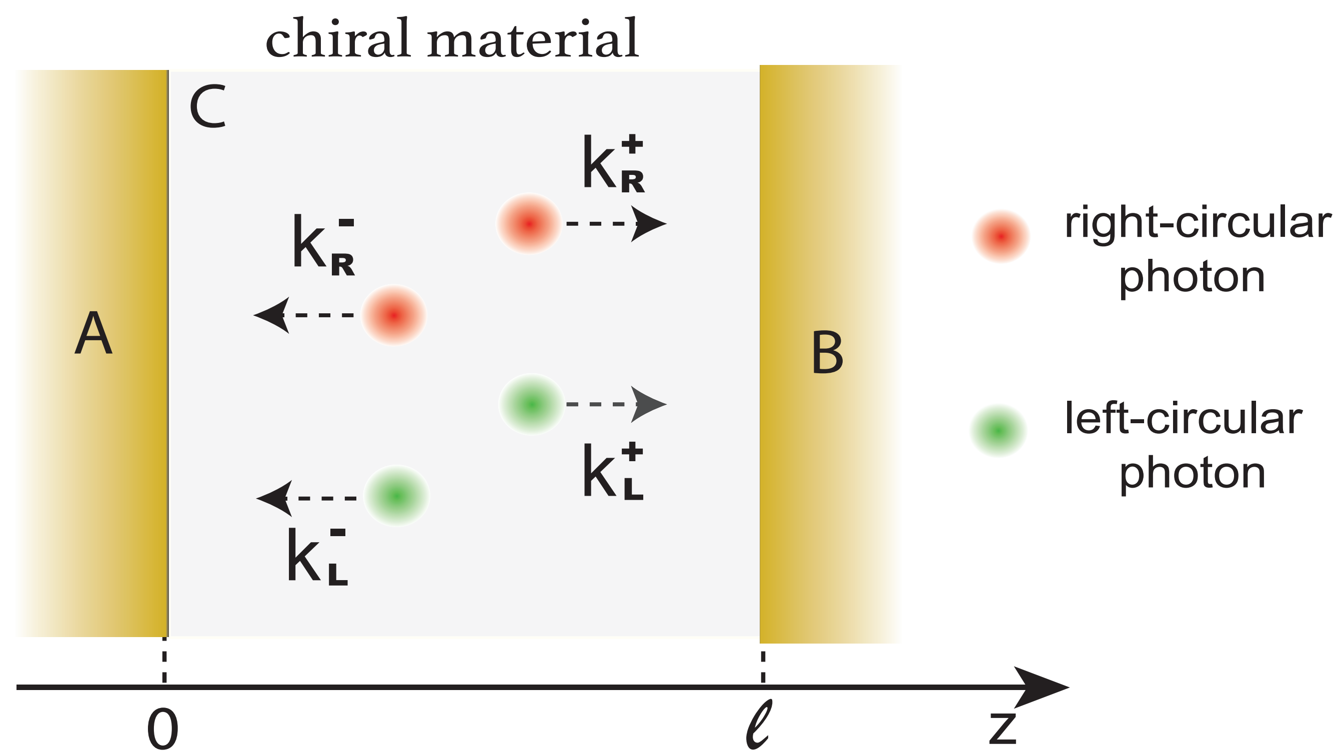
[T. H. Boyer, Phys. Rev. 174, 1764 (1968)]

This property is used in the bag models of hadrons:
A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974);
A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn,
Phys. Rev. D 10, 2599 (1974);
T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975); K. A. Milton, Phys. Rev. D 22, 1441 (1980)

Central no-go theorem on Casimir forces: The sign of Casimir force does not depend on the shape of symmetric boundaries

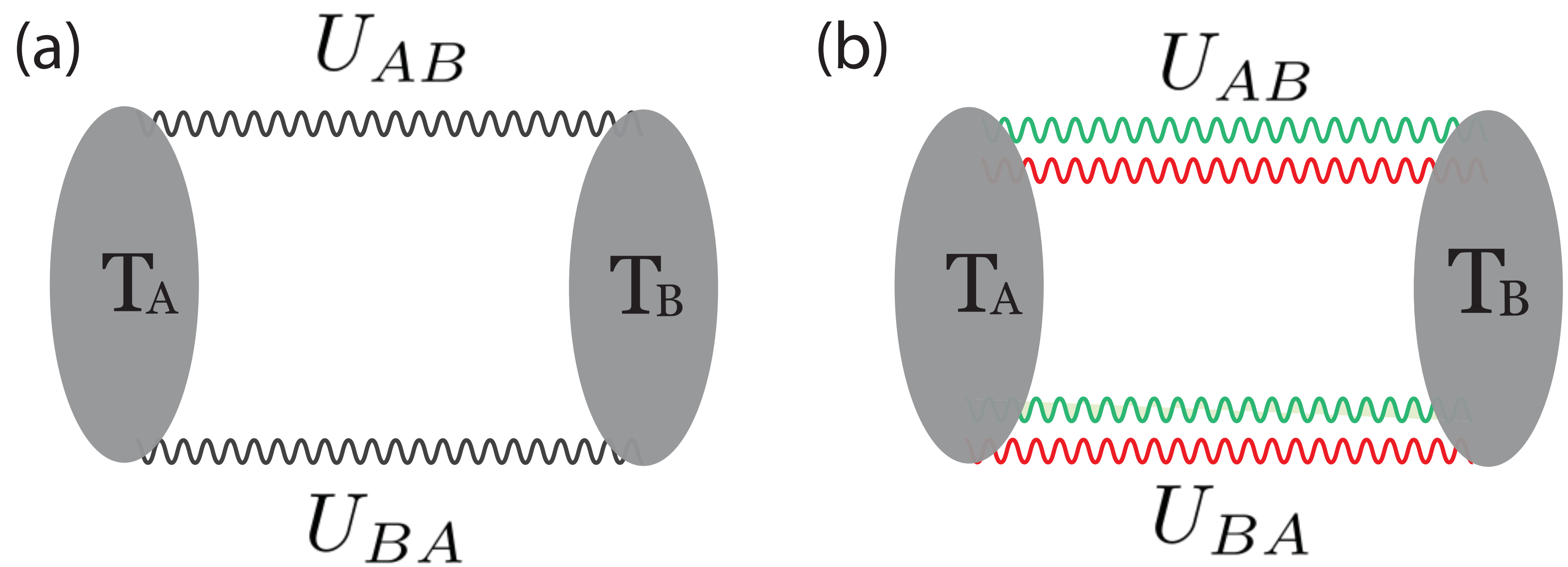
O. Kenneth and I. Klich, Phys. Rev. Lett. **97**, 160401 (2006).

Example of symmetry importance: effective chiral vacuum leads to repulsive Casimir effect

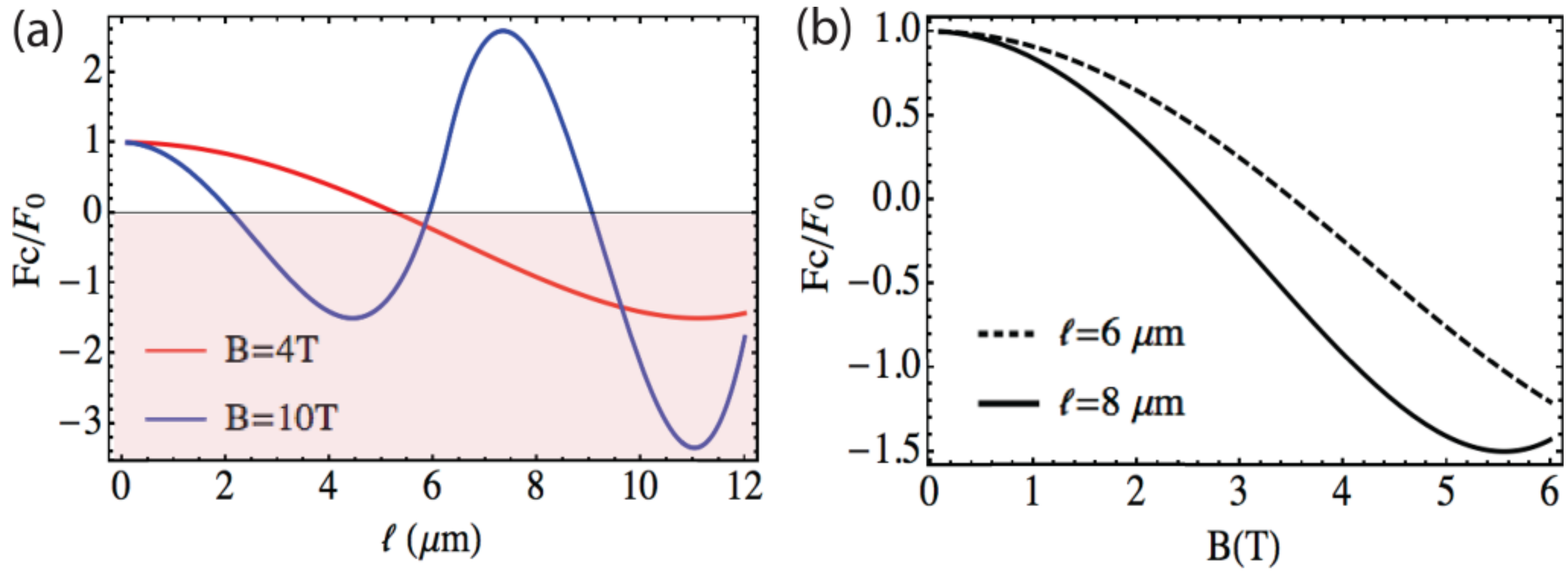


The key to realizing repulsive Casimir forces between similar objects is to insert an intermediate chiral material between them. The chiral Casimir force has several distinctive features: it can be oscillatory, its magnitude can be large (relative to the classic Casimir force), and it can vary in response to external magnetic fields.

Qing-Dong Jiang and Frank Wilczek
Phys. Rev. B **99**, 125403 (2019)

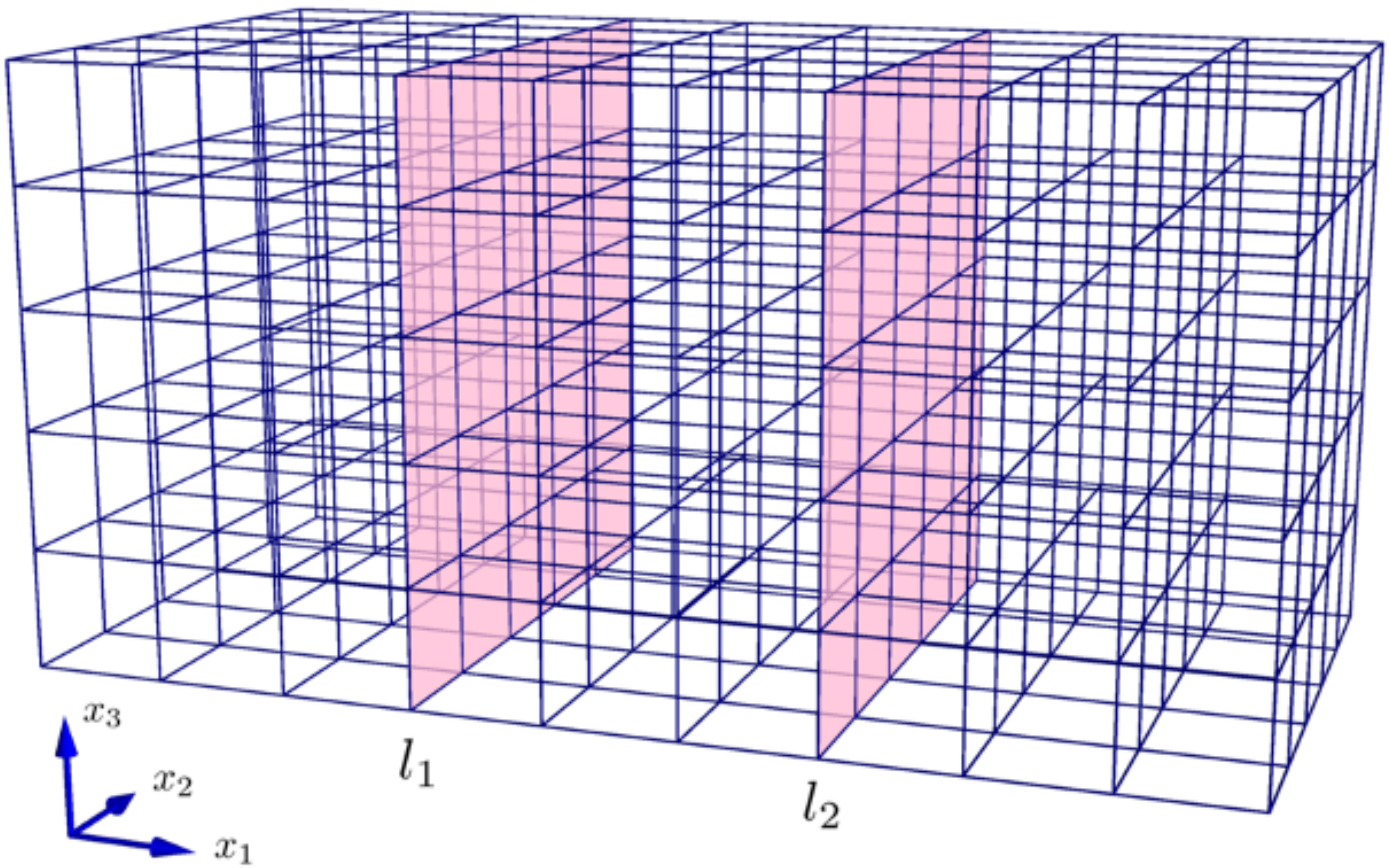


Feynman diagrams for normal Casimir energy and chiral Casimir energy. (a) Shows the Feynman diagram representation for normal Casimir energy when chiral symmetry of photon is kept. Black wavy lines represent photon propagator $\hat{D}_0$ and filled bubbles represent current-current correlation functions T_A and T_B. (b) Shows the Feynman diagram representation for Casimir energy when chiral symmetry is broken, namely, the velocity of photons depend on their chirality. Red and green wavy lines correspond to Green's functions for right-circular polarized photons and left-circular polarized photons, respectively.



Chiral Casimir force due to Faraday effect, normalized to the original metallic Casimir force per area $F0 = -\pi^2 \bar{h} c/(240 l^4)$. (a) Shows the Casimir force enhancement in different magnetic field. The red and blue curves represent Casimir force at magnetic field $B = 4$ T and $B = 10$ T, respectively. The shadow region corresponds to repulsive Casimir force regime. (b) Shows how the magnetic field B can control the Casimir force. The solid and dashed lines represent the Casimir force that is measured at the distance $l = 8$ and 6 μm, respectively.

Non-perturbative Casimir effect within Lattice gauge theories



The Casimir plates l_1 and l_2 are separated by the distance R. The space is compactified into a torus due to periodic boundary conditions.

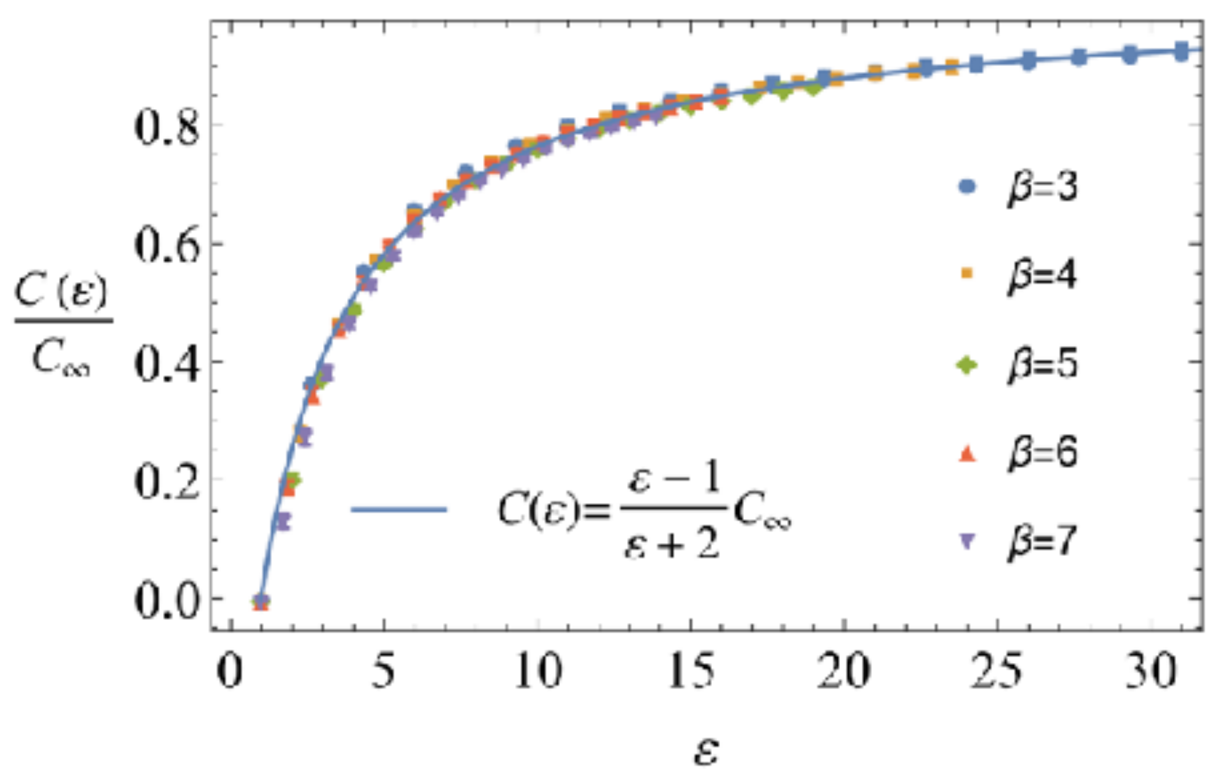
Take a lattice formulation of the theory and impose the appropriate conditions via the Lagrange multipliers at the boundaries.

$$\cos\theta_{x,23} = 1$$

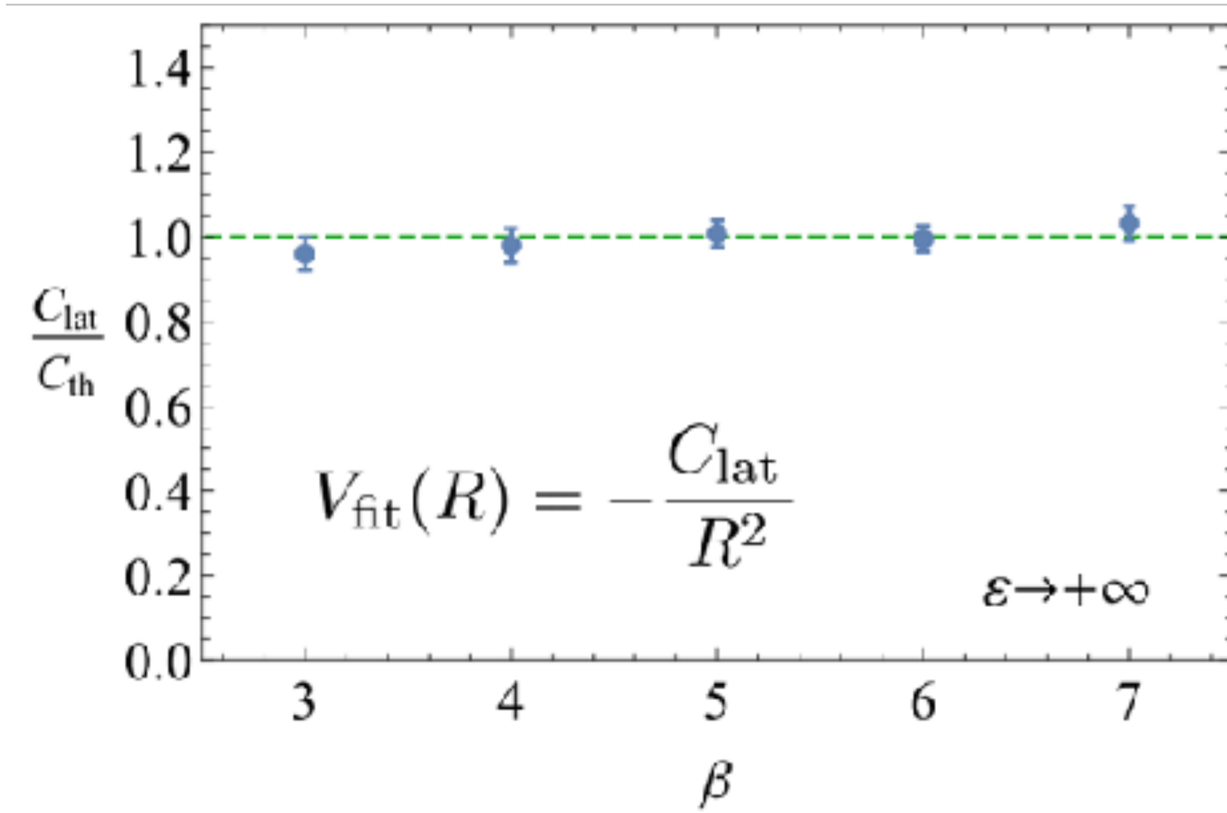
$$S_E[\theta; \mathscr{P}_S] = \sum_P \beta_P(\epsilon)\cos\theta_P$$

$$\beta_{P_{x,\mu\nu}} = \beta[1 + (\epsilon - 1)(\delta_{\mu,2}\delta_{\nu,3} + \delta_{\mu,3}\delta_{\nu,2}) \cdot (\delta_{x,l_1} + \delta_{x,l_2})]$$

Casimir energy for finite static permittivity



Check of the approach in a free theory (no monopoles, weak coupling regime):



Perfectly conducting wires

[= infinite static permittivity ε in (2+1)d]

[V.A. Goy, A.V. Molochkov, M.Chernodub., Phys.Rev. D94, 094504 (2016)]

Phase structure: deconfinement transition at $T = 0$

Electric charges exhibit a linear confinement in a Coulomb gas of monopoles

If the wires are close enough, then
- → between the wires, the dynamics of monopoles is dimensionally reduced;
- → the inter-monopole potential becomes log-confining;

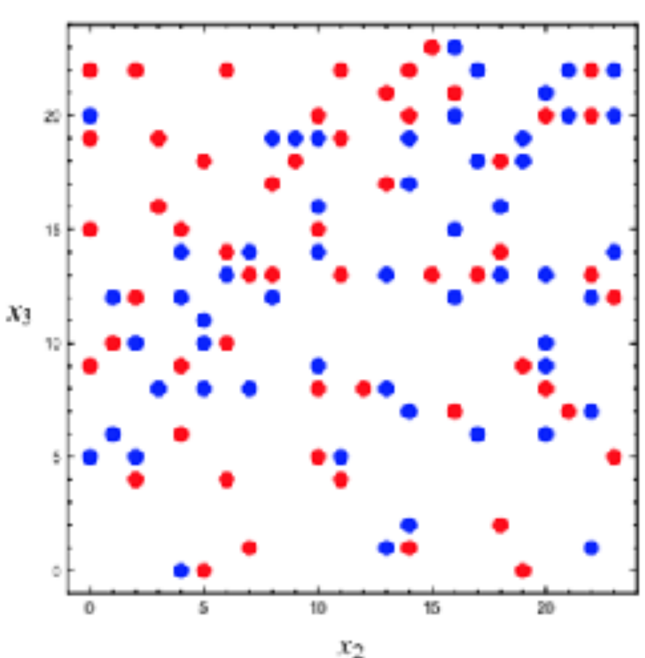
$$D_{3D}(\boldsymbol{x}) = -\frac{1}{4\pi|\boldsymbol{x}|} \to D_{2D}(\boldsymbol{x}) = \frac{2}{R} \ln \frac{|\boldsymbol{x}|}{R}$$

A very smooth transition, a BKT type or crossover?

- → the monopoles form magnetic-dipole pairs (and are suppressed);
- → the confinement of electric charges disappears (a deconfining transition).

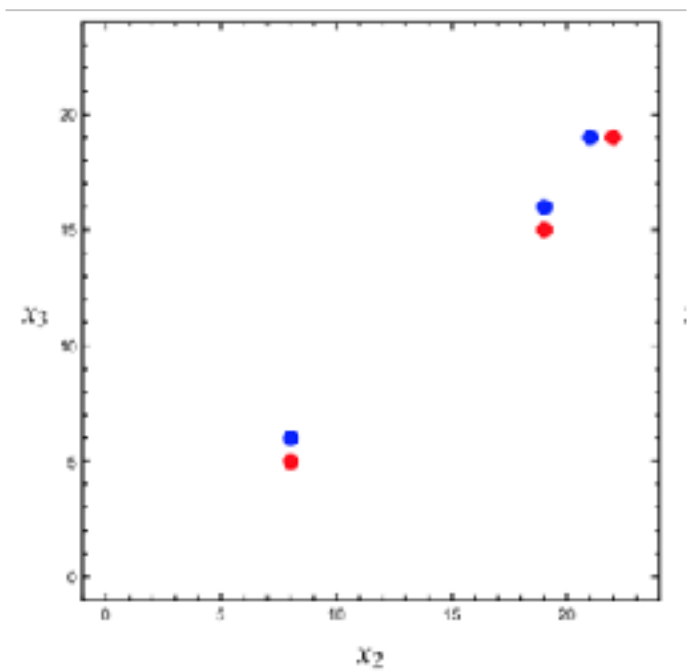
Examples of (anti-)monopole configurations

widely-spaced wires



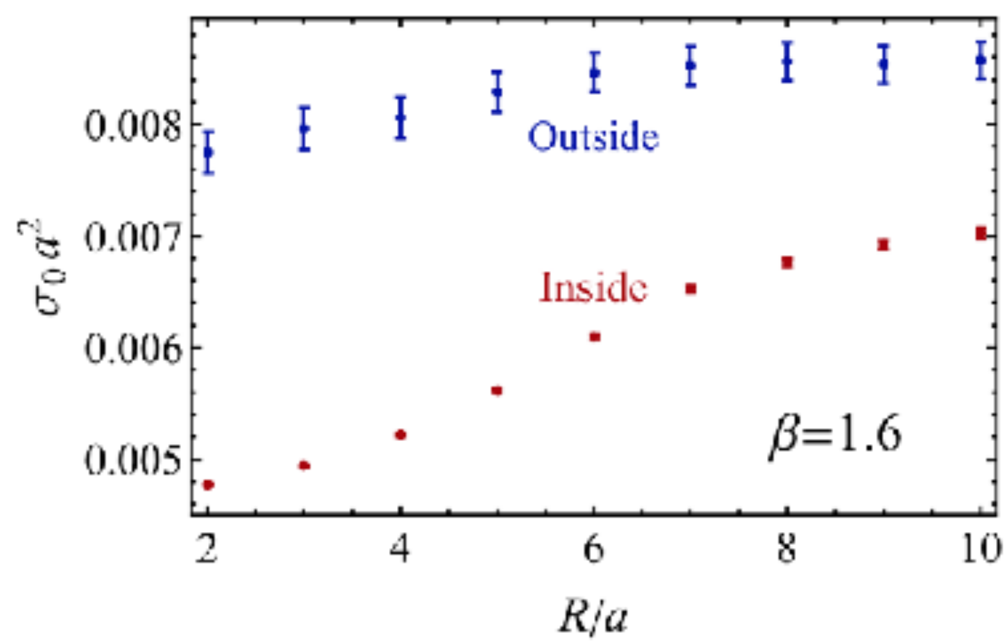
(a Coulomb gas of monopoles)

narrowly-spaced wires



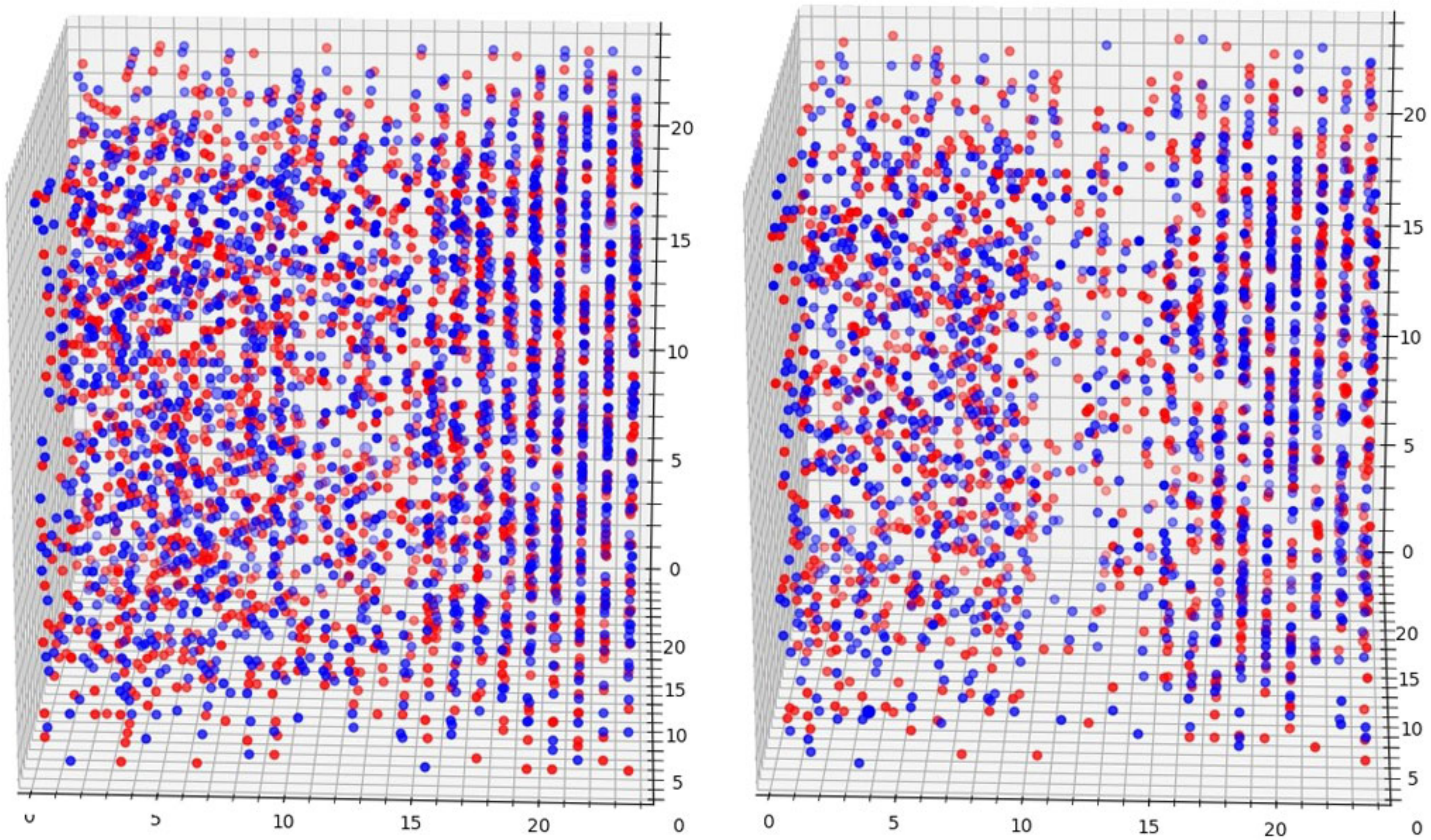
(a dilute gas of magnetic dipoles)

String tension inside and outside wires



V.A. Goy, A.V. Molochkov, M.Chernudub., Phys.Rev. D95, 074511 (2017); Phys.Rev. D96, 094507 (2017)]

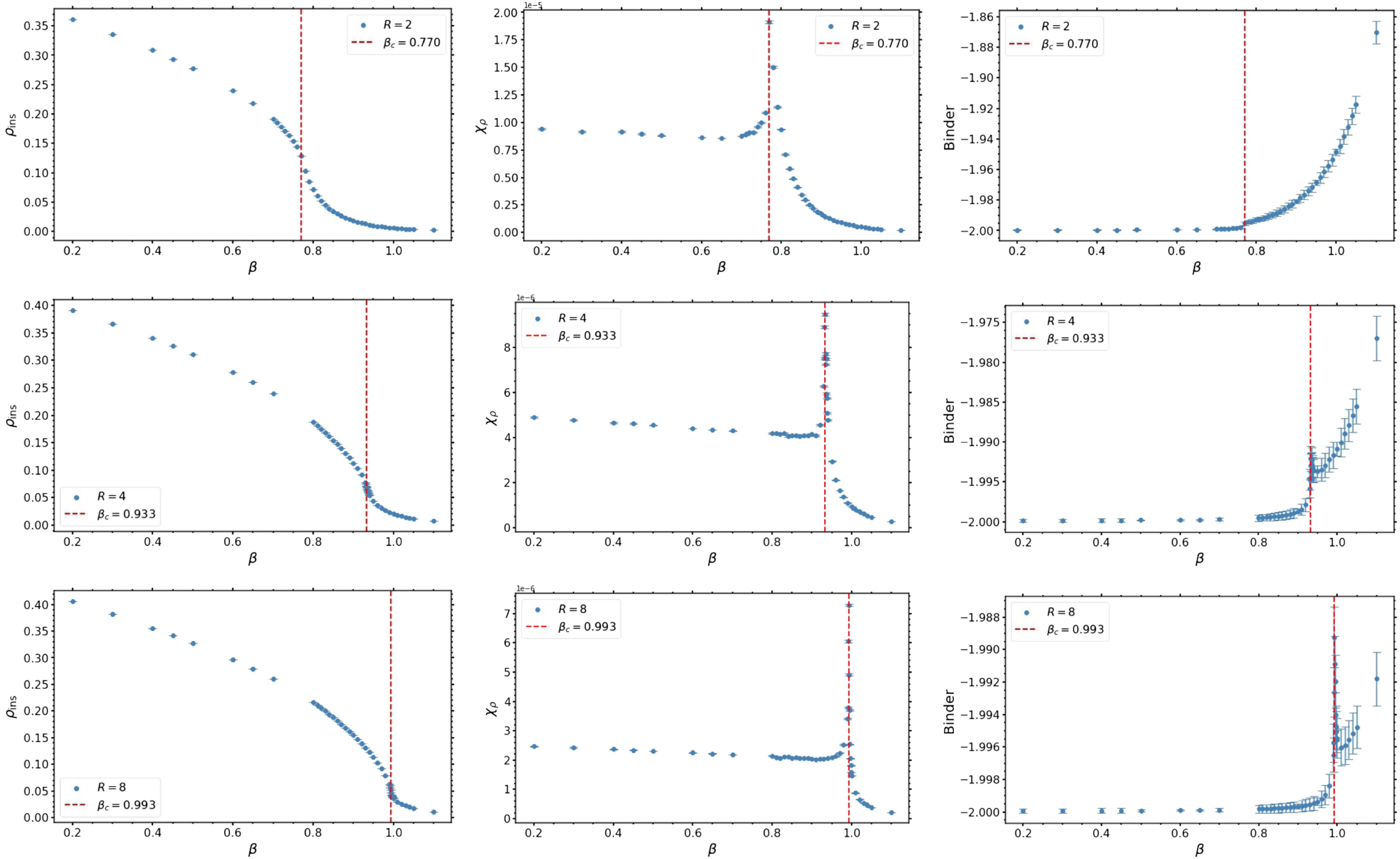
Compact U(1) monopole configurations in the presence of Casimir plates



(a) (b)

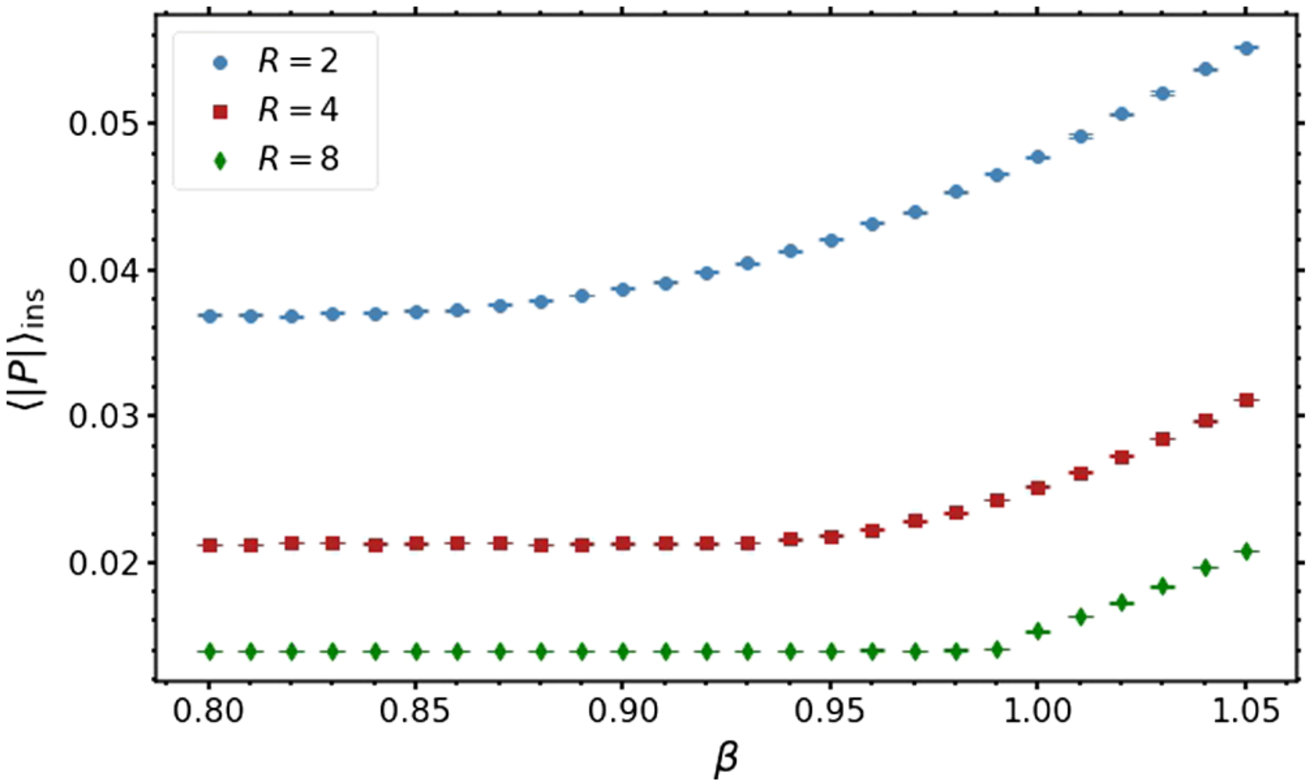
Typical examples of monopole configurations in (a) the confining phase ($\beta = 0.8$) and (b) the deconfining phase ($\beta = 0.9$) for the plates separated by the distance $R = 3$.

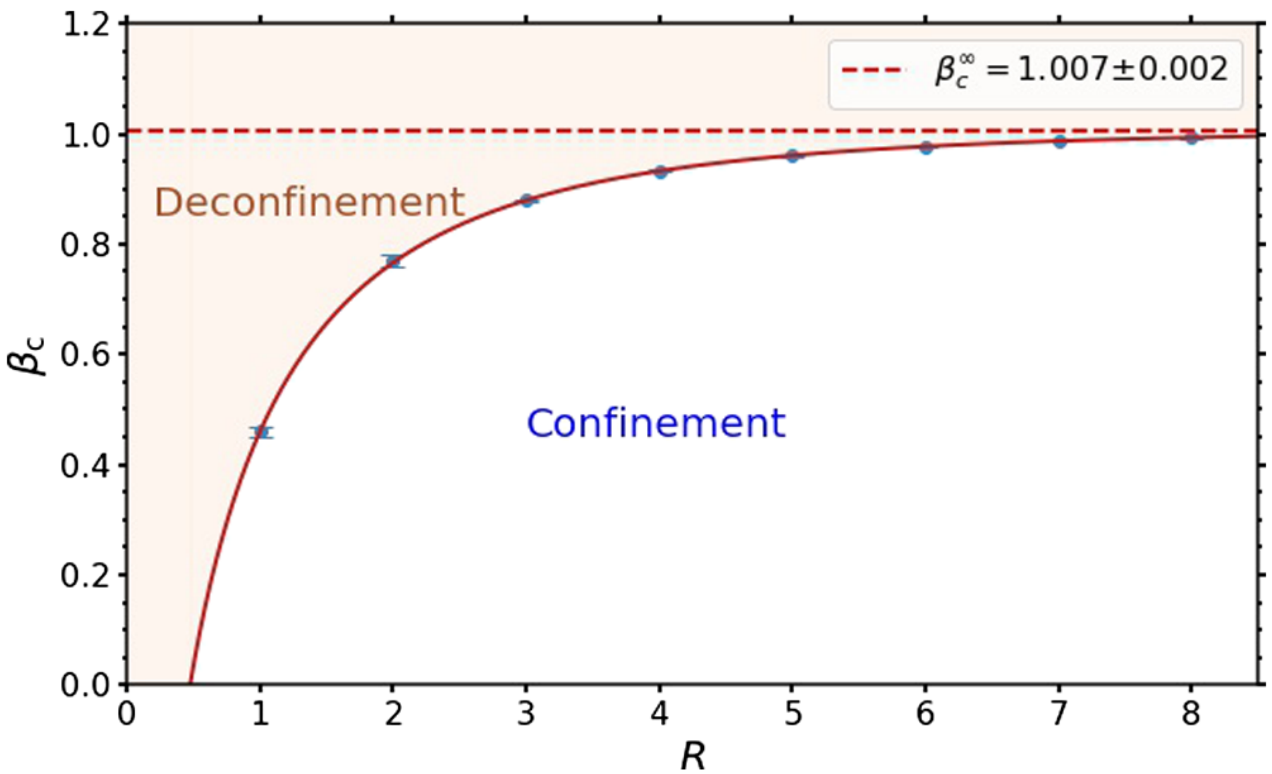
The monopoles and antimonopoles are represented by the red and blue dots, respectively. The plates, positioned vertically in the middle of the lattice, are not shown.

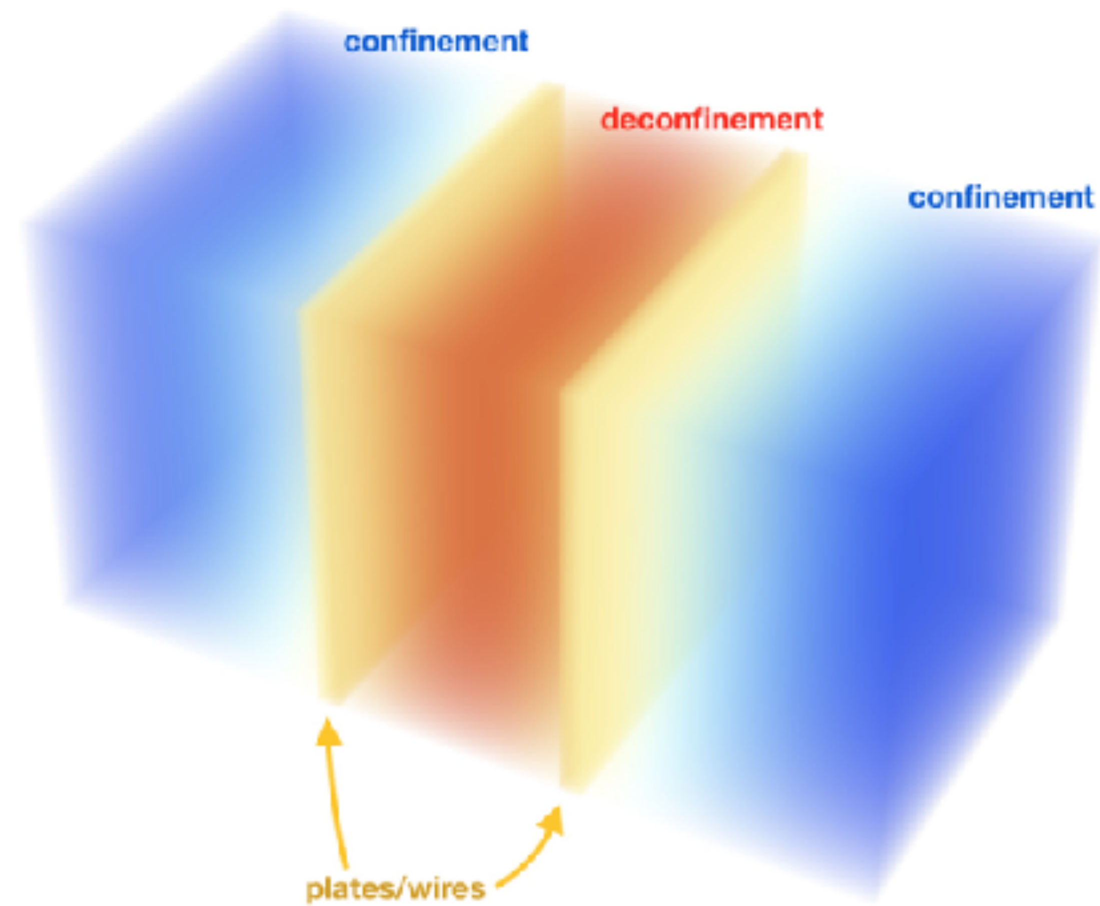


R = 2
βc = 0.770
R = 4
βc = 0.933
R = 8
βc = 0.993
ρins
χρ
Binder
β
1e−5
1e−6

Phase structure: deconfinement transition







The critical coupling β_c of the confinement-deconfinement transition as the function of the separation between the plates R in the ideal-metal limit ($\epsilon \to \infty$) in physical units.
(M.N. Chernodub, V.A. Goy, A.V. Molochkov, and A.S. Tanashkin Phys. Rev. D **105**, 114506)

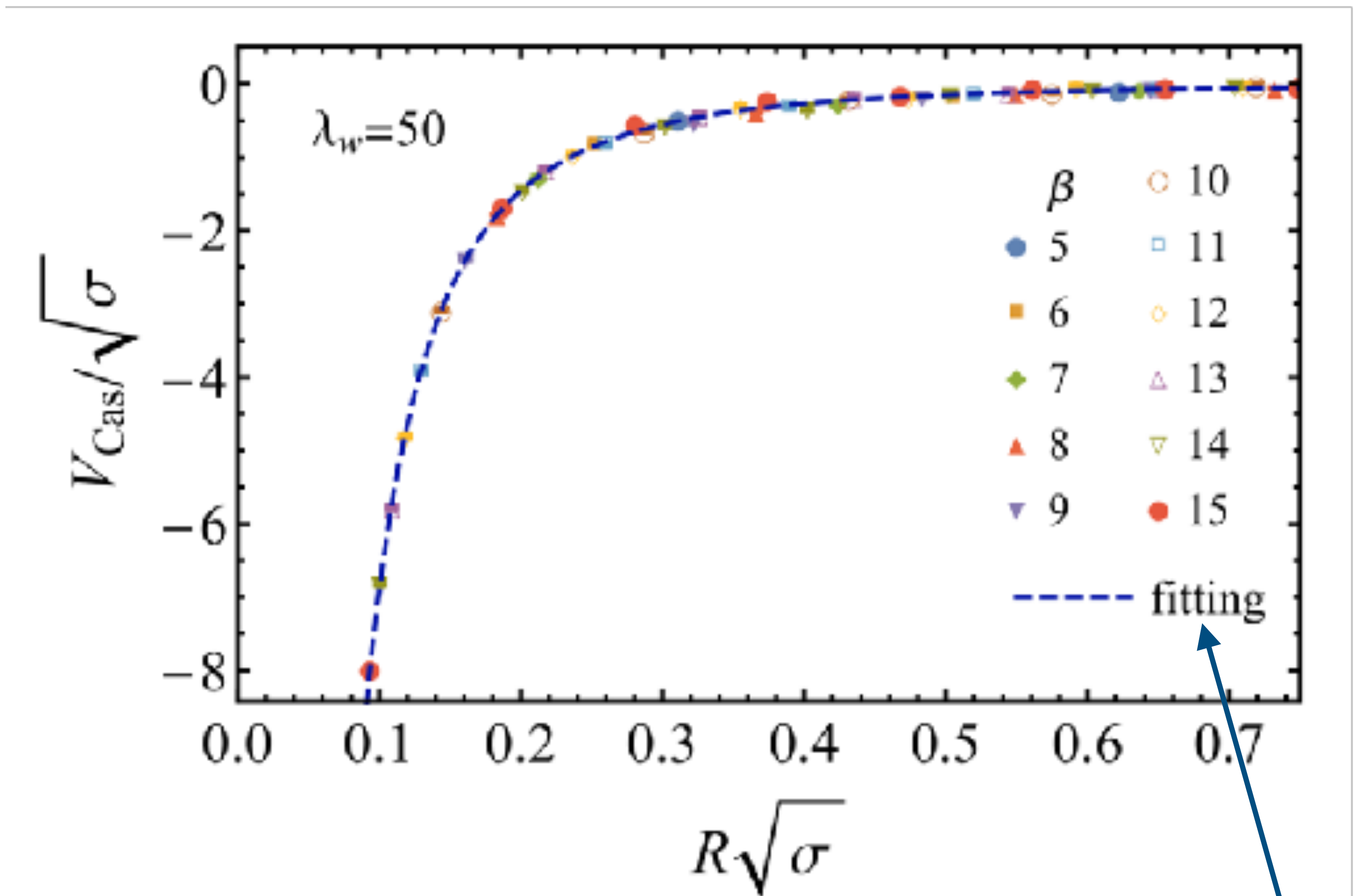
Non-Abelian Casimir effect: Casimir energy

The approach is easily generalizable to non-Abelian gauge groups.

Conditions at an ideal "chromo–metallic" boundary:

$$SU(N_c): \qquad F^{\mu\nu,a}(x) s_{\mu\nu}(x) = 0, \quad a = 1, \ldots N_c^2 - 1$$

The Casimir potential in (2+1)d for SU(2) gauge theory at $T = 0$:



σ is the fundamental string tension at $T = 0$

R is the distance between the wires (plates)

[V.A.Goy, A.V.Molochkov, H.Nguyen, M.Chernodub., PRL, arXiv:1805.11887]

Dimitra Karabali and V.P. Nair, Phys. Rev. D **98**, 105009 (2019)

Features:

- excellent scaling
- may be described by the function

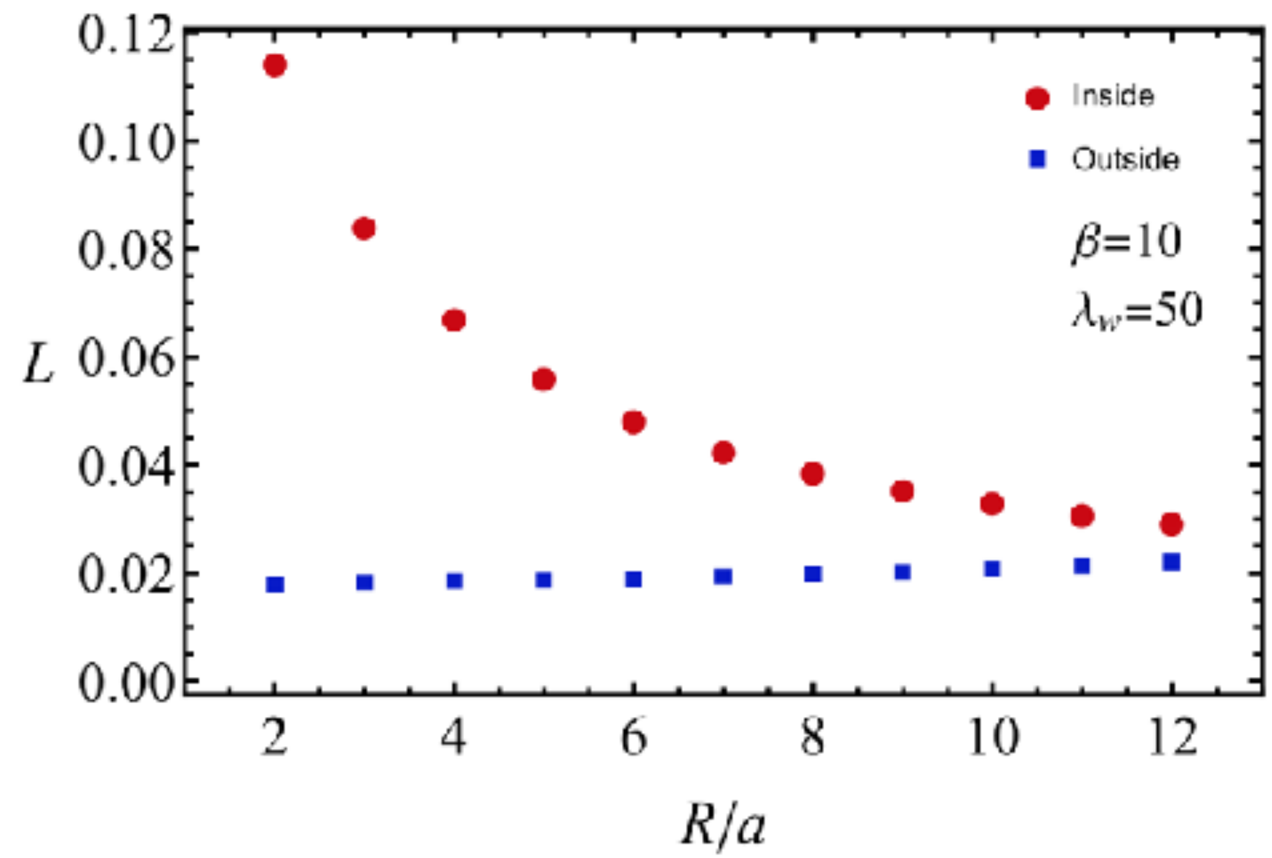
$$V_{\rm Cas}(R) = 3\frac{\zeta(3)}{16\pi}\frac{1}{R^2}\frac{1}{(\sqrt{\sigma}R)^{\nu}} e^{-M_{\rm Cas} R}$$

- Tree-level contribution
- (Perturbative) anomalous dimension $\nu = 0.05(2)$
- Nonperturbative Casimir mass $M_{\rm Cas} = 1.38(3)\sqrt{\sigma}$

(*cf.* the glueball mass $M_{0^{++}} \approx 4.7\sqrt{\sigma}$)

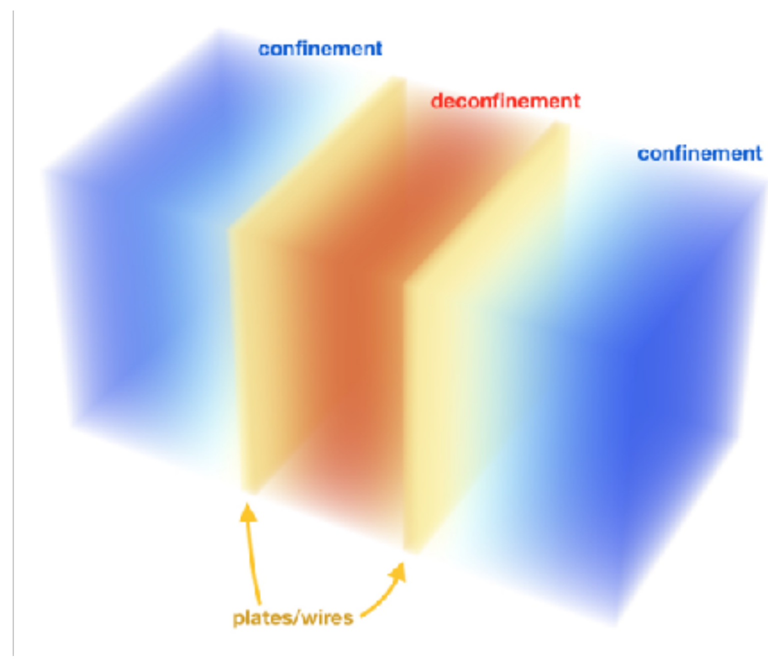
[M. J. Teper, Phys.Rev. D59, 014512 (1999)]

- The expectation value of the Polyakov line indicates deconfinement in between the plates (wires).



- No signal for a phase transition at finite R. (an infinite-order BKT–type transition or a crossover?)

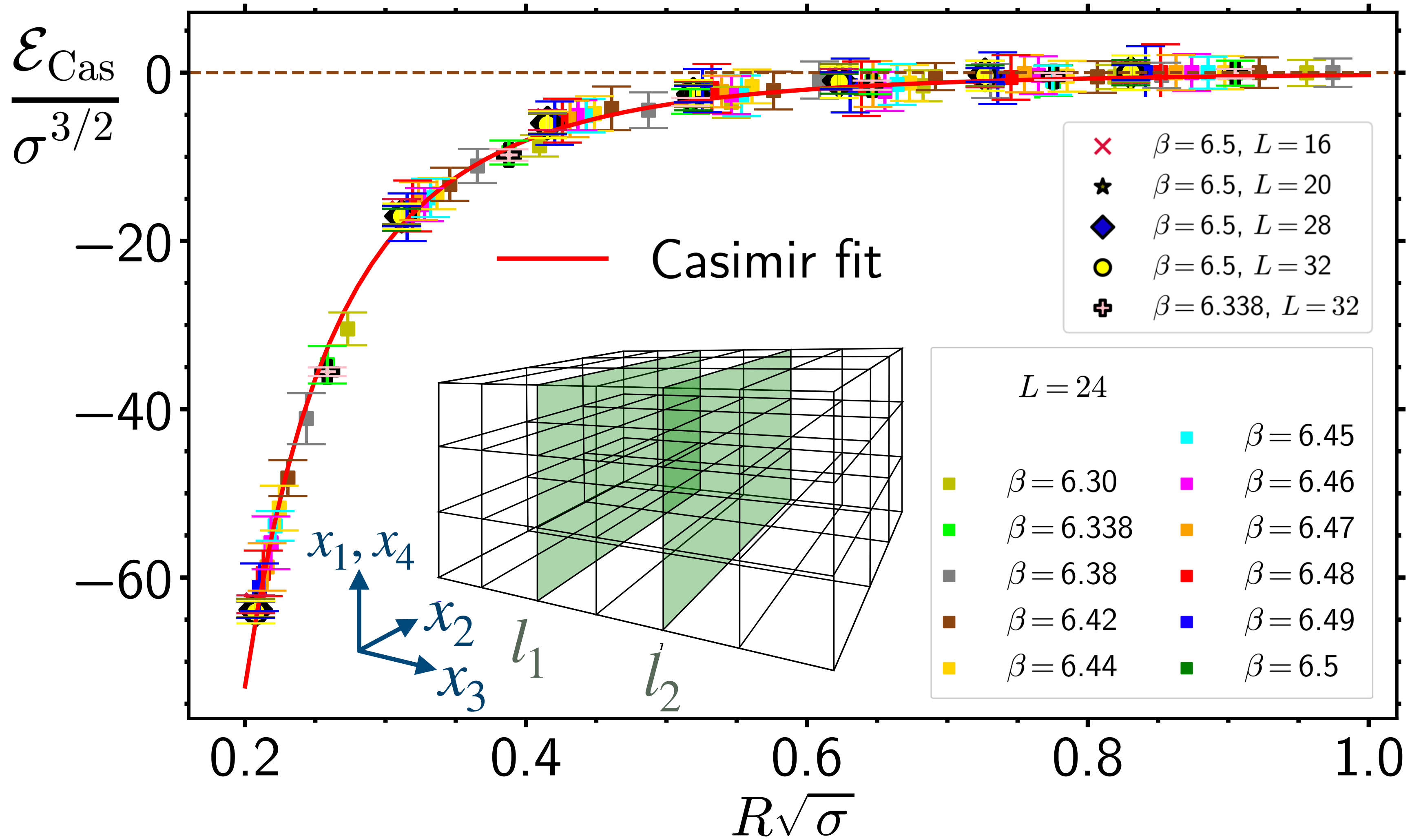
The finite Casimir geometry leads to a very smooth deconfinement transition in between the plates. The absence of a thermodynamic transition marks the difference with the finite temperature case.



[V.A.Goy, A.V.Molochkov, H.Nguyen, M.Ch., arXiv:1805.11887, PRL (2018)]

In a finite-temperature SU(2) gauge theory the phase transition is of the second order (Ising-type) [M. Teper, Phys.Lett. B313,417 (1993)].

A new gluonic excitation: the glueton - a colourless bound state of a gluon with its image in a chromometallic mirror.

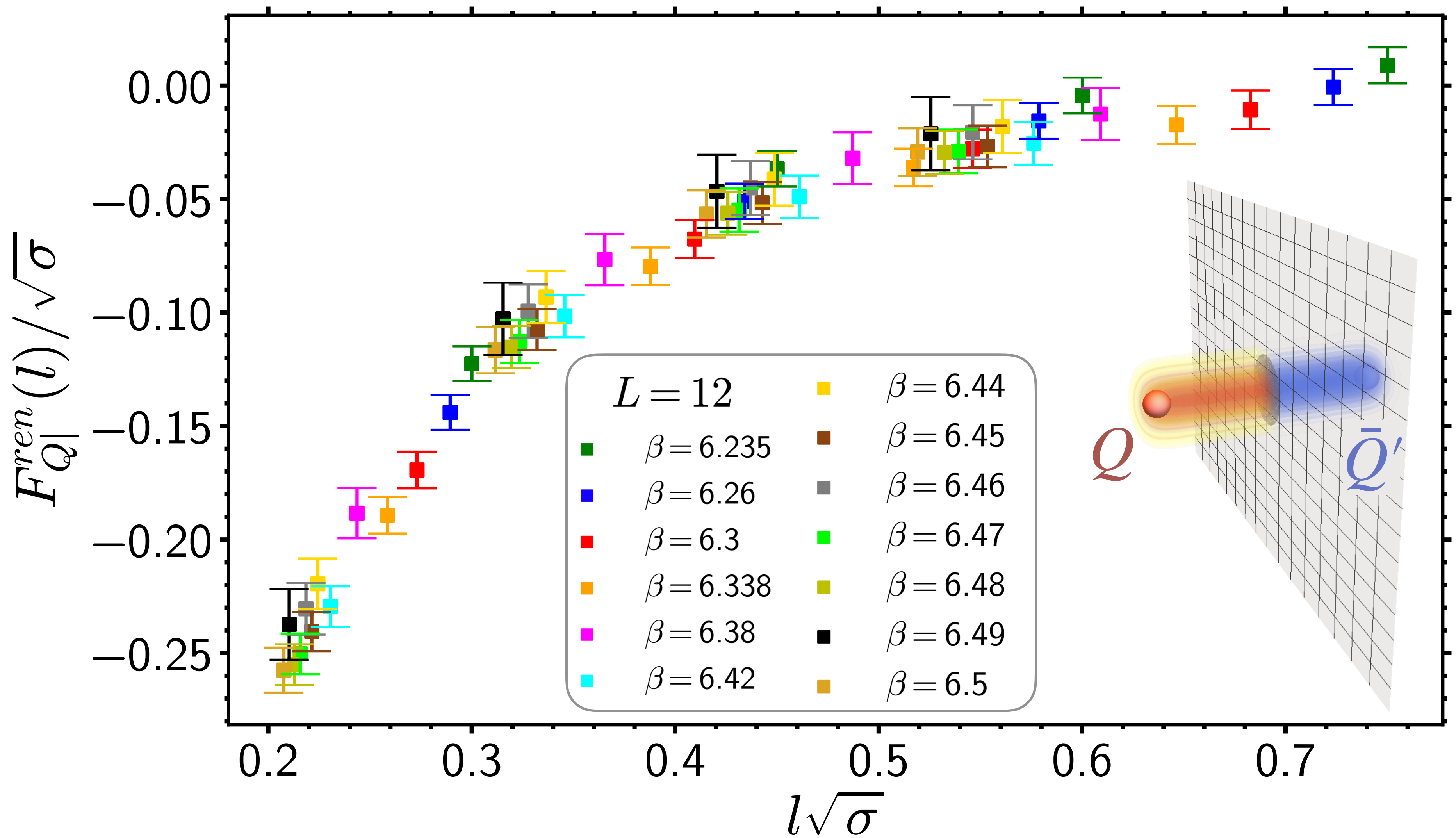


Casimir energy of a scalar field with certain mass mgt:

$$\mathcal{E}_{\rm Cas} = -C_0 \frac{2(N_c^2-1)m_{\rm gt}^2}{8\pi^2 R} \sum_{n=1}^{\infty} \frac{K_2(2n m_{\rm gt} R)}{n^2} \,.$$

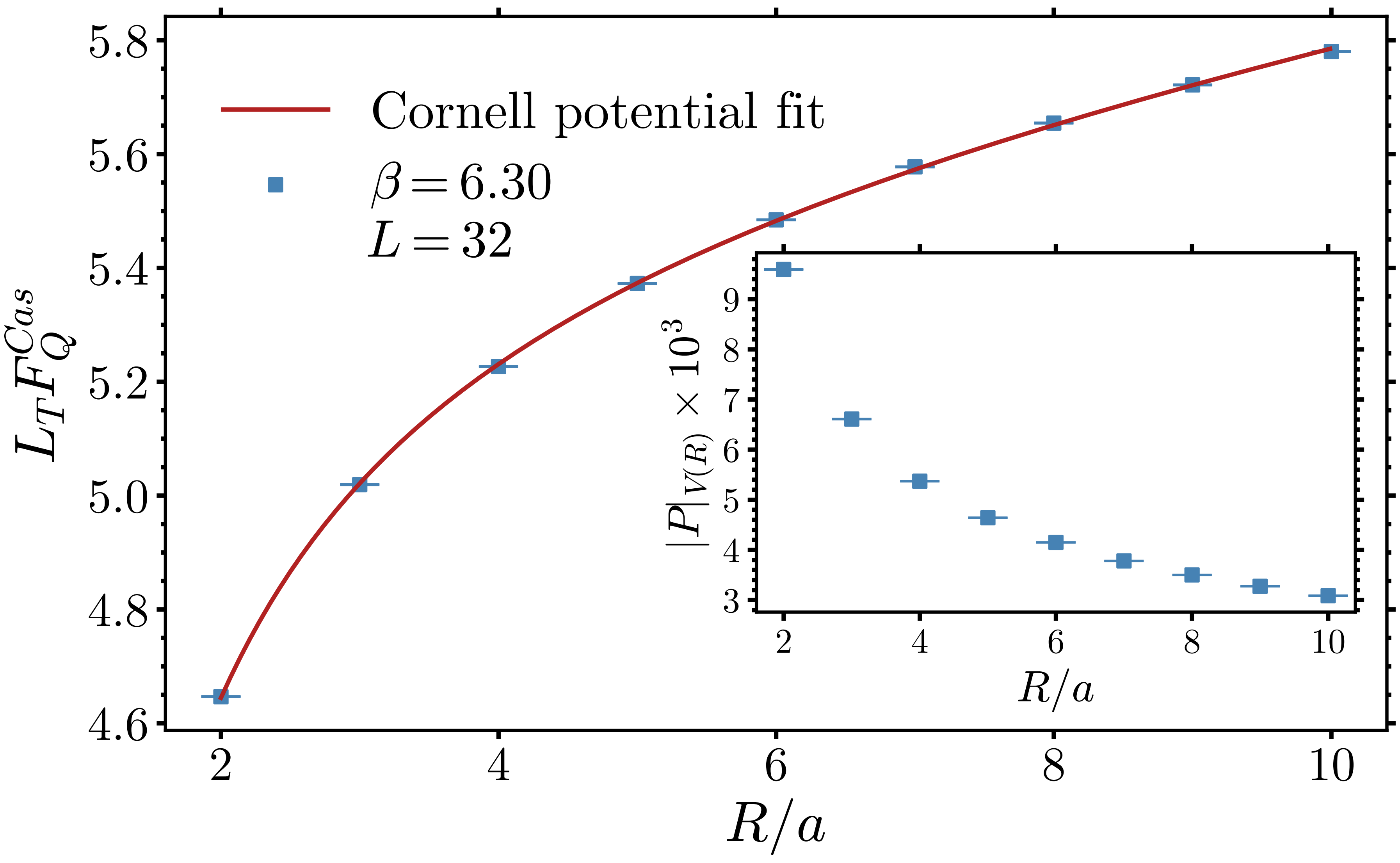
$m_{\rm gt} = 1.0(1)\sqrt{\sigma} = 0.49(5)\,{\rm GeV}$ $M_{0^{++}} = 3.405(21)\sqrt{\sigma} = 1.653(26)\,{\rm GeV}$

Quarkiton: quark colorless bound state with its negative image in a chromometallic mirror.



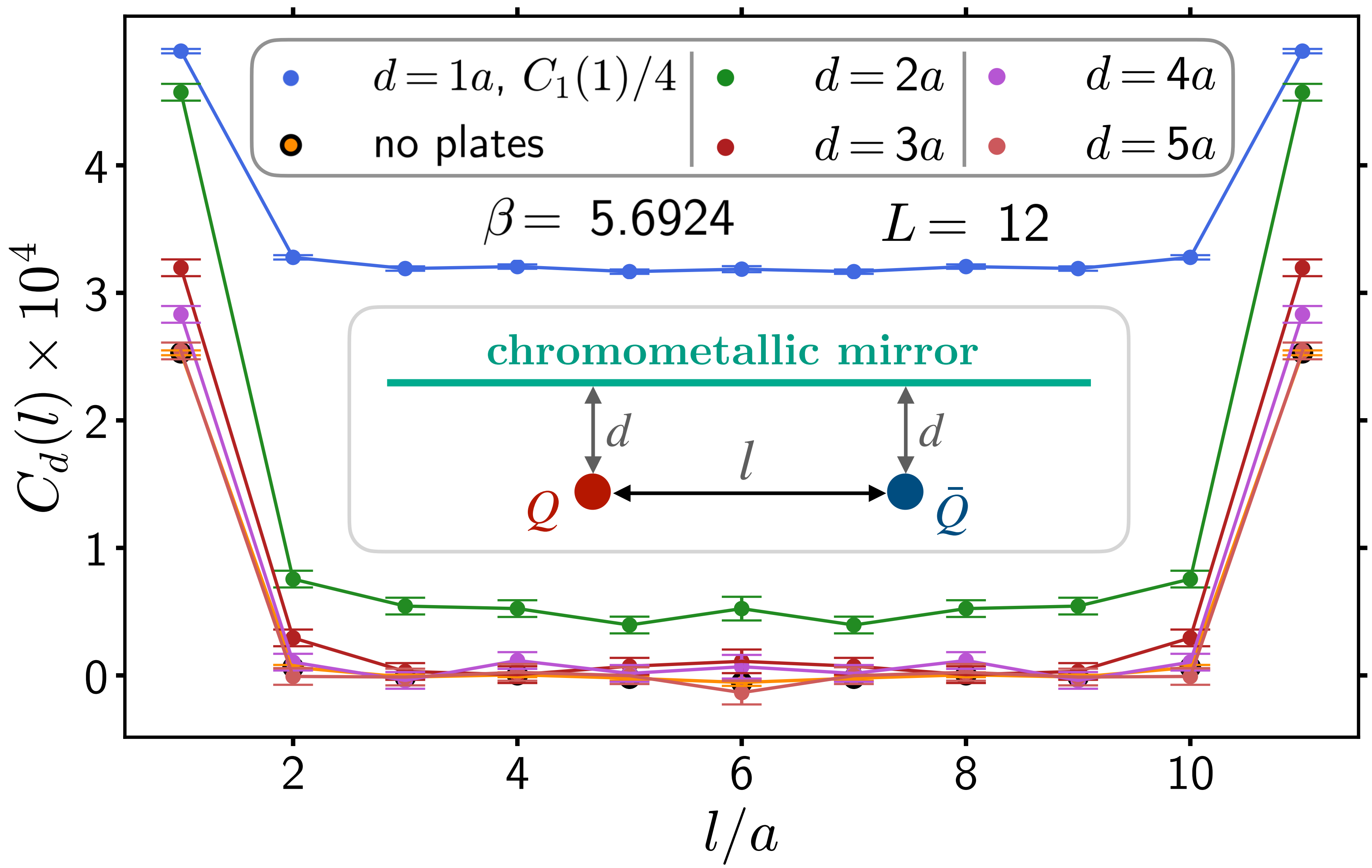
The renormalised free heavy-quark energy at a distance l from the chromometallic mirror, plotted in physical units, for various lattice coupling constants β at 12^3 lattice. The inset visualizes a quarkiton with the quark Q and its negative image in the chromometallic mirror. The anti-quark $\bar{Q}'$ (negative image), connected by a confining string (the "mirror" part of the string is shown in blue).

A single heavy quark in between the mirrors



A single isolated quark can exist in the hadronic phase of QCD near (and confined to) a large perfect chromometallic mirror, forming the quarkiton.

Quark - antiquark pair in the presence of chromometallic mirror



The correlator of the Polyakov loops for a quark and an antiquark located at the fixed distance d from the chromometallic mirror and separated by the distance l

Summary

- Does the quantum vacuum contribute to the cosmological constant? **It is an open question, still.**
- Quantum vacuum non-perturbative interaction with boundaries. **It plays a significant role in the Casimir effect, especially in the case of non-Abelian fields: New vacuum quasiparticle states at the boundaries.**
- Role of the quantum vacuum's condensates, topology and symmetry. **Condensates and non-trivial symmetry properties can change signs of the Casimir energy density.**
- Phase transitions. **Casimir plate can lead to phase transitions: different phases inside and outside of the plates.**
- Finite volume effects. **The effects are extremely important. They can lead to phase transitions, and vacuum structure changes inside the Casimir plates.**
- Casimir effect: Is it a quantum vacuum effect or just classical forces in a finite volume?