Gravitational Waves from Coalescing Black Holes

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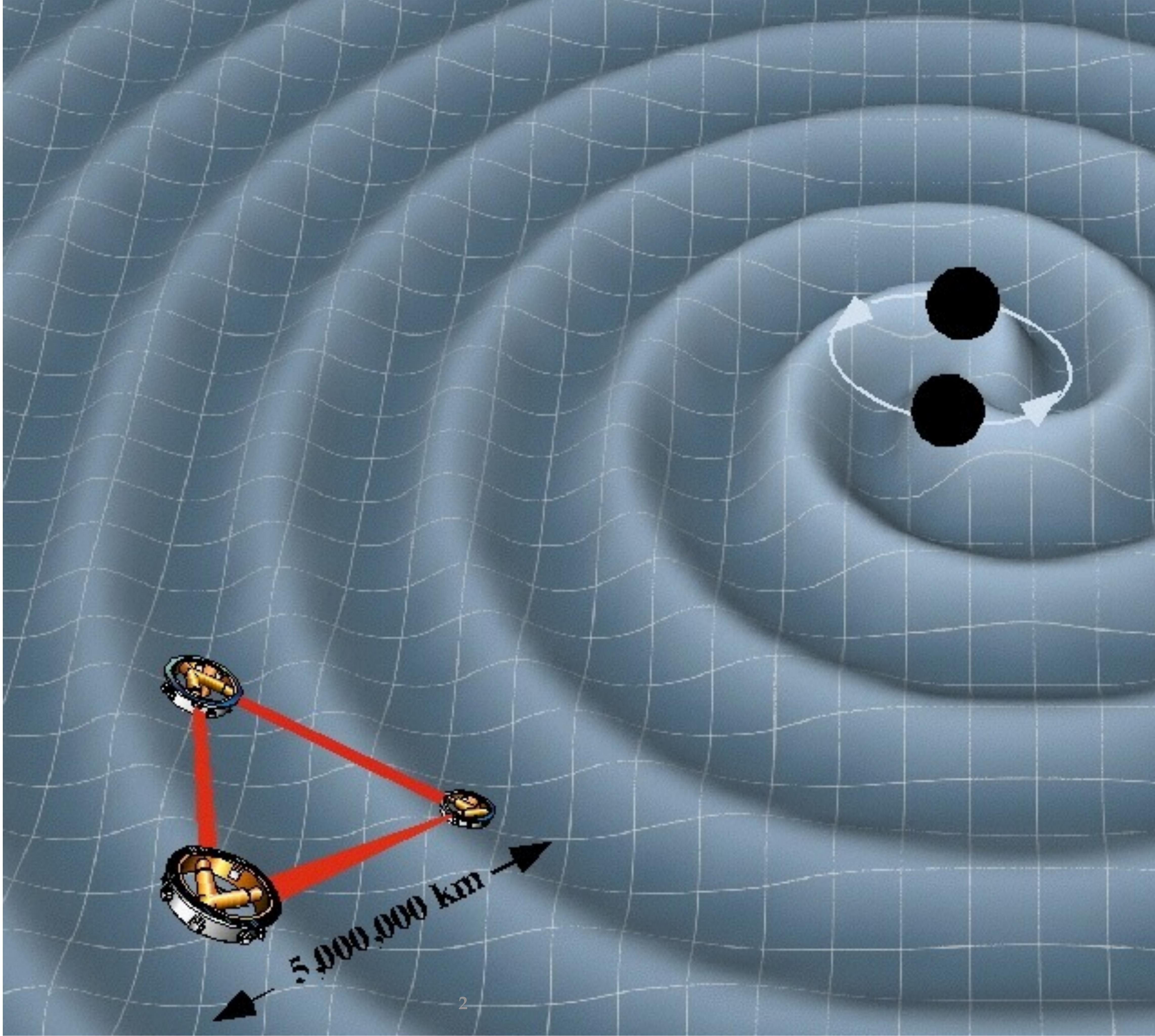
GARYFEST:
Gravitation, Solitons and Symmetries

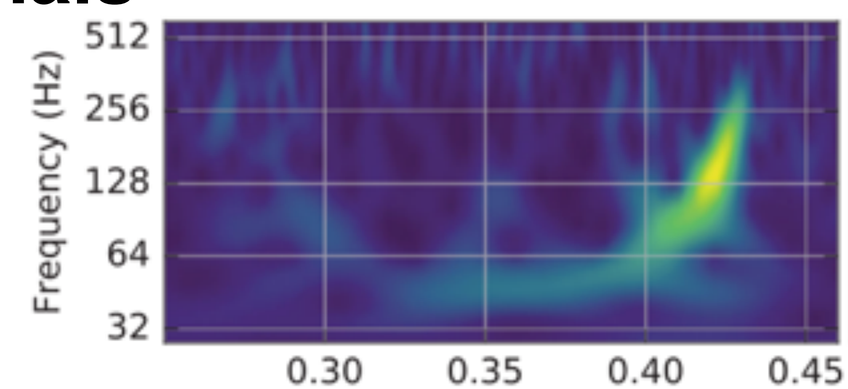
Le Studium, LMTP
Université François Rabelais
Tours, 22-24 March 2017

$$m_1 = 36^{+5}_{-4} M_\odot$$
$$m_2 = 29^{+4}_{-4} M_\odot$$
$$\chi_{\rm eff} = -0.06^{+0.17}_{-0.18}$$
$$D_{\rm L} = 410^{+160}_{-180} \text{Mpc}$$



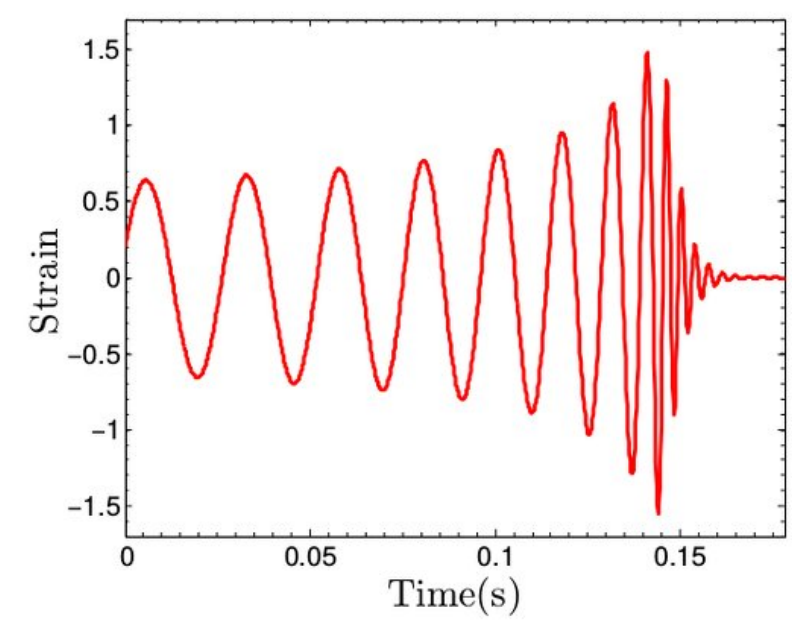
LIGO-Virgo data analysis

Various levels of search and analysis:
online/offline
unmodelled searches/matched-filter searches
online trigger
offline searches
significance assessment of candidate signals
parameter estimation



Online trigger searches:

CoherentWaveBurst Time-frequency
(Wilson, Meyer, Daubechies-Jaffard-Journe, Klimenko et al.)
Omicron-LALInference sine-Gaussians
Gabor-type wavelet analysis **(Gabor,…,Lynch et al.)**
Matched-filter:
PyCBC (f-domain), gstLAL (t-domain)



Offline data analysis:

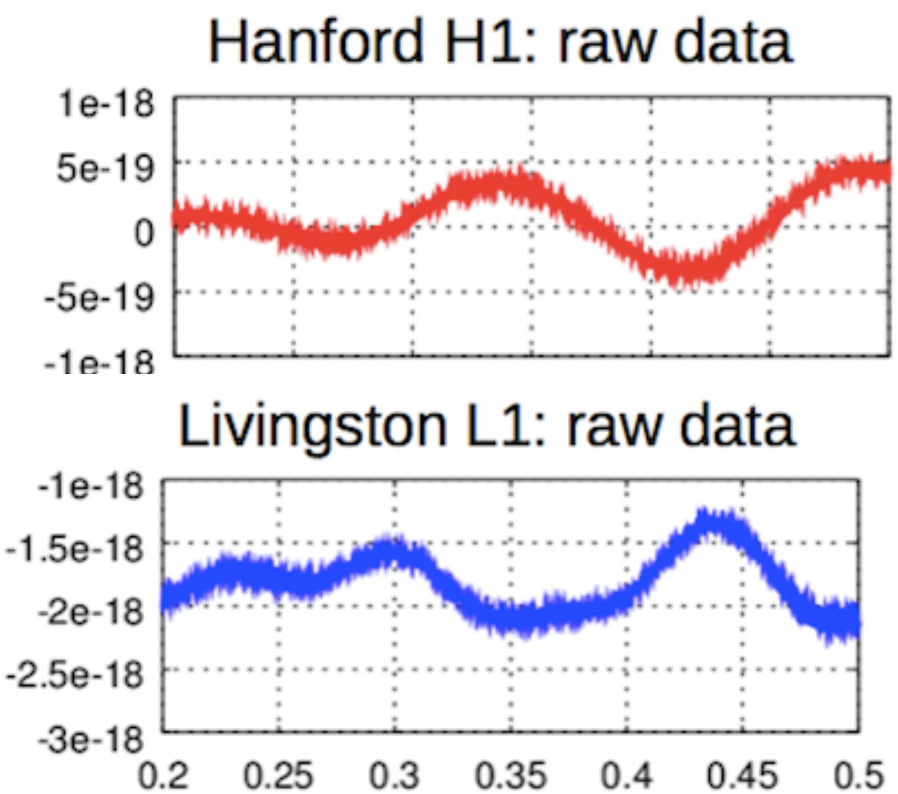
Generic transient searches
Binary coalescence searches

Here: focus on matched-filter definition

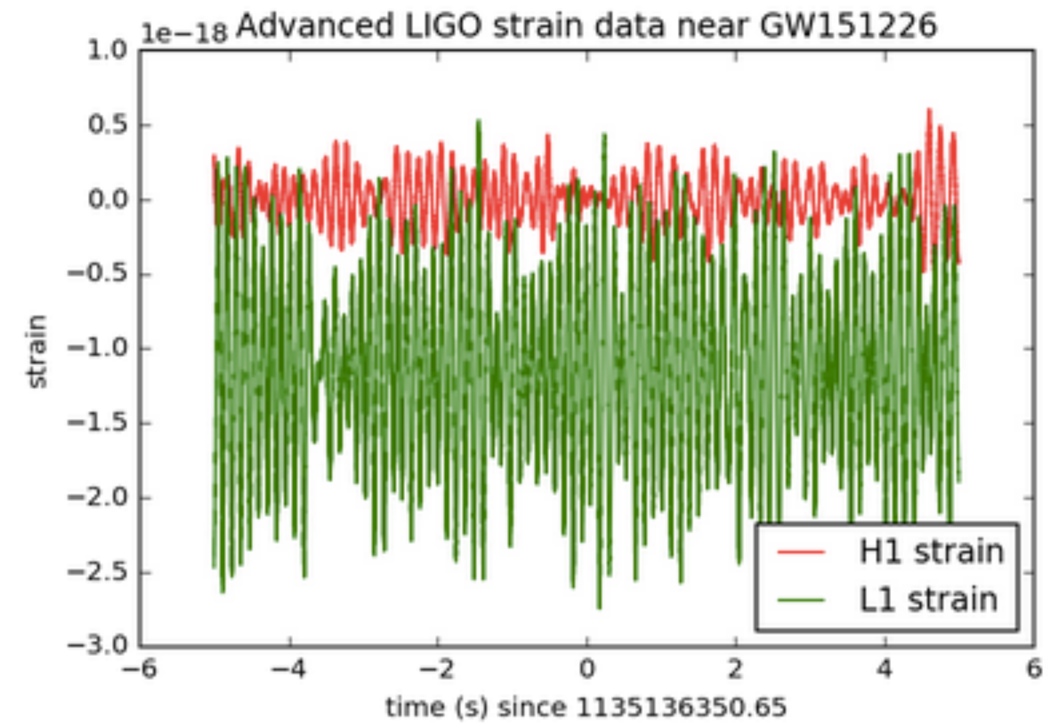
(crucial for high SNR, significance assessment, and parameter estimation)

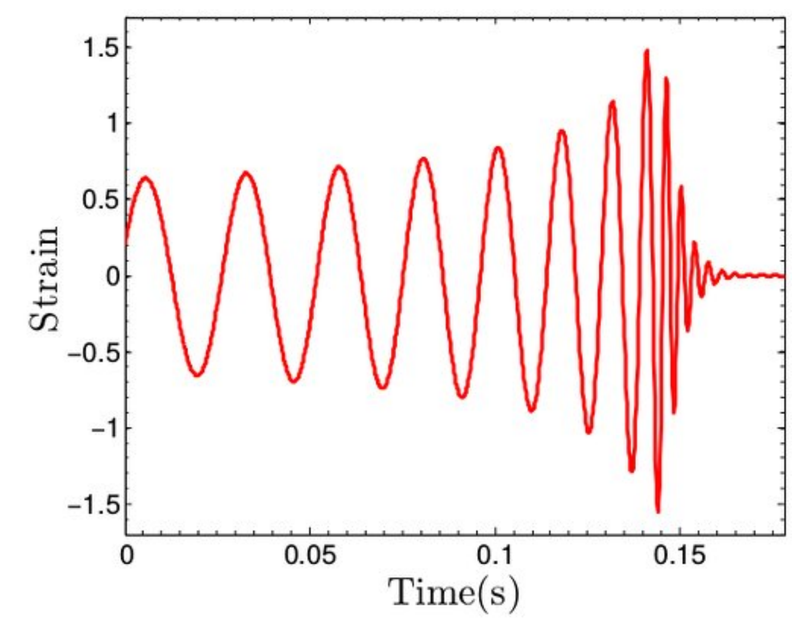
GW150914 and GW151226: incredibly small signals lost in the broad-band noise

GW150914, from Chassande-Mottin 5 April 2016



GW151226 from LIGO open data





$$h_{GW}^{\max} \sim 10^{-21} \sim 10^{-3}\, h_{LIGO}^{\text{broadband}}$$

$$\delta L/L = 10^{-21} \rightarrow \delta L \sim 10^{-9}\,\text{atom}\,!$$

$$\frac{\delta L^{tot}}{\lambda} \sim \mathcal{F}\frac{L}{\lambda}\frac{\delta L}{L} \sim 10^{11} h \sim 10^{-10}\text{fringe}$$

Matched Filtering

$$\langle output | h_{\text{template}} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\text{template}}(f)$$

MATCHED FILTERING SEARCH AND DATA ANALYSIS

Precomputed bank of ~ 200 000 EOB templates for inspiralling and coalescing BBH GW waveforms: m1, m2, chi1=S1/m1^2, chi2=S2/m2^2 for m1+m2> 4Msun; + ~ 50 000 PN inspiralling templates for m1+m2< 4 Msun

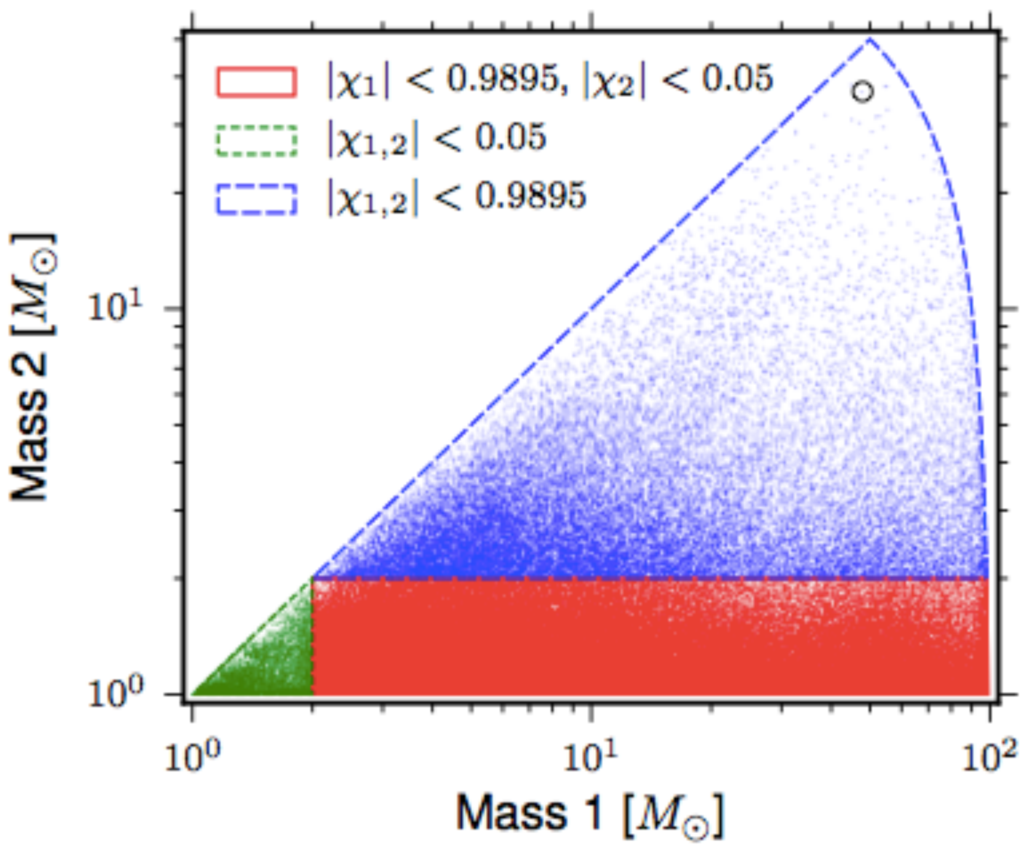
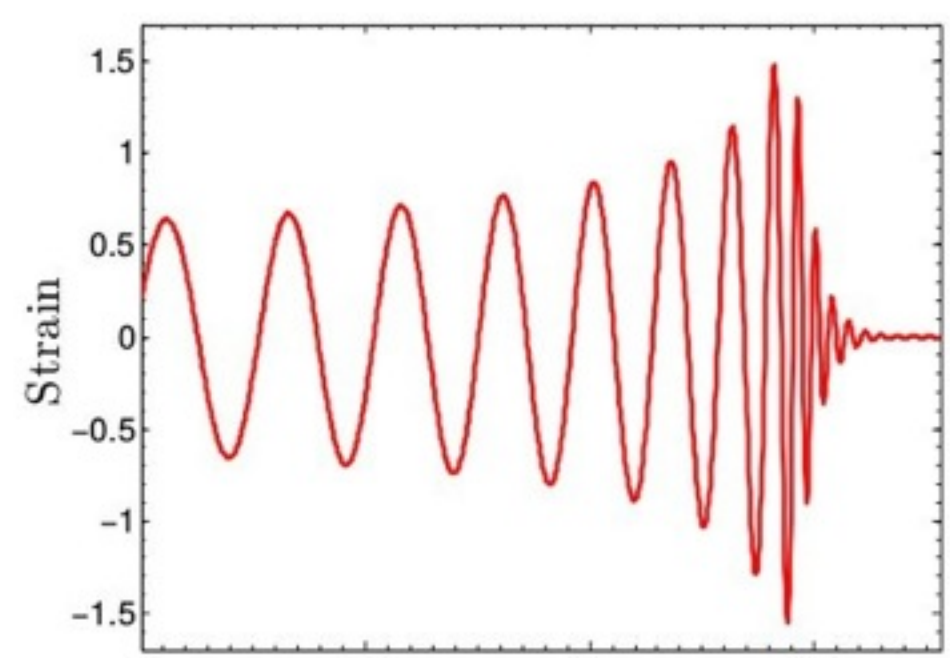


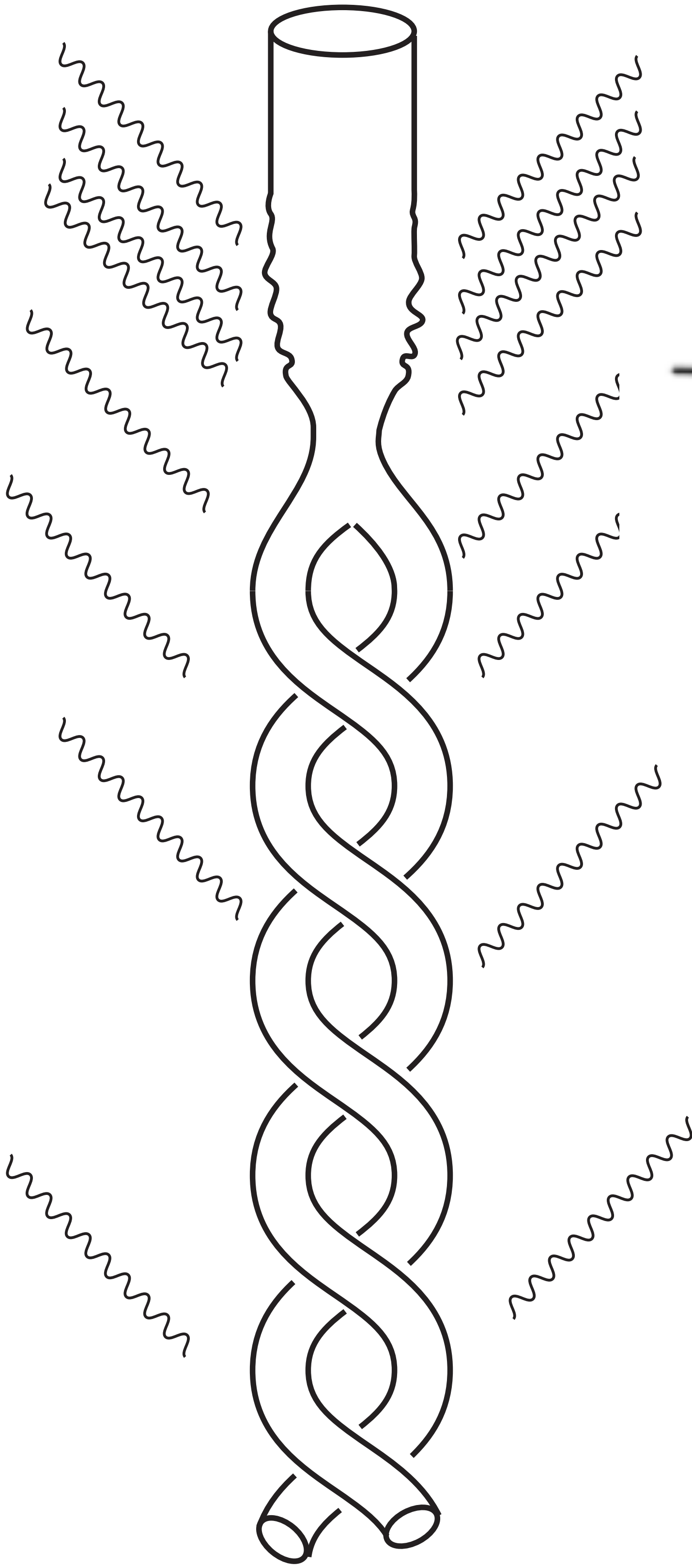
FIG. 1. The four-dimensional search parameter space covered by the template bank shown projected into the component-mass plane, using the convention $m_1 > m_2$. The lines bound mass regions with different limits on the dimensionless aligned-spin parameters χ_1 and χ_2. Each point indicates the position of a template in the bank. The circle highlights the template that best matches GW150914. This does not coincide with the best-fit parameters due to the discrete nature of the template bank.

Search template bank made of SpinningEOB[NR] templates (Buonanno-Damour 99, Damour '01…,Taracchini et al. 14) in ROM form; Recently improved (Bohé et al '16') by including leading 4PN terms (Bini-Damour '13), spin-dependent terms (Pan-Buonnano et al. '13), and calibrating against 141 NR simulations. [post-computed NR waveform for GW151226 took three months and 70 000 CPU hours !]



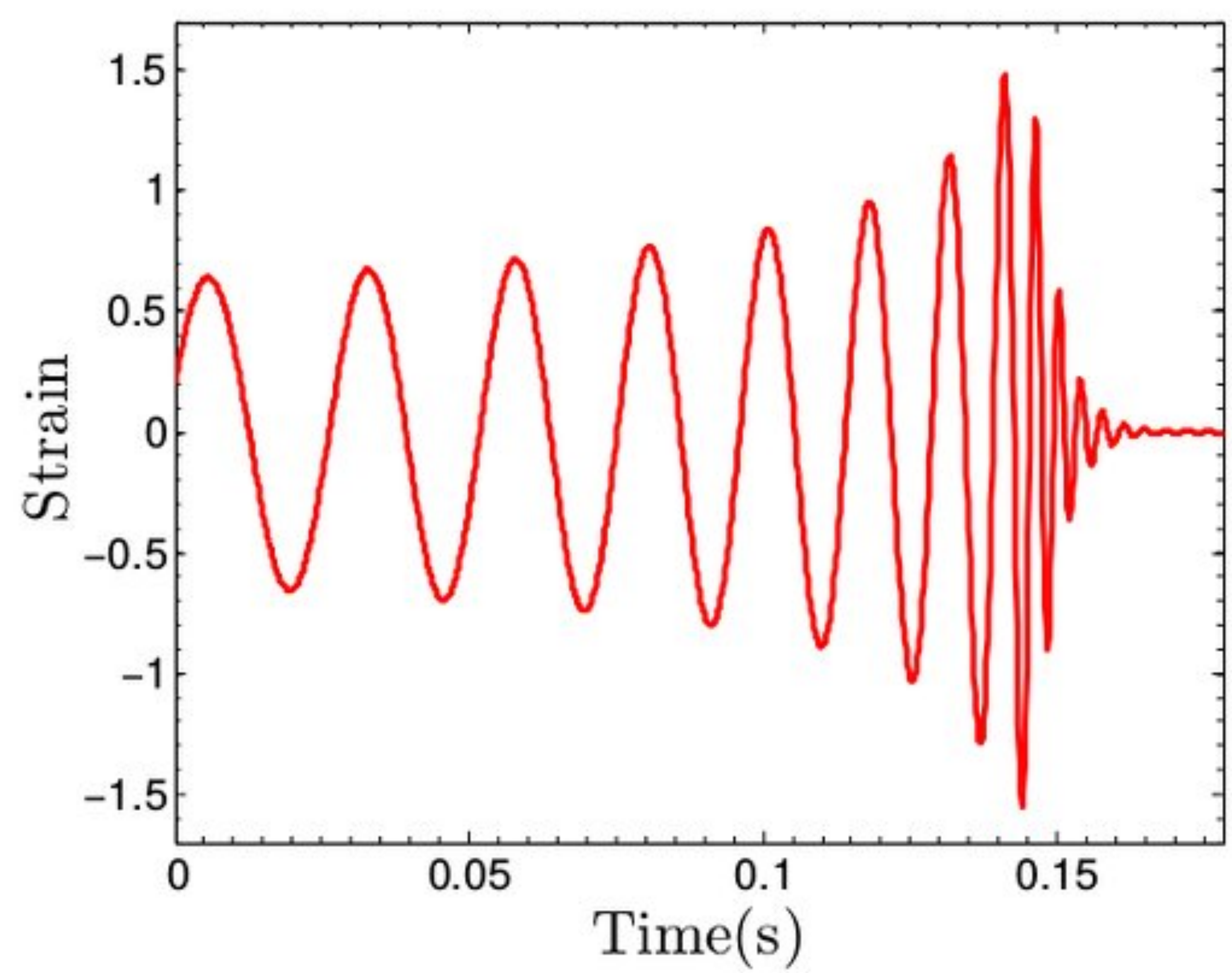
$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} = 0$$

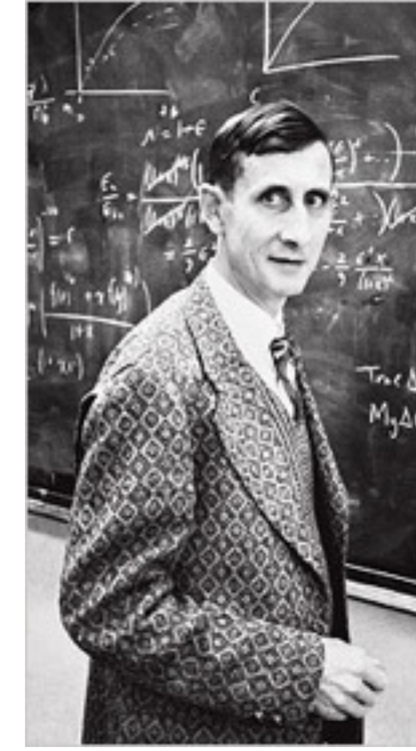


$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$

$$-g^{\mu\nu} g_{\alpha\beta,\,\mu\nu} + g^{\mu\nu} g^{\rho\sigma} (g_{\alpha\mu,\,\rho} g_{\beta\nu,\,\sigma} - g_{\alpha\mu,\,\rho} g_{\beta\sigma,\,\nu} + g_{\alpha\mu,\,\rho} g_{\nu\sigma,\,\beta} + g_{\beta\mu,\,\rho} g_{\nu\sigma,\,\alpha} - \frac{1}{2} g_{\mu\rho,\,\alpha} g_{\nu\sigma,\,\beta}) = 0$$

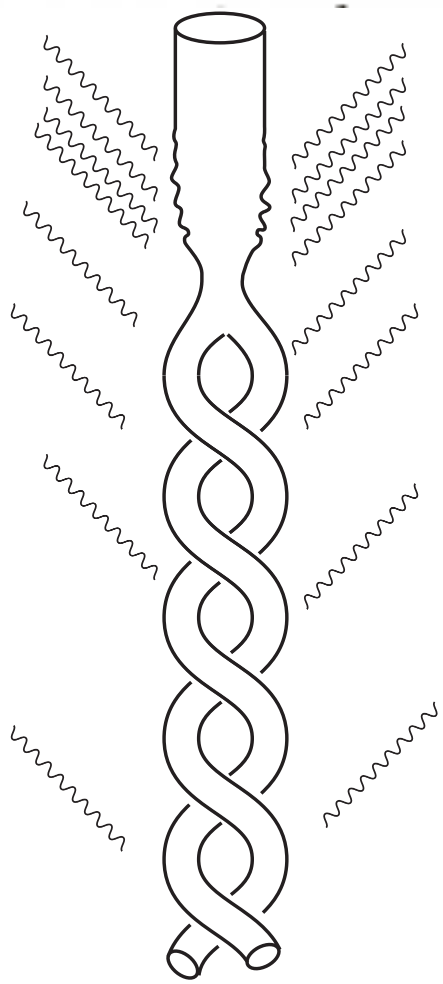


Pioneering the GWs from coalescing compact binaries



Freeman Dyson 1963, using Einstein 1918 + Landau-Lifshitz 1951 (+ Peters '64)
first vision of an intense GW flash from coalescing binary NS

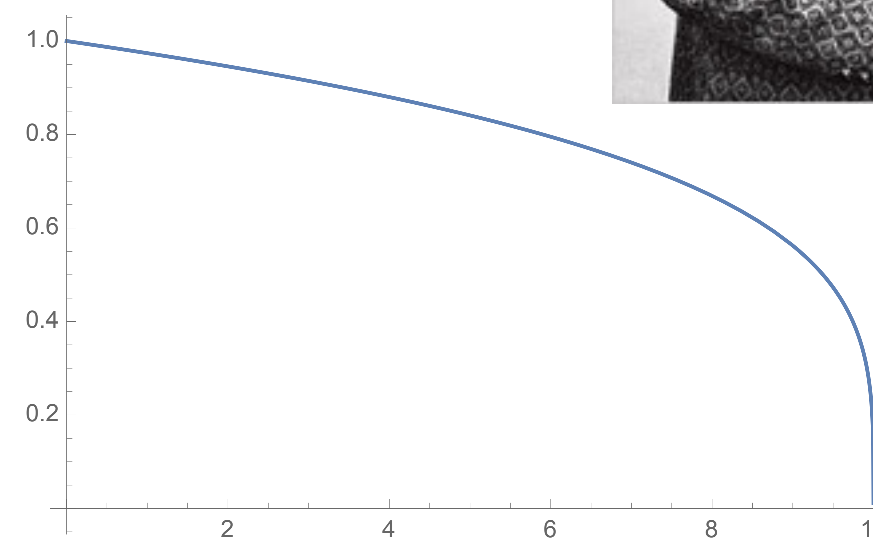
but the final end will be the same. According to (11), the loss of energy by gravitational radiation will bring the two stars closer with ever-increasing speed, until in the last second of their lives they plunge together and release a gravitational flash at a frequency of about 200 cycles and of unimaginable intensity.

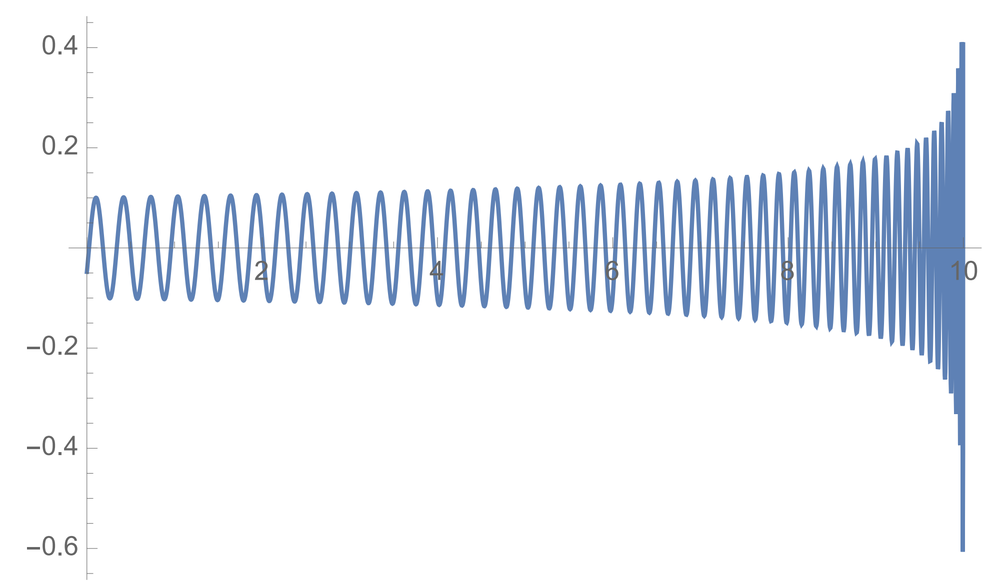


$$E = -\frac{G\, m_1\, m_2}{2r}$$

$$\frac{d}{dt} E = -F$$

$$F = \frac{32\, G^4}{5\, c^5} \frac{m_1^2\, m_2^2 (m_1 + m_2)}{r^5}$$

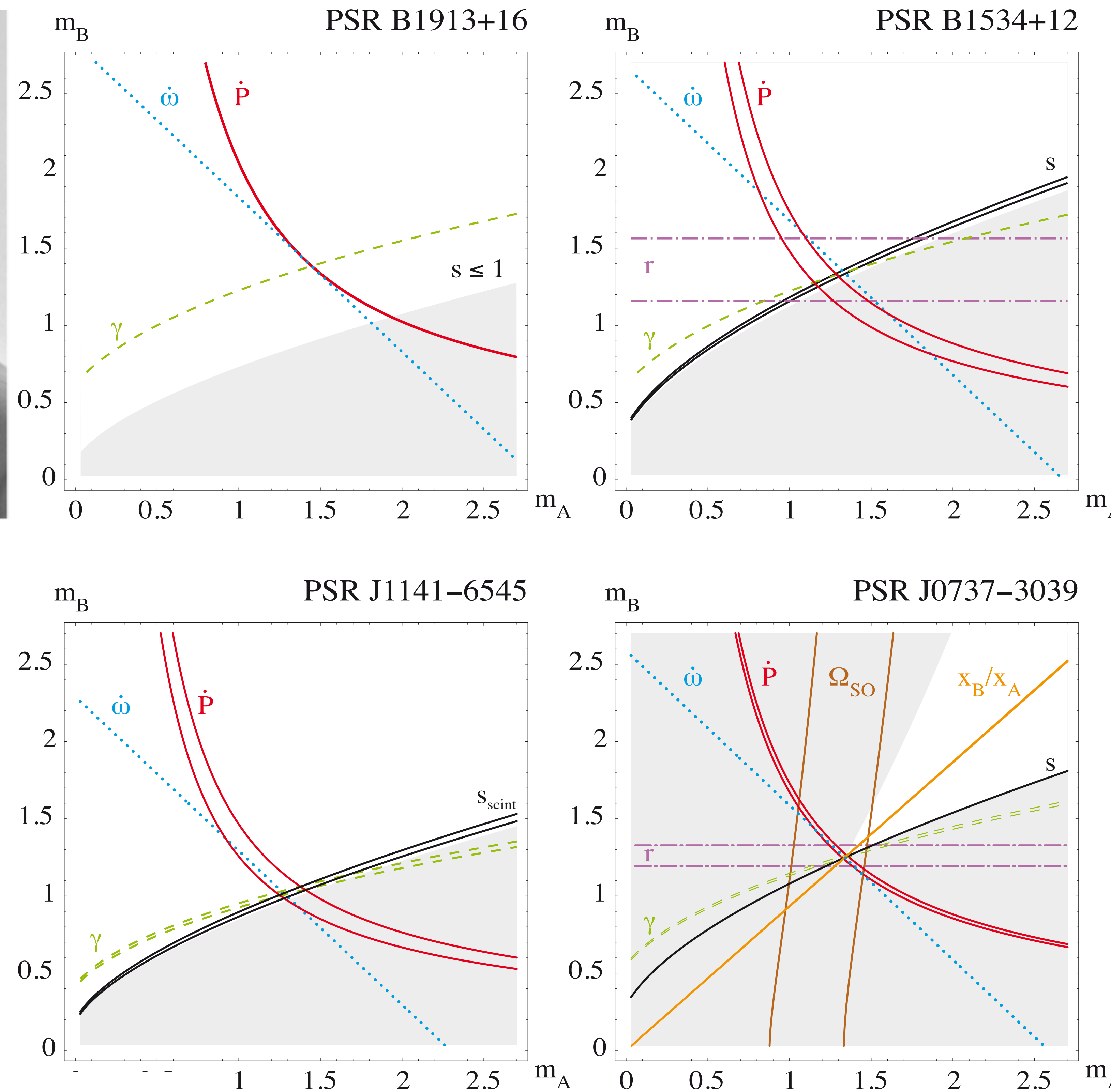


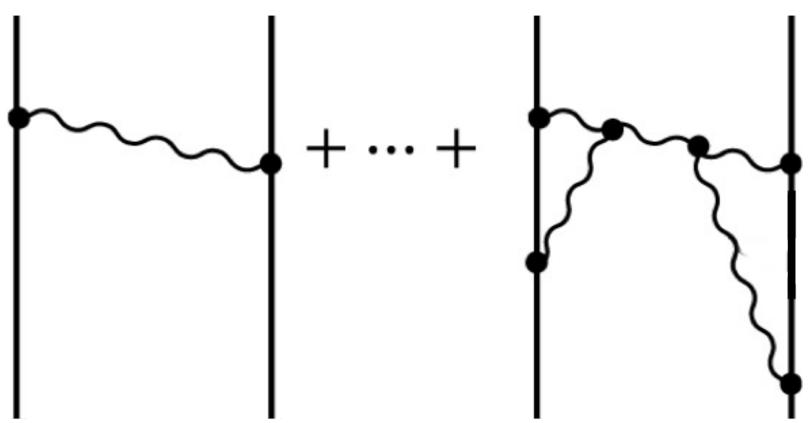


Challenge: describe the intense flash of GWs emitted by the last orbits and the merger of a binary BH, when v~c and r~GM/c^2

Binary pulsars: proof of radiative and strong-field gravity + existence of coalescing NS binaries

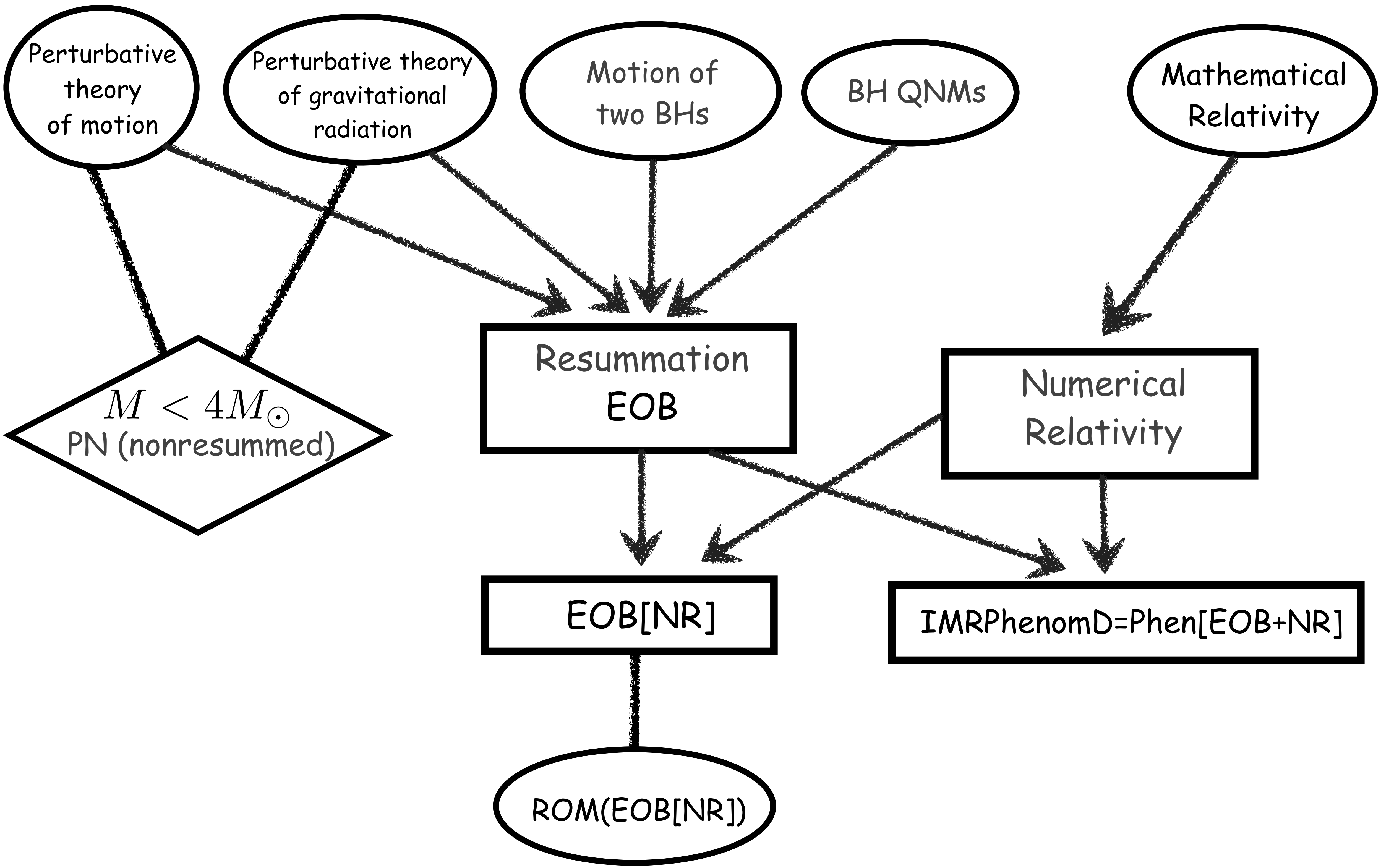
Russell Hulse Joseph Taylor

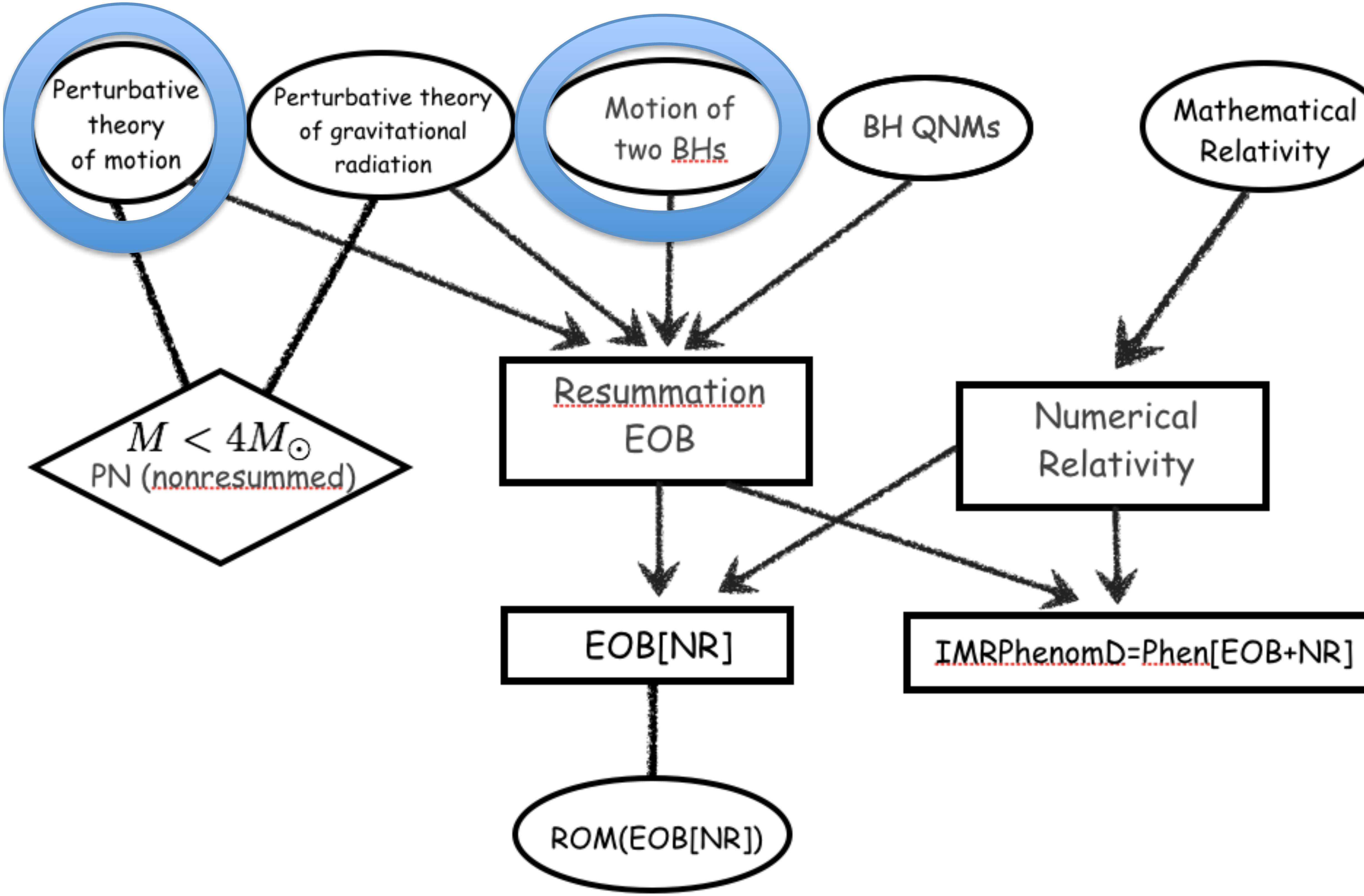




Damour-Deruelle'81 Damour'82
EOM (v/c)^5

$$\frac{dP}{dt} = -\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P}\right)^{\frac{5}{3}} \mu M^{\frac{2}{3}} \frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{\frac{7}{2}}}$$


Perturbative theory of motion
Perturbative theory of gravitational radiation
Motion of two BHs
BH QNMs
Mathematical Relativity
$M < 4M_\odot$
PN (nonresummed)
Resummation EOB
Numerical Relativity
EOB[NR]
IMRPhenomD=Phen[EOB+NR]
ROM(EOB[NR])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Long History of the GR Problem of Motion

Einstein 1912 : geodesic principle $$-\int m\sqrt{-g_{\mu\nu}\, dx^\mu\, dx^\nu}$$

Einstein 1913-1916 post-Minkowskian $$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)\ ,\ h_{\mu\nu} \ll 1$$

Einstein, Droste : post-Newtonian $$h_{00} \sim h_{ij} \sim \frac{v^2}{c^2}\ ,\ h_{0i} \sim \frac{v^3}{c^3}\ ,\ \partial_0 h \sim \frac{v}{c}\partial_i h$$

Weakly self-gravitating bodies:

$$\nabla_\nu\, T^{\mu\nu} = 0 \quad ; \quad T^{\mu\nu} = \rho'\, u^\mu u^\nu + p\, g^{\mu\nu} \Rightarrow \nabla_u\, u^\mu = O(\nabla p)$$

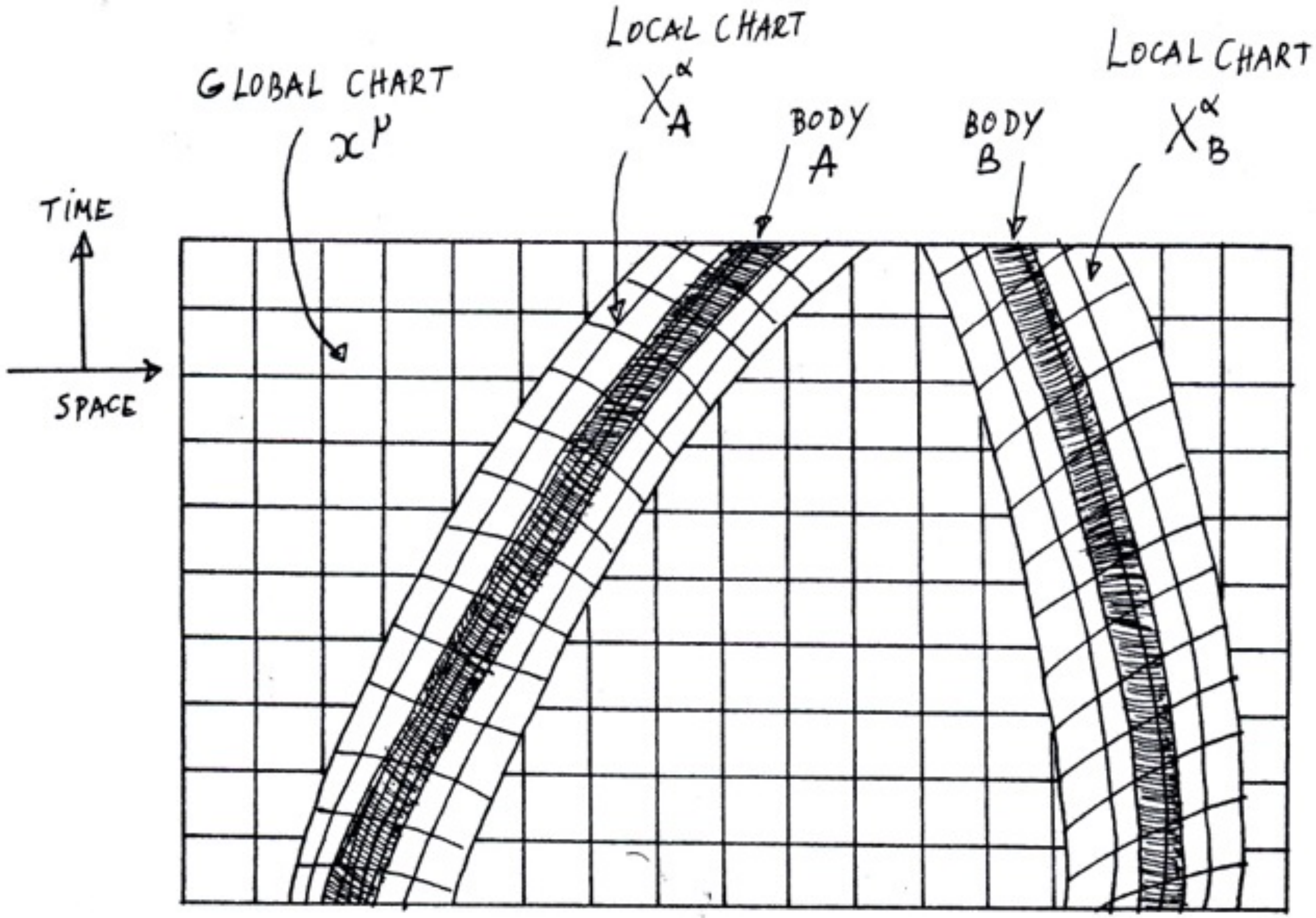
Einstein-Grossmann ’13,
1916 post-Newtonian: Droste, Lorentz, Einstein (visiting Leiden), De Sitter ;
Lorentz-Droste ‘17, Chazy ‘28, Levi-Civita ’37 ….,

Eddington’ 21, …, Lichnerowicz ‘39, Fock ‘39, Papapetrou ‘51,
… Dixon ‘64, Bailey-Israël ‘75, Ehlers-Rudolph ‘77….

Strongly Self-gravitating Bodies (NS, BH)

- Multi-chart approach and matched asymptotic expansions: necessary for strongly self-gravitating bodies (NS, BH) Manasse (Wheeler) ‘63, Demianski-Grishchuk ‘74, D’Eath ‘75, Kates ‘80, Damour ‘82

Useful even for weakly self-gravitating bodies, i.e.“relativistic celestial mechanics”, Brumberg-Kopeikin ’89, Damour-Soffel-Xu ‘91-94



Practical Techniques for Computing the Motion of Compact Bodies (NS or BH)

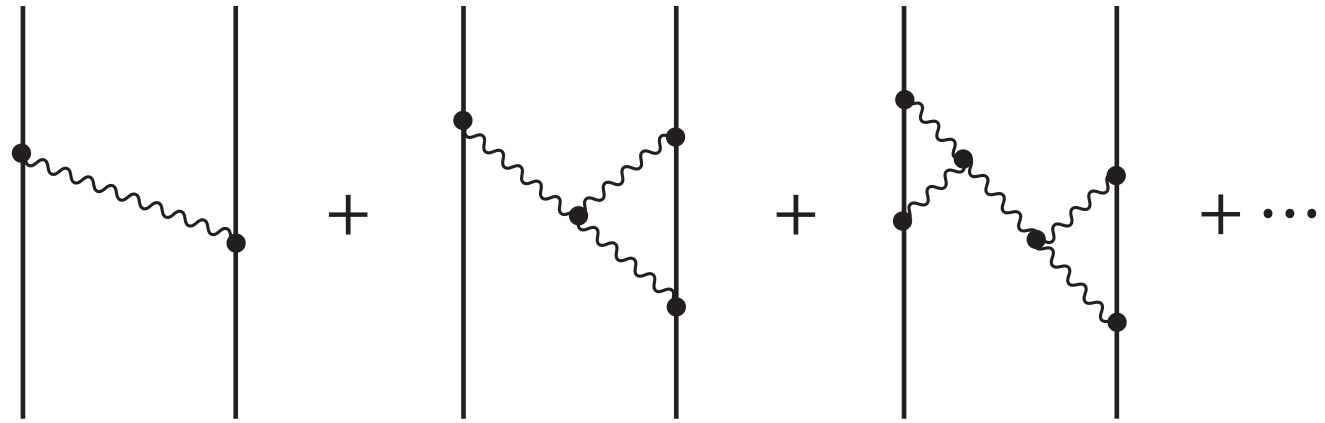
Skeletonization : $T_{\mu\nu} \longrightarrow$ point-masses (Mathisson '31)

delta-functions in GR : Infeld '54, Infeld-Plebanski '60

justified by Matched Asymptotic Expansions (« Effacing Principle » Damour '83)

QFT's analytic (Riesz '49) or dimensional regularization (Bollini-Giambiagi '72, t'Hooft-Veltman '72) imported in GR (Damour '80, Damour-Jaranowski-Schäfer '01, …)

Feynman-like diagrams and « Effective Field Theory » techniques



Bertotti-Plebanski '60,

Damour-Esposito-Farèse '96,

Goldberger-Rothstein '06, Porto '06, Gilmore-Ross' 08, Levi '10,

Foffa-Sturani '11 '13, Levi-Steinhoff '14, '15, Foffa-Mastrolia-Sturani-Sturm'16,

Damour-Jaranowski '17

Reduced (Fokker 1929) Action for Conservative Dynamics

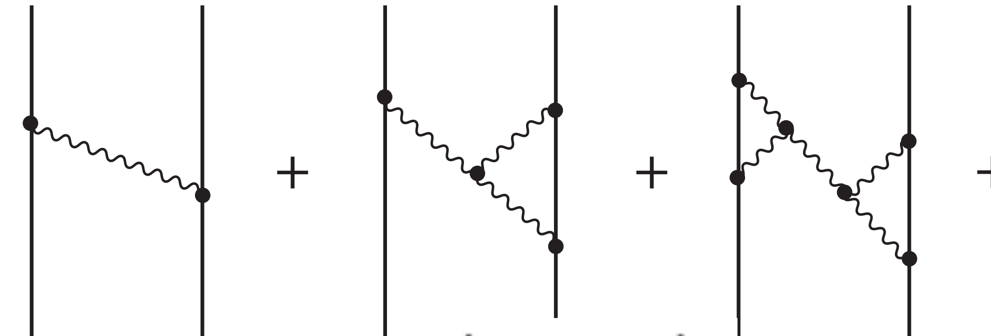
Needs gauge-fixed* action and time-symmetric Green function G.
*E.g. Arnowitt-Deser-Misner Hamiltonian formalism or harmonic coordinates.
Perturbatively solving (in dimension D=4 - eps) Einstein's equations
to get the equations of motion and the action for the conservative dynamics

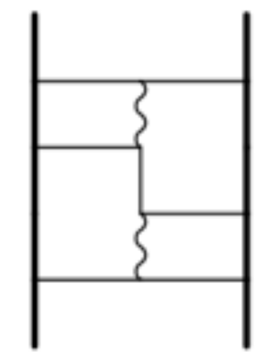
$$g = \eta + h$$

$$S(h,T) = \int \left(\frac{1}{2} h \Box h + \partial\partial hhh + ... + (h + hh + ...)T \right)$$

$$\Box h = -T + ... \rightarrow h = G\,T + ...$$

$$S_{\rm red}(T) = \frac{1}{2} T\,G\,T + V_3(G\,T, G\,T, G\,T) + ...$$

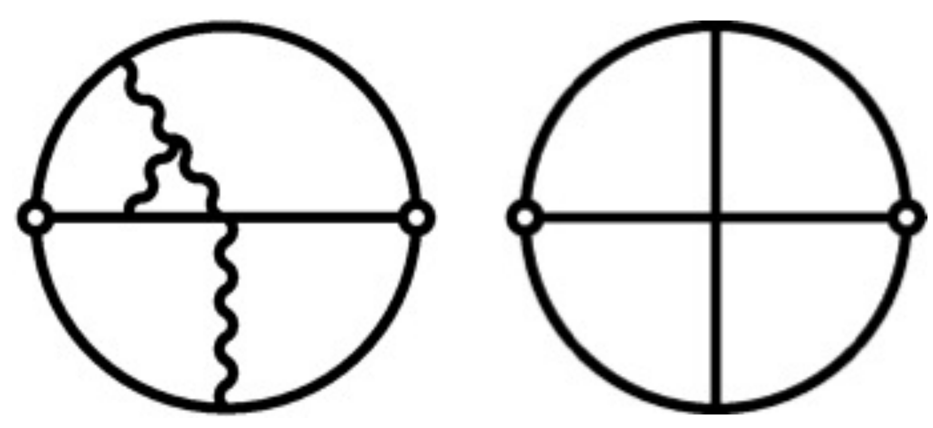




Beyond 1-loop order needs to use PN-expanded Green function for explicit computations.
Introduces IR divergences on top of the UV divergences linked to the point-particle description.
UV is (essentially) finite in dim.reg. and IR is linked to 4PN non-locality (Blanchet-Damour '88).

$$\Box^{-1} = (\Delta - \frac{1}{c^2}\partial_t^2)^{-1} = \Delta^{-1} + \frac{1}{c^2}\partial_t^2 \Delta^{-2} + ...$$

Recently (Damour-Jaranowski '17) found errors
in the EFT computation (by Foffa-Mastrolia-Sturani-Sturm'16)
of some of the static 4-loop contributions,and found a way of
analytically computing a 2-point 4-loop master integral previously
only numerically computed (Lee-Mingulov '15)



Post-Newtonian Equations of Motion [2-body, wo spins]

- 1PN (including v^2/c^2) [Lorentz-Droste ’17], Einstein-Infeld-Hoffmann ’38
- 2PN (inc. v^4/c^4) Ohta-Okamura-Kimura-Hiida ‘74, Damour-Deruelle ’81
  Damour ’82, Schäfer ’85, Kopeikin ‘85
- 2.5 PN (inc. v^5/c^5) Damour-Deruelle ‘81, Damour ‘82, Schäfer ’85,
  Kopeikin ‘85
- 3 PN (inc. v^6/c^6) Jaranowski-Schäfer ‘98, Blanchet-Faye ‘00,
  Damour-Jaranowski-Schäfer ‘01, Itoh-Futamase ‘03,
  Blanchet-Damour-Esposito-Farèse’ 04, Foffa-Sturani ‘11
- 3.5 PN (inc. v^7/c^7) Iyer-Will ’93, Jaranowski-Schäfer ‘97, Pati-Will ‘02,
  Königsdörffer-Faye-Schäfer ‘03, Nissanke-Blanchet ‘05, Itoh ‘09
- 4PN (inc. v^8/c^8) Jaranowski-Schäfer ’13, Foffa-Sturani ’13,’16
  Bini-Damour ’13, Damour-Jaranowski-Schäfer ’14, Bernard et al’16

New feature : non-locality in time

2-body Taylor-expanded N + 1PN + 2PN Hamiltonian

$$H_{\rm N}(\mathbf{x}_a, \mathbf{p}_a) = \frac{\mathbf{p}_1^2}{2m_1} - \frac{1}{2}\frac{Gm_1m_2}{r_{12}} + (1 \leftrightarrow 2)$$

$$\begin{aligned} c^2 H_{\rm 1PN}(\mathbf{x}_a, \mathbf{p}_a) = &-\frac{1}{8}\frac{(\mathbf{p}_1^2)^2}{m_1^3} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(-12\frac{\mathbf{p}_1^2}{m_1^2} + 14\frac{(\mathbf{p}_1 \cdot \mathbf{p}_2)}{m_1m_2} + 2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right) \\ &+ \frac{1}{4}\frac{Gm_1m_2}{r_{12}}\frac{G(m_1+m_2)}{r_{12}} + (1 \leftrightarrow 2), \end{aligned}$$

$$\begin{aligned} c^4 H_{\rm 2PN}(\mathbf{x}_a, \mathbf{p}_a) = &\frac{1}{16}\frac{(\mathbf{p}_1^2)^3}{m_1^5} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(5\frac{(\mathbf{p}_1^2)^2}{m_1^4} - \frac{11}{2}\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2} - \frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2m_2^2} + 5\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right. \\ &\left. - 6\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2} - \frac{3}{2}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right) \\ &+ \frac{1}{4}\frac{G^2m_1m_2}{r_{12}^2}\left(m_2\left(10\frac{\mathbf{p}_1^2}{m_1^2} + 19\frac{\mathbf{p}_2^2}{m_2^2}\right) - \frac{1}{2}(m_1+m_2)\frac{27(\mathbf{p}_1\cdot\mathbf{p}_2) + 6(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right) \\ &- \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\frac{G^2(m_1^2 + 5m_1m_2 + m_2^2)}{r_{12}^2} + (1 \leftrightarrow 2), \end{aligned}$$

2-body Taylor-expanded 3PN Hamiltonian [JS 98, DJS 01]

$$
\begin{aligned}
c^6 H_{3\mathrm{PN}}(\mathbf{x}_a,\mathbf{p}_a) = & -\frac{5}{128}\frac{(\mathbf{p}_1^2)^4}{m_1^7} + \frac{1}{32}\frac{G m_1 m_2}{r_{12}}\left(-14\frac{(\mathbf{p}_1^2)^3}{m_1^6} + 4\frac{((\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 4\mathbf{p}_1^2\mathbf{p}_2^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 6\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^4 m_2^2}\right.\\
& - 10\frac{(\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2 + \mathbf{p}_2^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 24\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^4 m_2^2}\\
& + 2\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3} + \frac{(7\mathbf{p}_1^2\mathbf{p}_2^2 - 10(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2^3}\\
& + \frac{(\mathbf{p}_1^2\mathbf{p}_2^2 - 2(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2^3} + 15\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3}\\
& \left. - 18\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3} + 5\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3}\right) + \frac{G^2 m_1 m_2}{r_{12}^2}\left(\frac{1}{16}(m_1 - 27m_2)\frac{(\mathbf{p}_1^2)^2}{m_1^4}\right.\\
& - \frac{115}{16}m_1\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2} + \frac{1}{48}m_2\frac{25(\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 371\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2} + \frac{17}{16}\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^3} + \frac{5}{12}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^3}\\
& - \frac{1}{8}m_1\frac{(15\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2) + 11(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1))(\mathbf{n}_{12}\cdot\mathbf{p}_1)}{m_1^3 m_2} - \frac{3}{2}m_1\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2}\\
& + \frac{125}{12}m_2\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2 m_2^2} + \frac{10}{3}m_2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2}\\
& \left. - \frac{1}{48}(220m_1 + 193m_2)\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2}\right) + \frac{G^3 m_1 m_2}{r_{12}^3}\left(-\frac{1}{48}\left(425m_1^2 + \left(473 - \frac{3}{4}\pi^2\right)m_1 m_2 + 150m_2^2\right)\frac{\mathbf{p}_1^2}{m_1^2}\right.\\
& + \frac{1}{16}\left(77(m_1^2 + m_2^2) + \left(143 - \frac{1}{4}\pi^2\right)m_1 m_2\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1 m_2} + \frac{1}{16}\left(20m_1^2 - \left(43 + \frac{3}{4}\pi^2\right)m_1 m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2}\\
& \left. + \frac{1}{16}\left(21(m_1^2 + m_2^2) + \left(119 + \frac{3}{4}\pi^2\right)m_1 m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1 m_2}\right)\\
& + \frac{1}{8}\frac{G^4 m_1 m_2^3}{r_{12}^4}\left(\left(\frac{227}{3} - \frac{21}{4}\pi^2\right)m_1 + m_2\right) + (1 \leftrightarrow 2).
\end{aligned}
$$

2-body Taylor-expanded 4PN Hamiltonian [DJS, 2014]

$$
\begin{aligned}
c^8 H_{4\mathrm{PN}}^{\mathrm{local}}(\mathbf{x}_a,\mathbf{p}_a) ={}& \frac{7(\mathbf{p}_1^2)^5}{256 m_1^9} + \frac{G m_1 m_2}{r_{12}} H_{48}(\mathbf{x}_a,\mathbf{p}_a) + \frac{G^2 m_1 m_2}{r_{12}^2} m_1 H_{46}(\mathbf{x}_a,\mathbf{p}_a) \\
&+ \frac{G^3 m_1 m_2}{r_{12}^3}\left(m_1^2 H_{441}(\mathbf{x}_a,\mathbf{p}_a) + m_1 m_2 H_{442}(\mathbf{x}_a,\mathbf{p}_a)\right) \\
&+ \frac{G^4 m_1 m_2}{r_{12}^4}\left(m_1^3 H_{421}(\mathbf{x}_a,\mathbf{p}_a) + m_1^2 m_2 H_{422}(\mathbf{x}_a,\mathbf{p}_a)\right) \\
&+ \frac{G^5 m_1 m_2}{r_{12}^5} H_{40}(\mathbf{x}_a,\mathbf{p}_a) + (1 \leftrightarrow 2).
\end{aligned}
\tag{A3}
$$

$$
\begin{aligned}
H_{48}(\mathbf{x}_a,\mathbf{p}_a) ={}& \frac{45(\mathbf{p}_1^2)^4}{128 m_1^8} - \frac{9(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{64 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^3}{64 m_1^6 m_2^2} - \frac{9(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{16 m_1^6 m_2^2} \\
&- \frac{3(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{32 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{21(\mathbf{p}_1^2)^3\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{35(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{256 m_1^5 m_2^3} \\
&+ \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{128 m_1^5 m_2^3} + \frac{33(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1^2)^2}{256 m_1^5 m_2^3} - \frac{85(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3} \\
&- \frac{45(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2^3} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3} + \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3} \\
&+ \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} + \frac{3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} + \frac{55(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} \\
&- \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{23(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} \\
&+ \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{64 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4(\mathbf{p}_1^2)^2}{64 m_1^4 m_2^4} \\
&- \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^4 m_2^4} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{64 m_1^4 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{64 m_1^4 m_2^4} \\
&- \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{16 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{16 m_1^4 m_2^4} \\
&- \frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_2^2)^2}{64 m_1^4 m_2^4} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{32 m_1^4 m_2^4} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_2^2)^2}{128 m_1^4 m_2^4}.
\end{aligned}
\tag{A4a}
$$

$$
\begin{aligned}
H_{46}(\mathbf{x}_a,\mathbf{p}_a) ={}& \frac{369(\mathbf{n}_{12}\cdot\mathbf{p}_1)^6}{160 m_1^6} - \frac{889(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_1^2}{192 m_1^6} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2}{16 m_1^6} - \frac{63(\mathbf{p}_1^2)^3}{64 m_1^6} - \frac{549(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{128 m_1^5 m_2} \\
&+ \frac{67(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{16 m_1^5 m_2} - \frac{167(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2}{128 m_1^5 m_2} + \frac{1547(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} - \frac{851(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2} \\
&+ \frac{1099(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} + \frac{3263(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2} + \frac{1067(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{480 m_1^4 m_2^2} - \frac{4567(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{3840 m_1^4 m_2^2} \\
&- \frac{3571(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{320 m_1^4 m_2^2} + \frac{3073(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{480 m_1^4 m_2^2} + \frac{4349(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2} \\
&- \frac{3461\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{3840 m_1^4 m_2^2} + \frac{1673(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_2^2}{1920 m_1^4 m_2^2} - \frac{1999(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} + \frac{2081(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} - \frac{13(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{8 m_1^3 m_2^3} \\
&+ \frac{191(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{192 m_1^3 m_2^3} - \frac{19(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3} \\
&+ \frac{11(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{192 m_1^3 m_2^3} + \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{96 m_1^3 m_2^3} + \frac{233(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{96 m_1^3 m_2^3} - \frac{47(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{32 m_1^3 m_2^3} \\
&+ \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{185\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4}{4 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{4 m_1^2 m_2^4} \\
&- \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1\cdot\mathbf{p}_2)}{2 m_1^2 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{6 m_1^2 m_2^4} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{48 m_1^2 m_2^4} \\
&- \frac{133(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{24 m_1^2 m_2^4} - \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{96 m_1^2 m_2^4} + \frac{197(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_2^2)^2}{96 m_1^2 m_2^4} - \frac{173\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{48 m_1^2 m_2^4} + \frac{13(\mathbf{p}_2^2)^3}{8 m_2^6}.
\end{aligned}
\tag{A4b}
$$

$$
\begin{aligned}
H_{441}(\mathbf{x}_a,\mathbf{p}_a) ={}& \frac{5027(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{384 m_1^4} - \frac{22993(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{960 m_1^4} - \frac{6695(\mathbf{p}_1^2)^2}{1152 m_1^4} - \frac{3191(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{640 m_1^3 m_2} \\
&+ \frac{28561(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{1920 m_1^3 m_2} + \frac{8777(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2} + \frac{752969\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{28800 m_1^3 m_2} \\
&- \frac{16481(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{960 m_1^2 m_2^2} + \frac{94433(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{4800 m_1^2 m_2^2} - \frac{103957(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{2400 m_1^2 m_2^2} \\
&+ \frac{791(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{400 m_1^2 m_2^2} + \frac{26627(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_2^2}{1600 m_1^2 m_2^2} - \frac{118261\mathbf{p}_1^2\mathbf{p}_2^2}{4800 m_1^2 m_2^2} + \frac{105(\mathbf{p}_2^2)^2}{32 m_2^4},
\end{aligned}
\tag{A4c}
$$

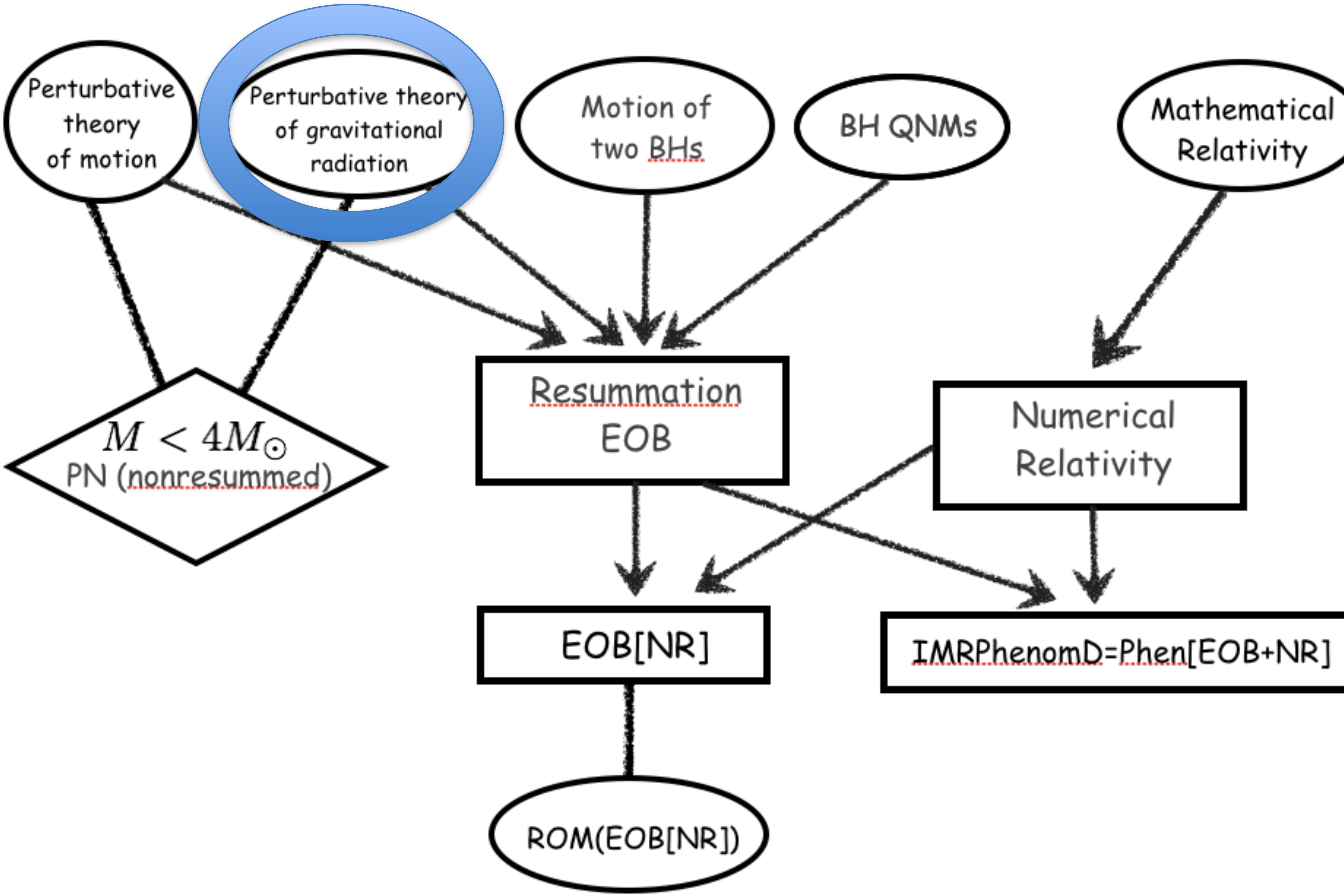
$$
\begin{aligned}
H_{442}(\mathbf{x}_a,\mathbf{p}_a) ={}& \left(\frac{2749\pi^2}{8192} - \frac{211189}{19200}\right)\frac{(\mathbf{p}_1^2)^2}{m_1^4} + \left(\frac{63347}{1600} - \frac{1059\pi^2}{1024}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{m_1^4} + \left(\frac{375\pi^2}{8192} - \frac{23533}{1280}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^4} \\
&+ \left(\frac{10631\pi^2}{8192} - \frac{1918349}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{13723\pi^2}{16384} - \frac{2492417}{57600}\right)\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2} \\
&+ \left(\frac{1411429}{19200} - \frac{1059\pi^2}{512}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{m_1^2 m_2^2} + \left(\frac{248991}{6400} - \frac{6153\pi^2}{2048}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^2 m_2^2} \\
&- \left(\frac{30383}{960} + \frac{36405\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{1243717}{14400} - \frac{40483\pi^2}{16384}\right)\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2} \\
&+ \left(\frac{2369}{60} + \frac{35655\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2} + \left(\frac{43101\pi^2}{16384} - \frac{391711}{6400}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{m_1^3 m_2} \\
&+ \left(\frac{56955\pi^2}{16384} - \frac{1646983}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2}.
\end{aligned}
\tag{A4d}
$$

$$
H_{421}(\mathbf{x}_a,\mathbf{p}_a) = \frac{64861\mathbf{p}_1^2}{4800 m_1^2} - \frac{91(\mathbf{p}_1\cdot\mathbf{p}_2)}{8 m_1 m_2} + \frac{105\mathbf{p}_2^2}{32 m_2^2} - \frac{9841(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{1600 m_1^2} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{2 m_1 m_2}.
\tag{A4e}
$$

$$
\begin{aligned}
H_{422}(\mathbf{x}_a,\mathbf{p}_a) ={}& \left(\frac{1937033}{57600} - \frac{199177\pi^2}{49152}\right)\frac{\mathbf{p}_1^2}{m_1^2} + \left(\frac{176033\pi^2}{24576} - \frac{2864917}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1 m_2} + \left(\frac{282361}{19200} - \frac{21837\pi^2}{8192}\right)\frac{\mathbf{p}_2^2}{m_2^2} \\
&+ \left(\frac{698723}{19200} + \frac{21745\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2} + \left(\frac{63641\pi^2}{24576} - \frac{2712013}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1 m_2} \\
&+ \left(\frac{3200179}{57600} - \frac{28691\pi^2}{24576}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_2^2}.
\end{aligned}
\tag{A4f}
$$

$$
H_{40}(\mathbf{x}_a,\mathbf{p}_a) = -\frac{m_1^4}{16} + \left(\frac{6237\pi^2}{1024} - \frac{169799}{2400}\right)m_1^3 m_2 + \left(\frac{44825\pi^2}{6144} - \frac{609427}{7200}\right)m_1^2 m_2^2.
\tag{A4g}
$$

$$
\begin{aligned}
H_{4\mathrm{PN}}^{\mathrm{nonloc}}(t) ={}& -\frac{1}{5}\frac{G^2 M}{c^8} I_{ij}^{(3)}(t) \\
&\times \mathrm{Pf}_{2r_{12}/c}\int_{-\infty}^{+\infty}\frac{\mathrm{d}v}{|v|} I_{ij}^{(3)}(t+v),
\end{aligned}
$$


Perturbative theory of motion
Perturbative theory of gravitational radiation
Motion of two BHs
BH QNMs
Mathematical Relativity
$M < 4M_\odot$
PN (nonresummed)
Resummation EOB
Numerical Relativity
EOB[NR]
IMRPhenomD=Phen[EOB+NR]
ROM(EOB[NR])

Perturbative Theory of the Generation of Gravitational Radiation

Einstein ’16, ’18 (+ Landau-Lifshitz 41, and Fock ’55) : h_+, h_x and quadrupole formula

Relativistic, multipolar extensions of LO quadrupole radiation :

Sachs-Bergmann ’58, Sachs ’61, Mathews ’62, Peters-Mathews ’63, Pirani '64

Bonnor-Rotenberg ’66,

Campbell-Morgan ’71,

Campbell et al ’75,

Epstein-Wagoner-Will ’75-76

Thorne ’80, .., Will et al 00

MPM Formalism:

Blanchet-Damour ’86,

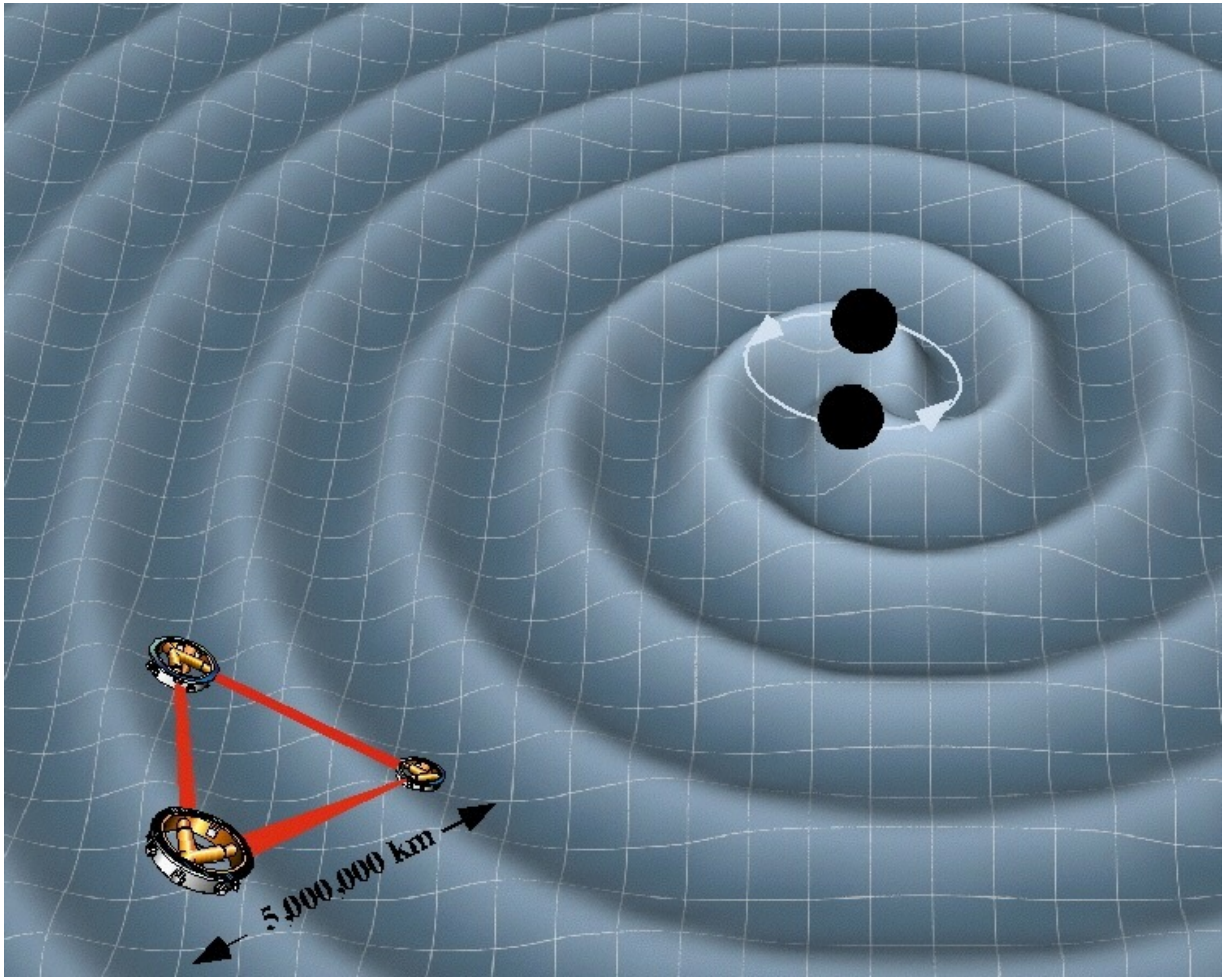
Damour-Iyer ’91,

Blanchet ’95 ‘98

Combines multipole exp.

Post Minkowkian exp.

and analytic continuation



MULTIPOLAR POST-MINKOWSKIAN FORMALISM

(BLANCHET-DAMOUR-IYER)

Decomposition of space-time in various overlapping regions:

1. near-zone: r << lambda : PN theory
2. exterior zone: r >> r_source: MPM expansion
3. far wave-zone: Bondi-type expansion

followed by matching between the zones

in exterior zone, iterative solution of Einstein's vacuum field equations by means of a double expansion in non-linearity and in multipoles, with crucial use of analytic continuation (complex B) for dealing with formal UV divergences at r=0

$$g = \eta + Gh_1 + G^2 h_2 + G^3 h_3 + ...,$$

$$\Box h_1 = 0,$$

$$\Box h_2 = \partial\partial h_1 h_1,$$

$$\Box h_3 = \partial\partial h_1 h_1 h_1 + \partial\partial h_1 h_2,$$

$$h_1 = \sum_\ell \partial_{i_1 i_2 \ldots i_\ell} \left(\frac{M_{i_1 i_2 \ldots i_\ell}(t - r/c)}{r} \right) + \partial\partial \ldots \partial \left(\frac{\epsilon_{j_1 j_2 k} S_{k j_3 \ldots j_\ell}(t - r/c)}{r} \right),$$

$$h_2 = FP_B \Box^{-1}_{\text{ret}} \left(\left(\frac{r}{r_0} \right)^B \partial\partial h_1 h_1 \right) + ...,$$

$$h_3 = FP_B \Box^{-1}_{\text{ret}} \ldots$$

Perturbative computation of GW flux from binary systems

- lowest order : Einstein 1918 Peters-Mathews 63
- 1 + (v^2/c^2) : Wagoner-Will 76
- … + (v^3/c^3) : Blanchet-Damour 92, Wiseman 93
- … + (v^4/c^4) : Blanchet-Damour-Iyer Will-Wiseman 95
- … + (v^5/c^5) : Blanchet 96
- … + (v^6/c^6) : Blanchet-Damour-Esposito-Farèse-Iyer 2004
- … + (v^7/c^7) : Blanchet

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$x = \left(\frac{v}{c}\right)^2 = \left(\frac{G(m_1+m_2)\Omega}{c^3}\right)^{\frac{2}{3}} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{2}{3}}$$

$$\begin{aligned}
\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \Big\{ & 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} \\
& + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} \\
& + \Big[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_E - \frac{856}{105}\ln(16\,x) \\
& \quad + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\Big] x^3 \\
& + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Big\}.
\end{aligned}$$

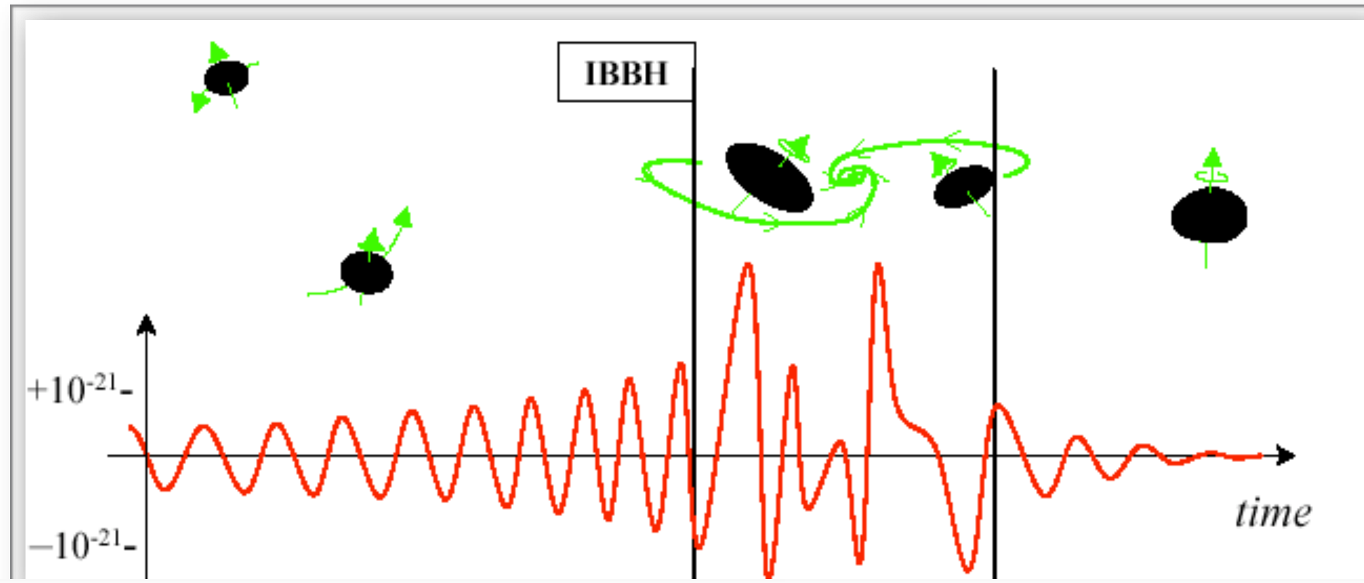
Analytical GW Templates for BBH Coalescences ?

PN corrections to Einstein's quadrupole frequency « chirping »
from PN-improved balance equation dE(f)/dt = - F(f)

$$\frac{d\phi}{d\ln f} = \frac{\omega^2}{d\omega/dt} = Q_\omega^N \widehat{Q}_\omega \qquad \frac{v}{c} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{1}{3}}$$

$$Q_\omega^N = \frac{5\,c^5}{48\,\nu\,v^5}\,;\; \widehat{Q}_\omega = 1 + c_2\left(\frac{v}{c}\right)^2 + c_3\left(\frac{v}{c}\right)^3 + \cdots \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

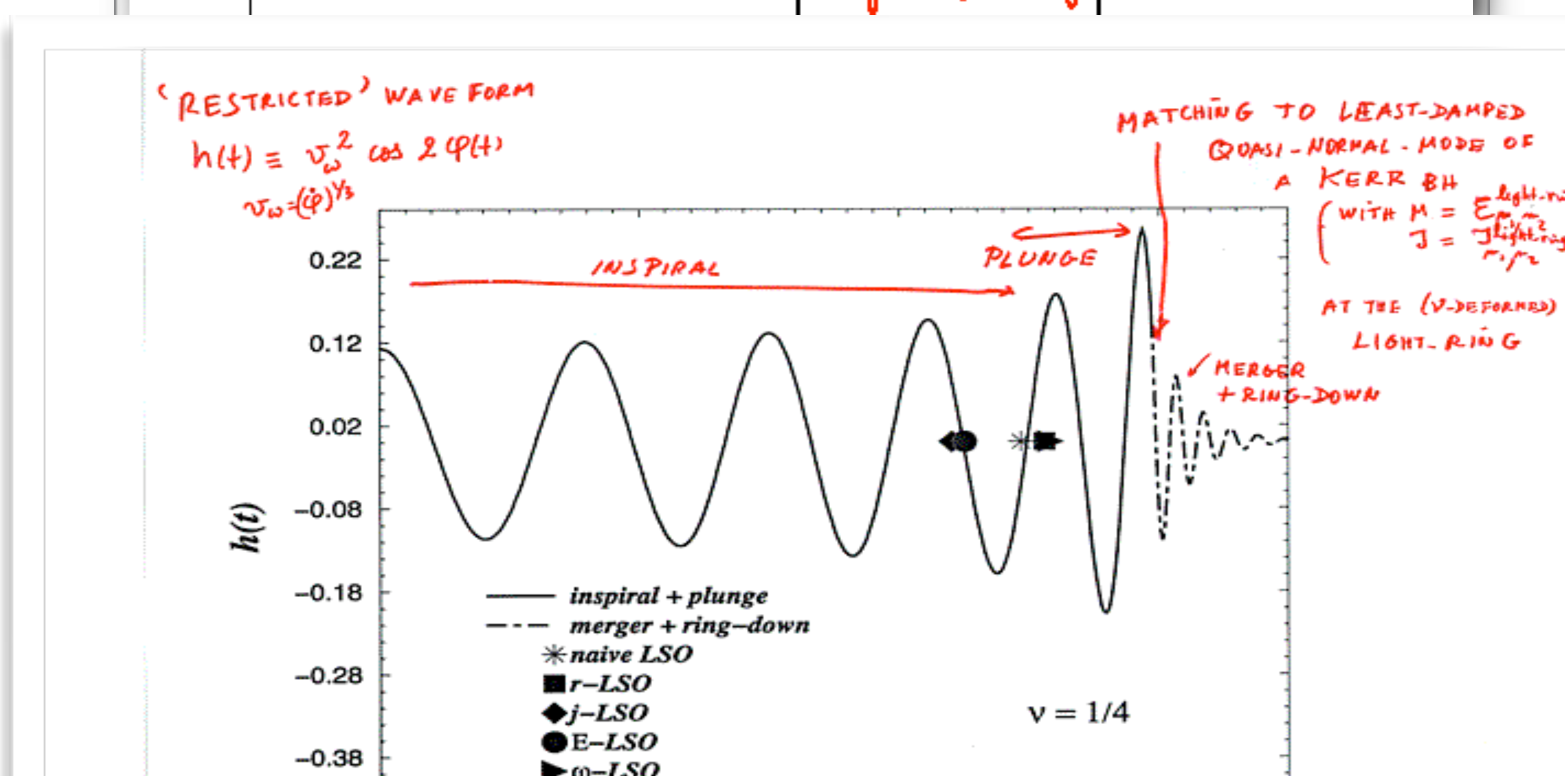
Cutler et al. '93:
« slow convergence of PN »

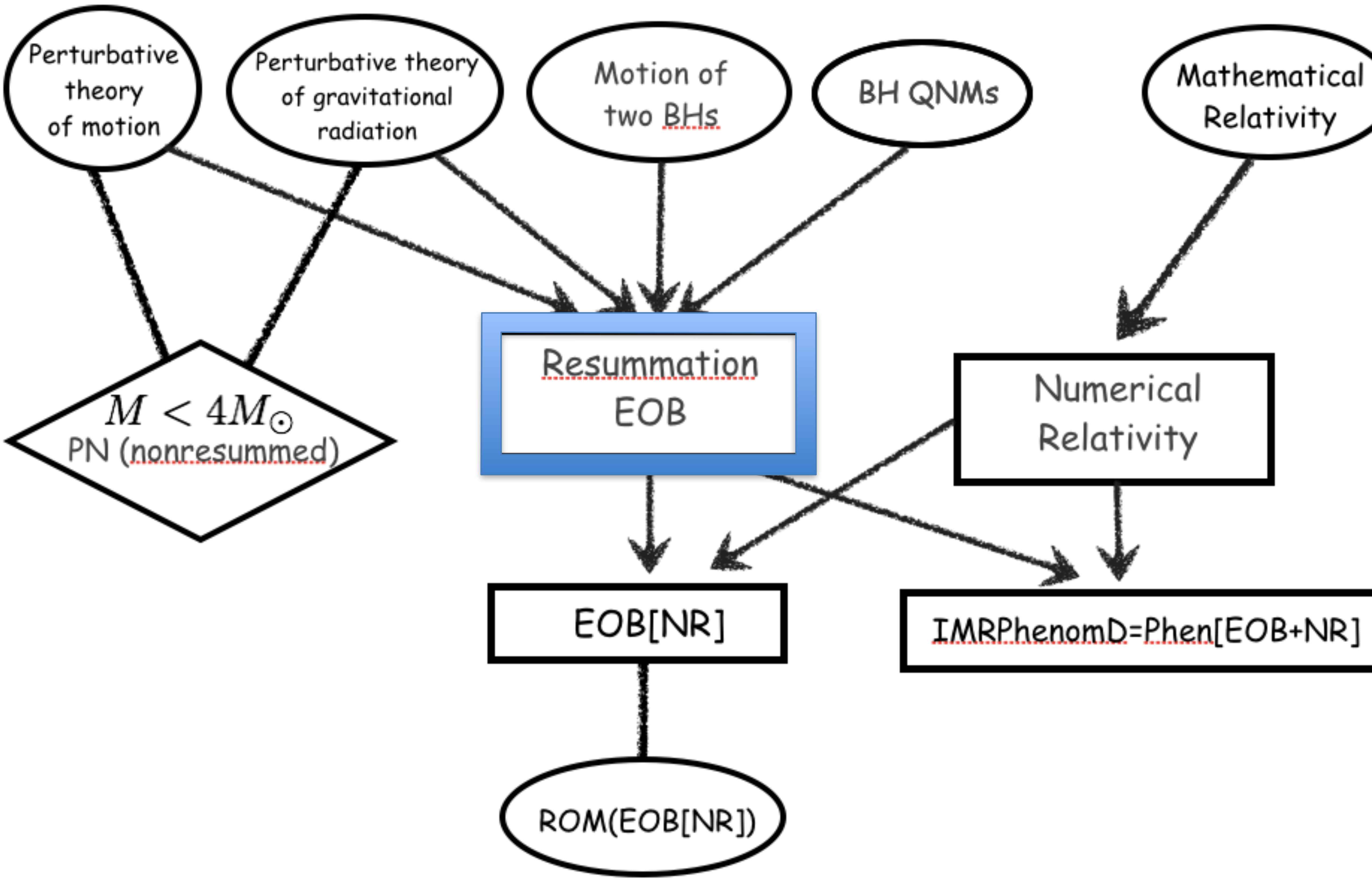


Brady-Creighton-Thorne'98:
« inability of current computational techniques to evolve a BBH through its last ~10 orbits of inspiral » and to compute the merger

Damour-Iyer-Sathyaprakash'98:
use resummation methods for E and F

Buonanno-Damour '99-00:
novel, resummed approach:
Effective-One-Body
analytical formalism



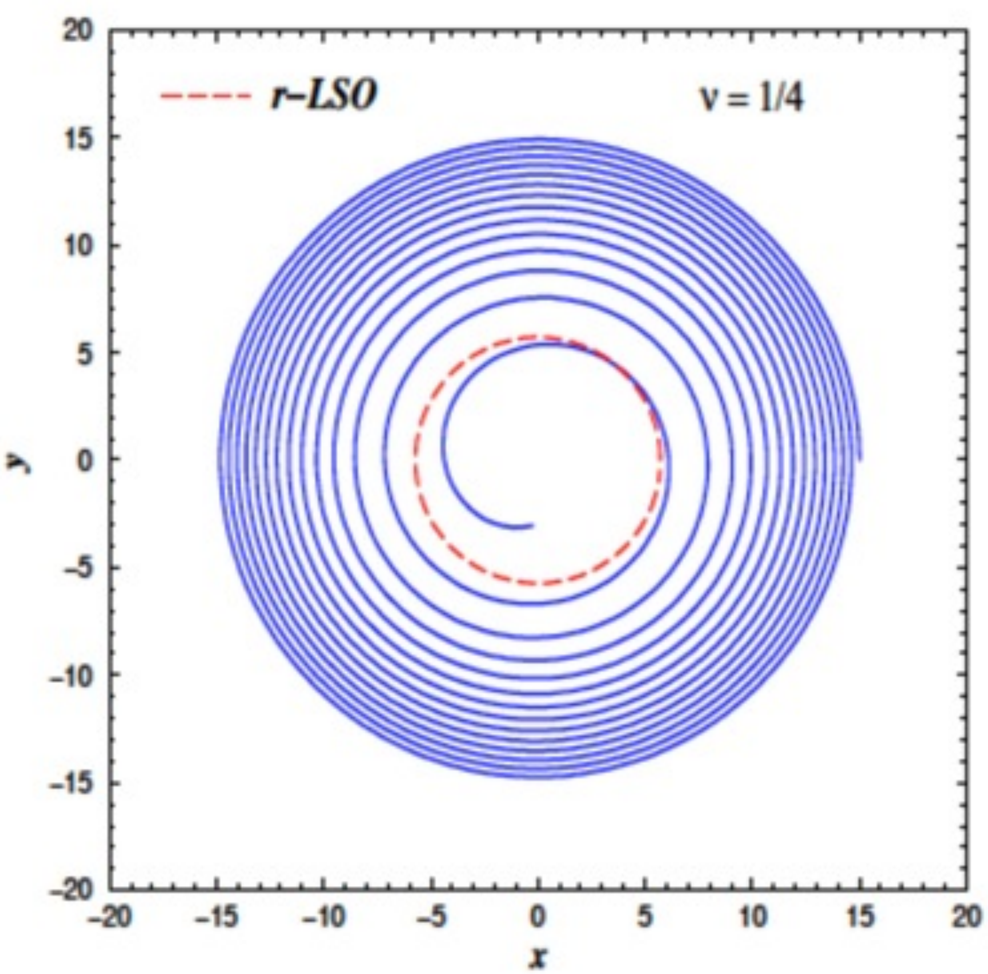

Perturbative theory of motion
Perturbative theory of gravitational radiation
Motion of two BHs
BH QNMs
Mathematical Relativity
$M < 4M_\odot$
PN (nonresummed)
Resummation EOB
Numerical Relativity
EOB[NR]
IMRPhenomD=Phen[EOB+NR]
ROM(EOB[NR])

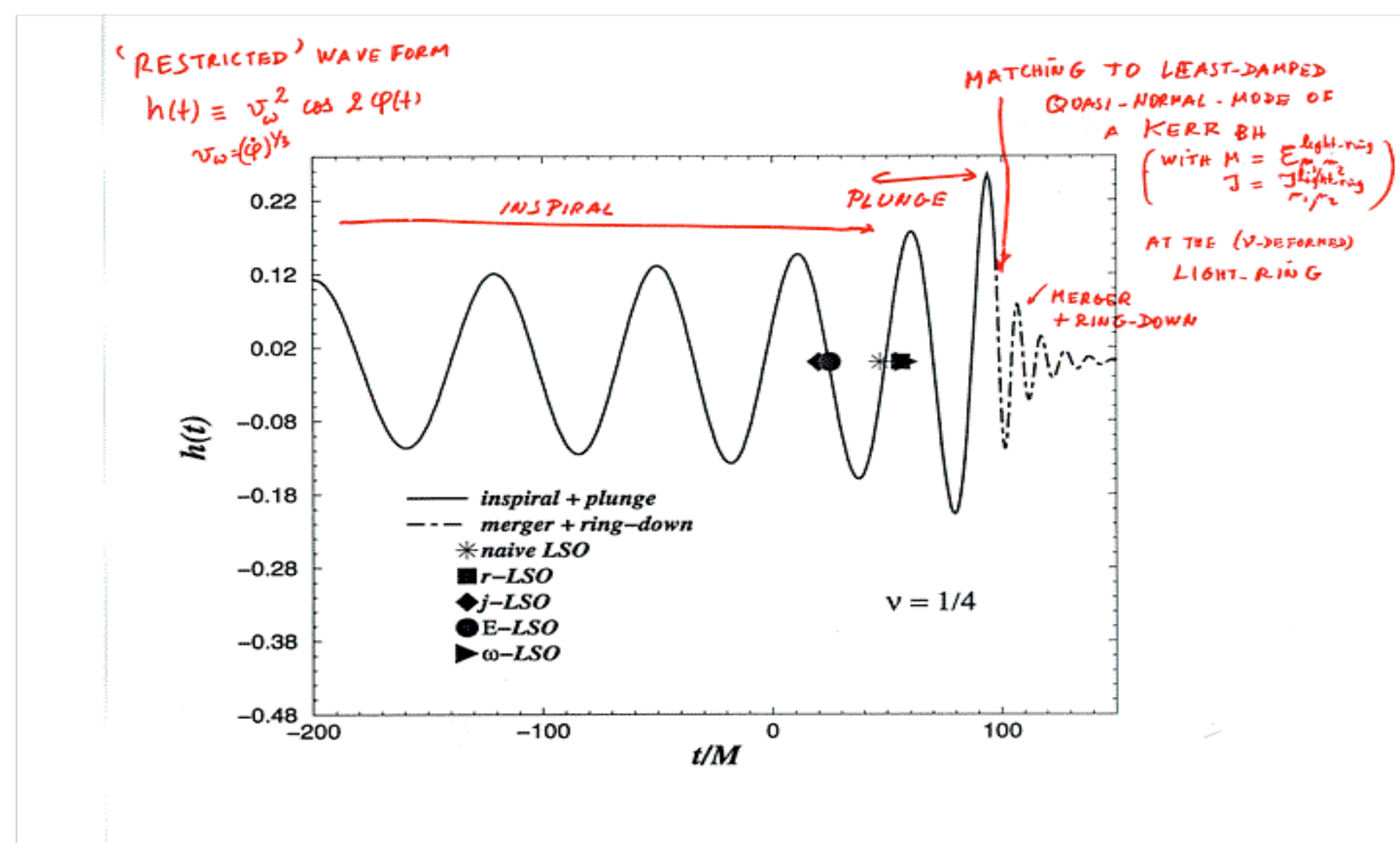
Effective One Body (EOB) Method)

Buonanno-Damour 1999, 2000; Damour-Jaranowski-Schaefer 2000; Damour 2001

Resummation of perturbative PN results → description of the coalescence
+ addition of ringdown (Vishveshwara 70, Davis-Ruffini-Tiomno 1972)

Buonanno-Damour 2000

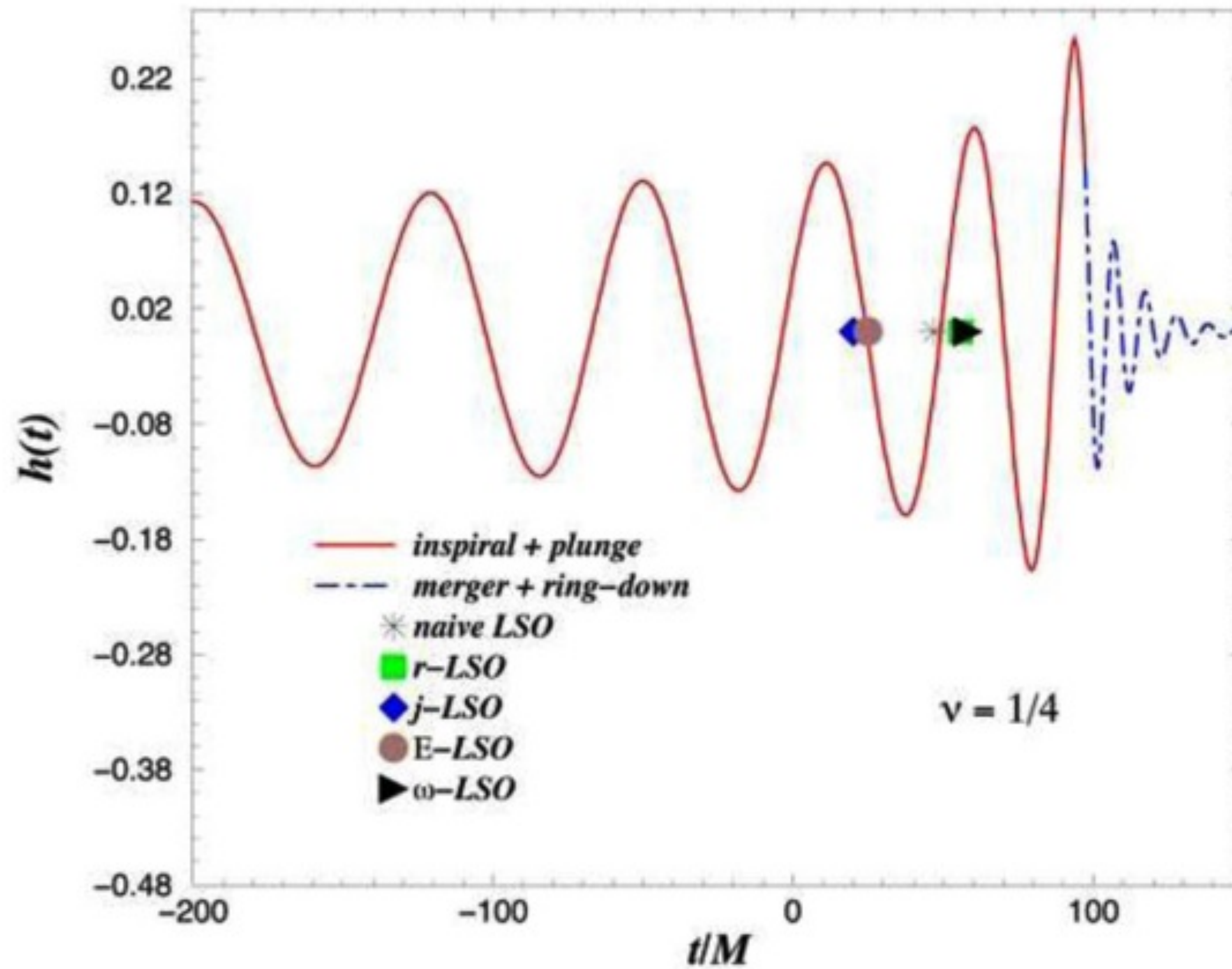




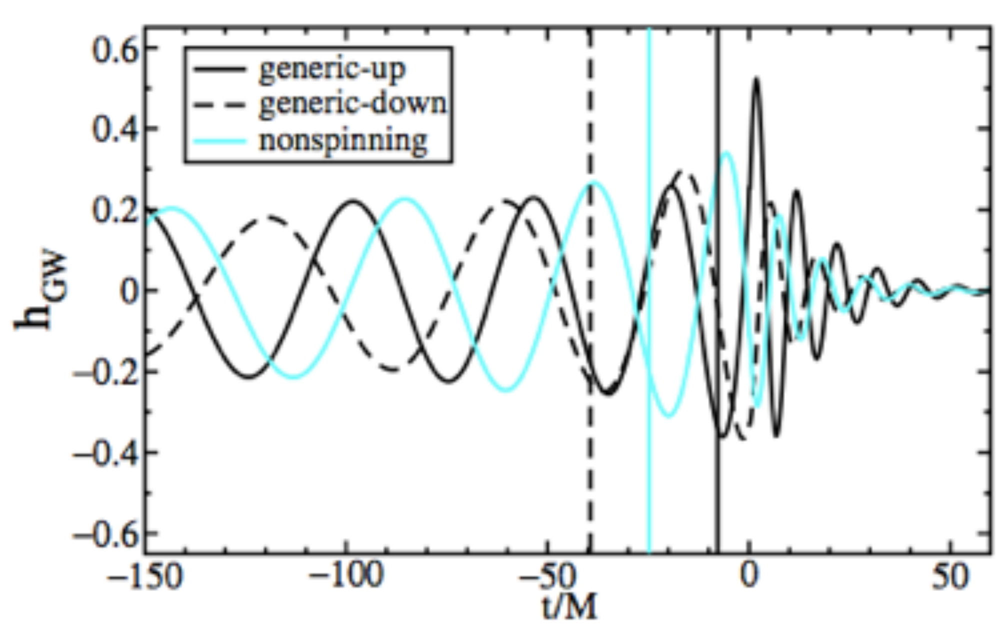
Predictions as early as 2000 :
continued transition, non adiabaticity, first complete waveform, final spin (OK within 10%), final mass

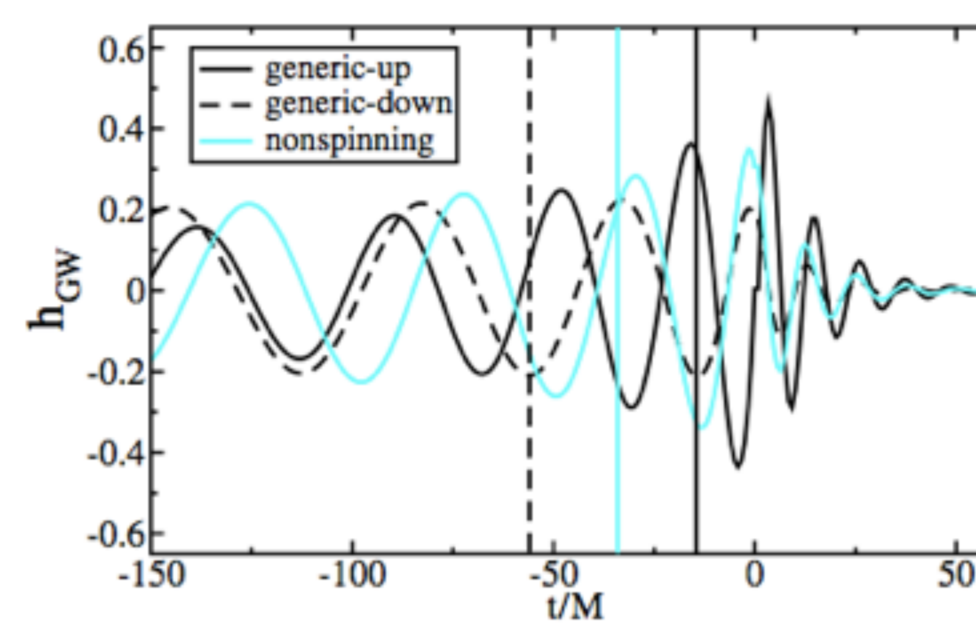
First complete waveforms for BBH coalescences: analytical EOB

Non-spinning BHs
Buonanno-Damour 2000



Spinning BHs
Buonanno-Chen-Damour
Nov 2005:
« to show the promise
of a purely analytical
EOB-based approach »





EOB THEORY + EOB[NR] + EOB[SF] DEVELOPMENTS

Buonanno,Damour 99 (2 PN Hamiltonian)

Buonanno,Damour 00 (Rad.Reac. full waveform)

Damour, Jaranowski,Schäfer 00 (3 PN Hamiltonian)

Damour 01,
Buonanno, Chen, Damour 05,
Damour-Jaranowski,Schäfer 08,
Barausse, Buonanno, 10,
Nagar 11,
Balmelli-Jetzer 12,
Taracchini et al 12,14,
Damour,Nagar 14
(spinning bodies)

Damour, Nagar 07,
Damour, Iyer, Nagar 08,
Pan et al. 11
(factorized waveform)

Damour, Nagar 10
Bini-Damour-Faye 12
(tidal effects)

Bini, Damour 13, Damour, Jaranowski, Schäfer 15 (4 PN Hamiltonian)

EOB vs NR and EOB[NR]
Buonanno, Cook, Pretorius 07,
Buonanno, Pan, Taracchini 08-
Damour-Nagar 08-

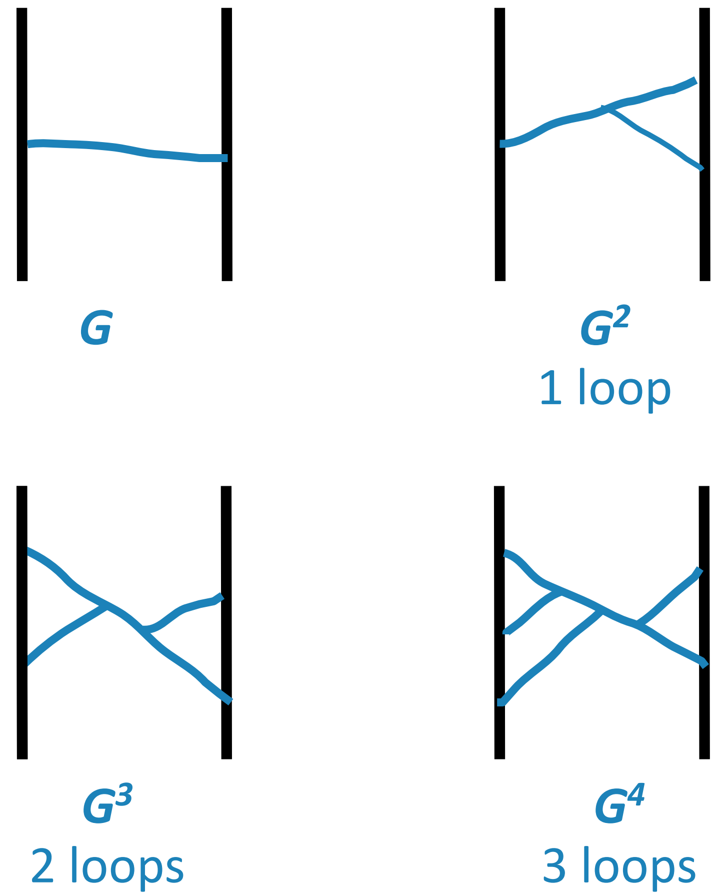
EOB vs SF and EOB[SF]
Damour 09
Barack-Sago-Damour 10
Barausse-Buonanno-LeTiec 12
Akcay-Barack-Damour-Sago 12
Bini-Damour 13-16
LeTiec 15
Bini-Damour-Geralico 16
Hopper-Kavanagh-Ottewill 16
Akcay-vandeMeent 16

Reduced Order Model version (Pürrer 2014, 2016) of EOB[NR] (Taracchini et al 2014)

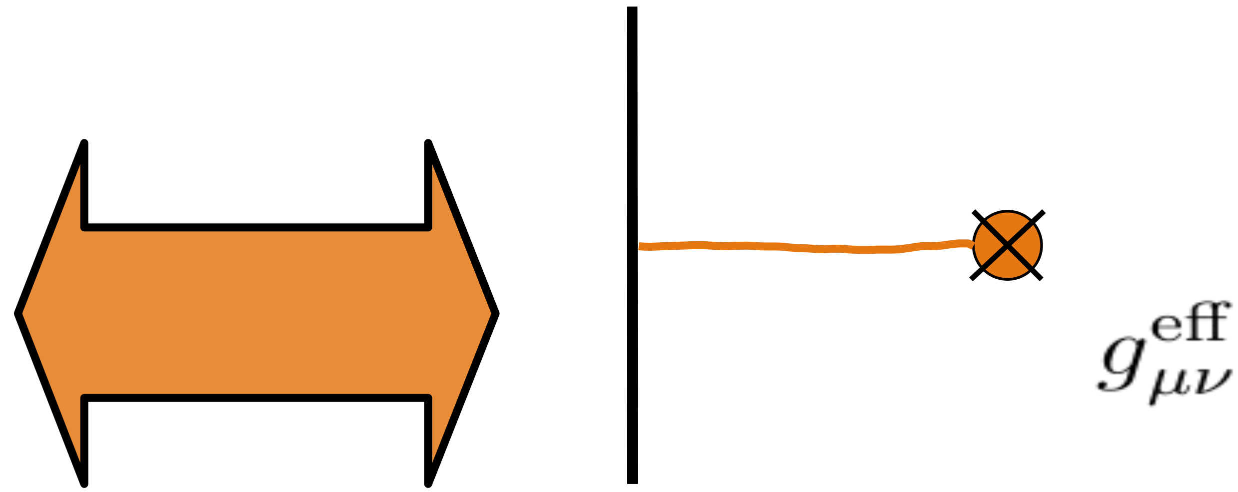
Phenomenological model (Ajith et al 2007, Hannam et al 2014, Husa et al 2016, Kahn et al 2016) of FFT of hybrids EOB + NR

Real dynamics versus Effective dynamics

Real dynamics



Effective dynamics



$$S = -\int \mu ds + \ldots$$

$$H = H_0 + \left(G H_1 + \frac{G^2}{c^2} H_2 + \frac{G^3}{c^4} H_3 + \frac{G^4}{c^6} H_4 \right) \left(1 + \frac{1}{c^2} + \ldots \right)$$

Effective metric for non-spinning bodies: a nu-deformation of Schwarzschild

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$ds^2_{\text{eff}} = -A(r;\nu)\, dt^2 + B(r;\nu)\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2 \right)$$

TWO-BODY/EOB "CORRESPONDENCE":

THINK QUANTUM-MECHANICALLY (J.A. WHEELER)

Real 2-body system
(in the c.o.m. frame)
(m_1, m_2)

1:1 map

An effective particle
in some effective metric

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$g_{\mu\nu}^{\rm eff}$$

$$\mathcal{E} = f(E)$$

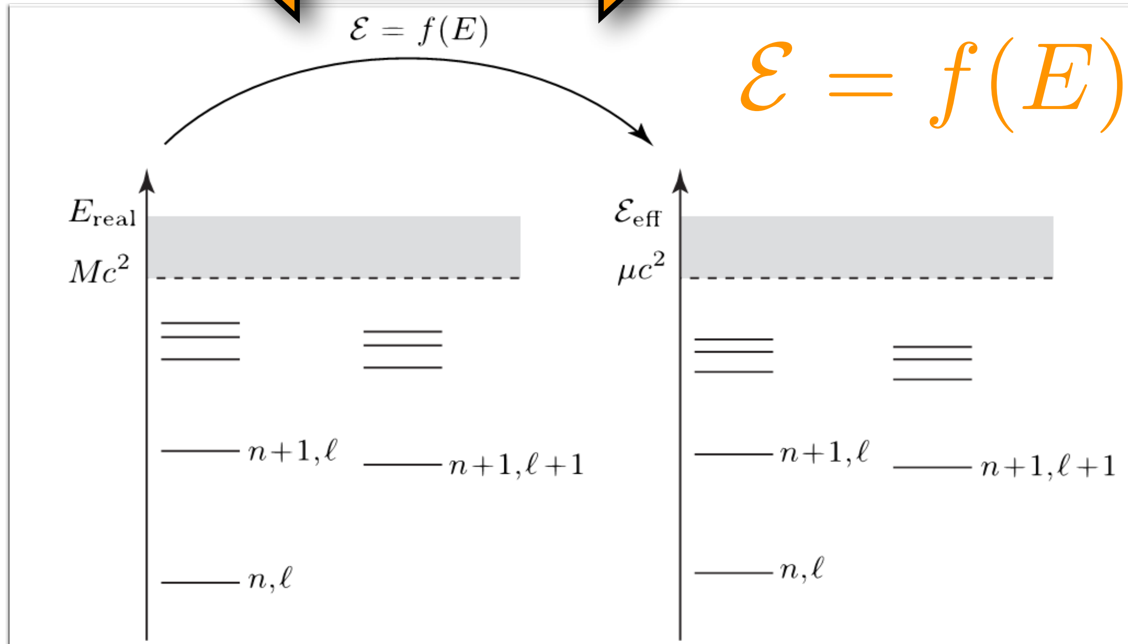


Figure 1: Sketch of the correspondence between the quantized energy levels of the real and effective conservative dynamics. n denotes the 'principal quantum

$$\mu^2 + g_{\rm eff}^{\mu\nu} \frac{\partial S_{\rm eff}}{\partial x^\mu} \frac{\partial S_{\rm eff}}{\partial x^\nu} + \mathcal{O}(p^4) = 0$$

Bohr-Sommerfeld's Quantization Conditions
(action-angle variables & Delaunay Hamiltonian)

$$J = \ell\hbar = \frac{1}{2\pi} \oint p_\varphi d\varphi$$

$$N = n\hbar = I_r + J$$

$$I_r = \frac{1}{2\pi} \oint p_r dr$$

$$H^{\rm classical}(q,p) \rightarrow H^{\rm classical}(I_a) \rightarrow \boxed{E^{\rm quantum}(I_a = n_a h) = f^{-1}[\mathcal{E}_{\rm eff}^{\rm quantum}(I_a^{\rm eff} = n_a h)]}$$

2-body CoM Newton + 1PN + 2PN + 3PN Hamiltonian

$$
\begin{aligned}
\widehat{H}_{\rm N}(\mathbf{r},\mathbf{p}) &= \frac{\mathbf{p}^2}{2} - \frac{1}{r},\\
\widehat{H}_{\rm 1PN}(\mathbf{r},\mathbf{p}) &= \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\Big\{(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r} + \frac{1}{2r^2},\\
\widehat{H}_{\rm 2PN}(\mathbf{r},\mathbf{p}) &= \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\Big\{\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\Big\}\frac{1}{r}\\
&\quad + \frac{1}{2}\Big\{(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r^2} - \frac{1}{4}(1+3\nu)\frac{1}{r^3},\\
\widehat{H}_{\rm 3PN}(\mathbf{r},\mathbf{p}) &= \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4\\
&\quad + \frac{1}{16}\Big\{\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3 + (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\Big\}\frac{1}{r}\\
&\quad + \left\{\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\right\}\frac{1}{r^2}\\
&\quad + \left\{\left(-\frac{25}{8} + \left(\frac{\pi^2}{64} - \frac{335}{48}\right)\nu - \frac{23\nu^2}{8}\right)\mathbf{p}^2 + \left(-\frac{85}{16} - \frac{3\pi^2}{64} - \frac{7\nu}{4}\right)\nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{r^3}\\
&\quad + \left\{\frac{1}{8} + \left(\frac{109}{12} - \frac{21}{32}\pi^2\right)\nu\right\}\frac{1}{r^4}.
\end{aligned}
$$

Resummed (non-spinning) EOB Hamiltonian

$$ds^2_{\text{eff}} = -A(r;\nu)\, dt^2 + B(r;\nu)\, dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$H_{\text{EOB}} = M\sqrt{1 + 2\nu\left(\frac{1}{\mu}\sqrt{A(r)\left(\mu^2 + \frac{p_r^2}{B(r)} + \frac{p_\phi^2}{r^2} + 2\nu(4-3\nu)\left(\frac{GM}{r}\right)^2\frac{p_r^4}{\mu^2}\right)} - 1\right)}$$

$$A^{3PN}(r; M, \nu) = \text{Pade}^1_3\left[1 - 2\frac{GM}{c^2 r} + 2\nu\left(\frac{GM}{c^2 r}\right)^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu\left(\frac{GM}{c^2 r}\right)^4\right]$$

$$M = m_1 + m_2\,, \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}\,, \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{\mu}{M}$$

Resummed EOB waveform

(Damour-Iyer-Sathyaprakash 1998) Damour-Nagar 2007, Damour-Iyer -Nagar 2008

$$h_{\ell m} \equiv h_{\ell m}^{(N,\epsilon)} \hat{h}_{\ell m}^{(\epsilon)} \hat{h}_{\ell m}^{\rm NQC}$$

$$\hat{h}_{\ell m}^{(\epsilon)} = \hat{S}_{\rm eff}^{(\epsilon)} T_{\ell m} e^{i\delta_{\ell m}} \rho_{\ell m}^{\ell}$$

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell+1)} e^{\pi\hat{\hat{k}}} e^{2i\hat{\hat{k}}\ln(2kr_0)}$$

$$\rho_{22}(x;\nu) = 1 + \left(\frac{55\nu}{84} - \frac{43}{42}\right)x + \left(\frac{19583\nu^2}{42336} - \frac{33025\nu}{21168} - \frac{20555}{10584}\right)x^2$$
$$+ \left(\frac{10620745\nu^3}{39118464} - \frac{6292061\nu^2}{3259872} + \frac{41\pi^2\nu}{192} - \frac{48993925\nu}{9779616} - \frac{428}{105}\mathrm{eulerlog}_2(x) + \frac{1556919113}{122245200}\right)x^3$$
$$+ \left(\frac{9202}{2205}\mathrm{eulerlog}_2(x) - \frac{387216563023}{160190110080}\right)x^4 + \left(\frac{439877}{55566}\mathrm{eulerlog}_2(x) - \frac{16094530514677}{533967033600}\right)x^5 + \mathcal{O}(x^6),$$

$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega}\sum_{\ell=2}^{\ell_{\max}}\sum_{m=1}^{\ell}(m\Omega)^2|Rh_{\ell m}^{(\epsilon)}|^2$$

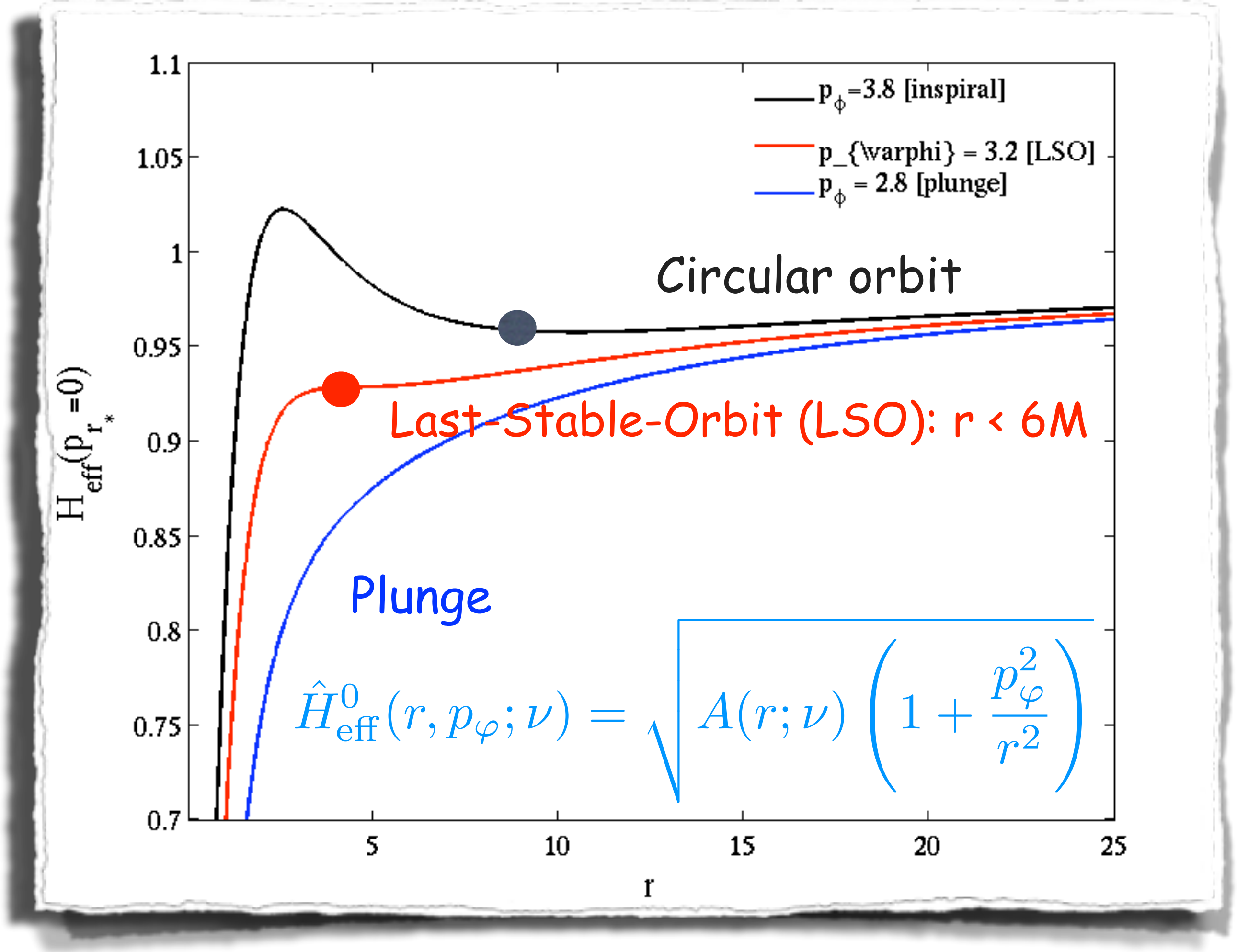
HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi$$

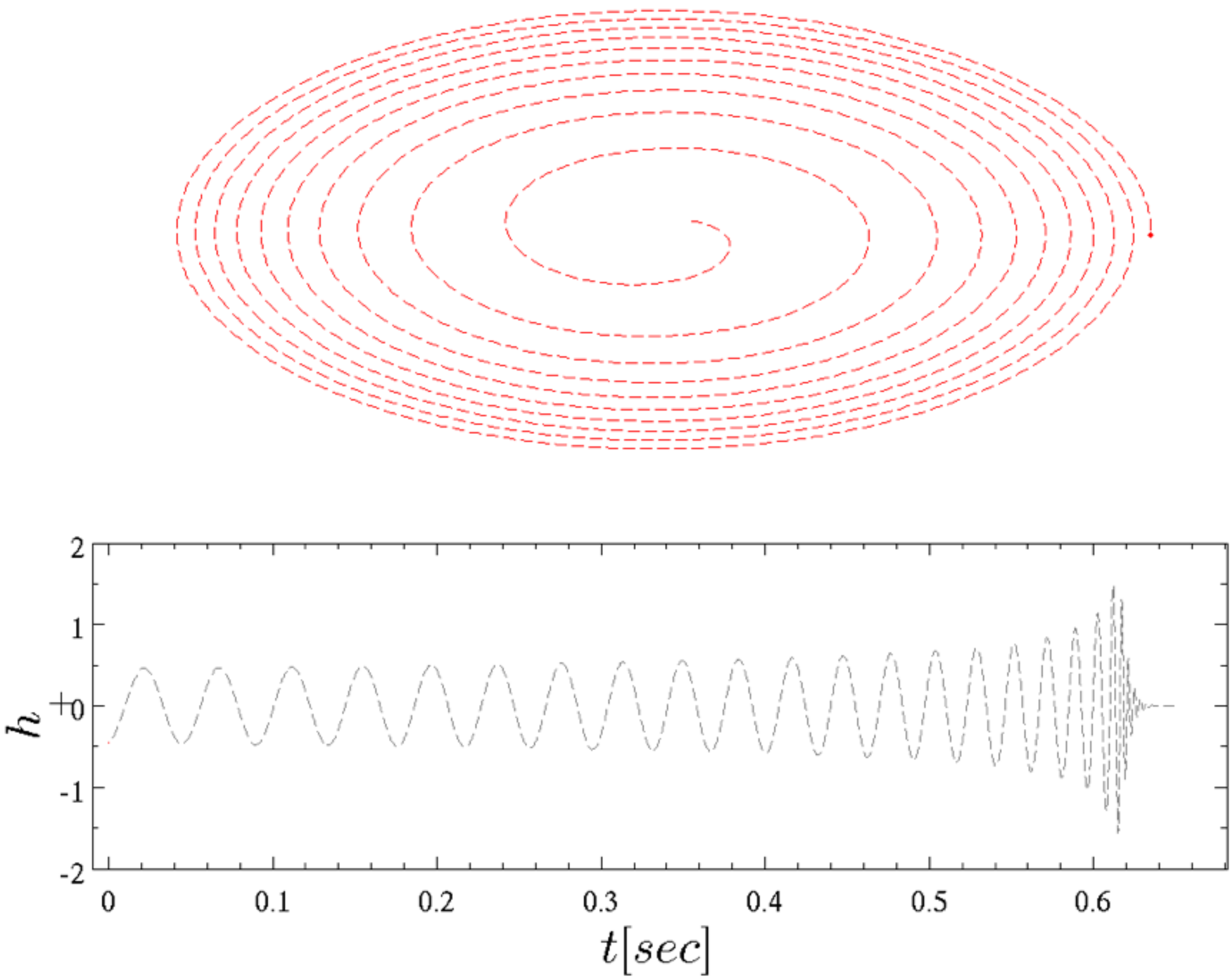


- The system must radiate angular momentum
- How?Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

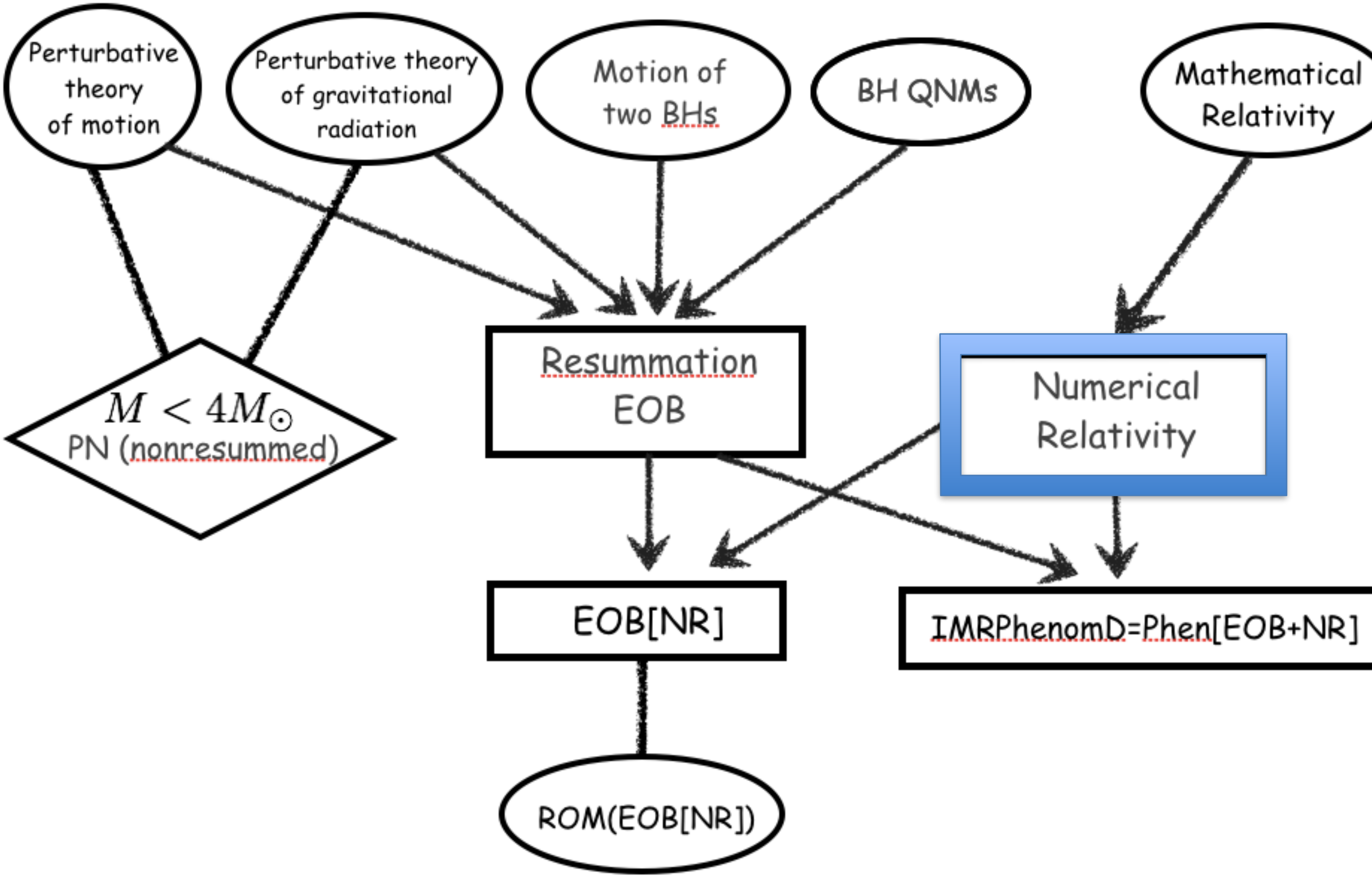
$$\hat{\mathcal{F}}_\varphi^{\rm Taylor} = -\frac{32}{5}\nu\Omega^5 r_\Omega^4 \hat{F}^{\rm Taylor}(v_\varphi)$$

Plus horizon contribution [Nagar&Akcay2012]

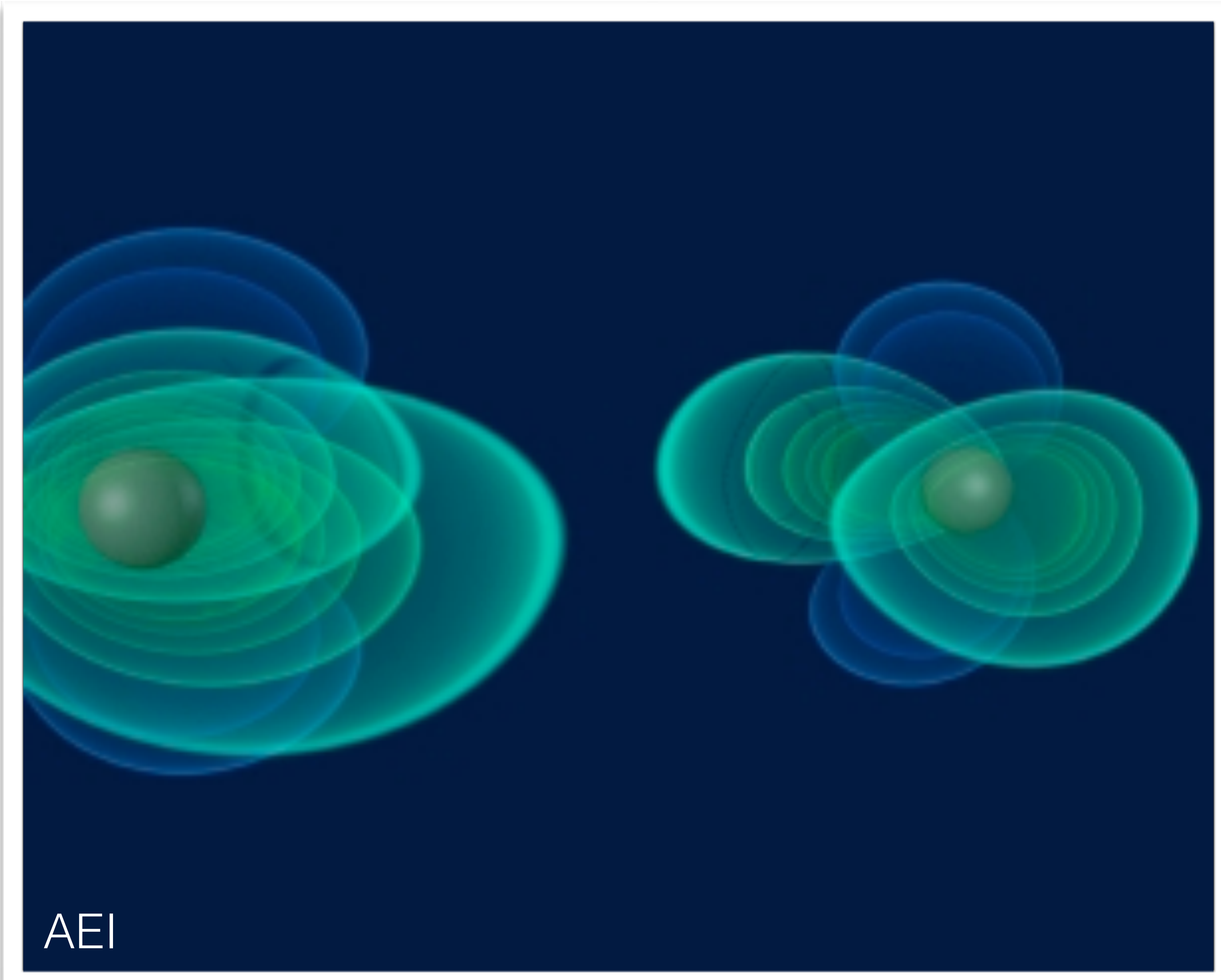
Resummation multipole by multipole
(Damour&Nagar 2007,
Damour, Iyer & Nagar 2008,
Damour & Nagar, 2009)



2
1
h_+ 0
-1
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
$t[sec]$


Perturbative theory of motion
Perturbative theory of gravitational radiation
Motion of two BHs
BH QNMs
Mathematical Relativity
$M < 4M_\odot$
PN (nonresummed)
Resummation EOB
Numerical Relativity
EOB[NR]
IMRPhenomD=Phen[EOB+NR]
ROM(EOB[NR])

Numerical Relativity



Numerical Relativity (NR)

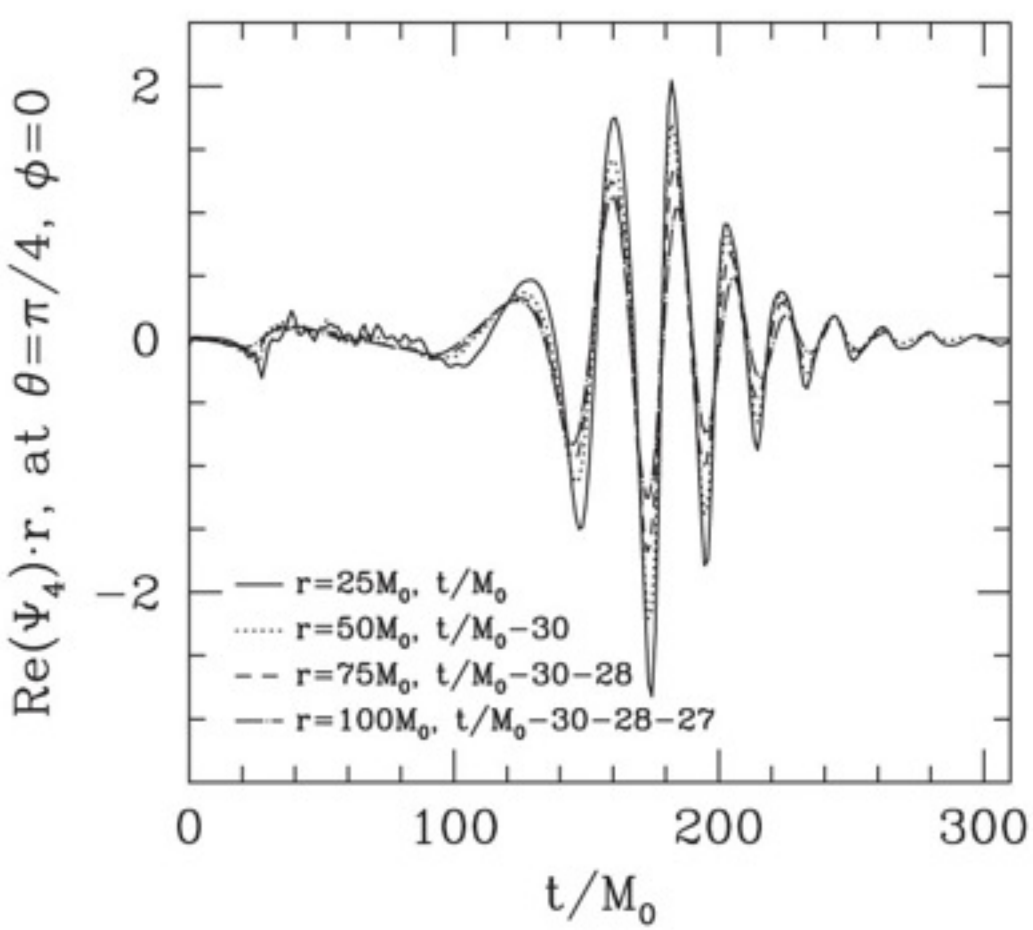
Mathematical foundations : Darmois 27, Lichnerowicz 43, Choquet-Bruhat 52-

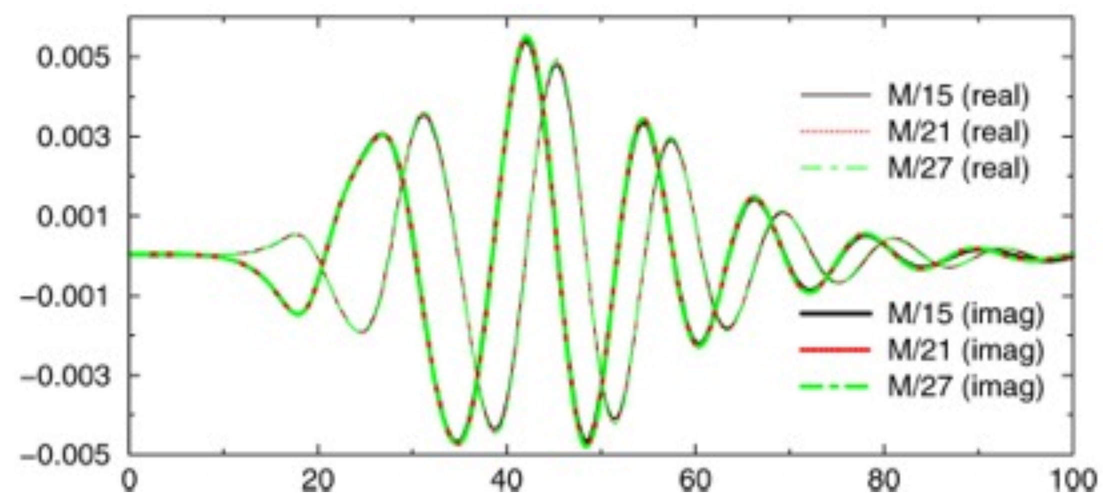
Breakthrough:

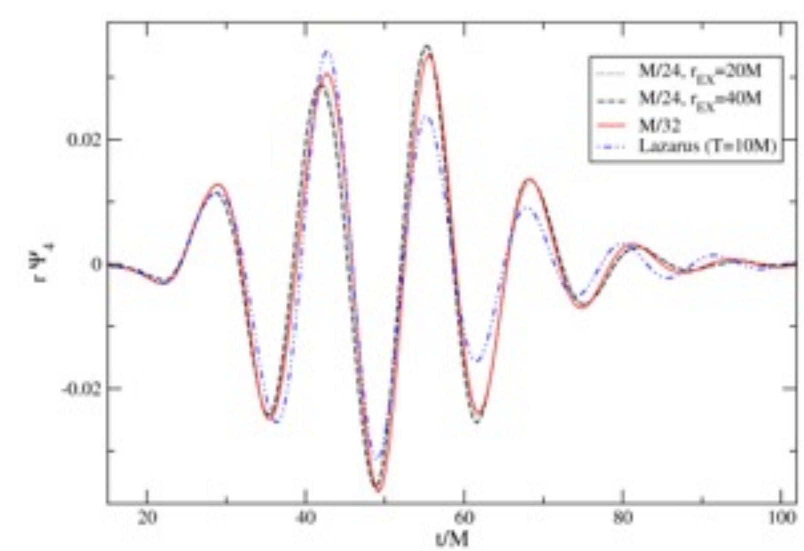
Pretorius 2005 generalized harmonic coordinates, constraint damping, excision

Campanelli-Lousto-Maronetti-Zlochover 2006
Baker-Centrella-Choi-Koppitz-van Meter 2006

Moving punctures



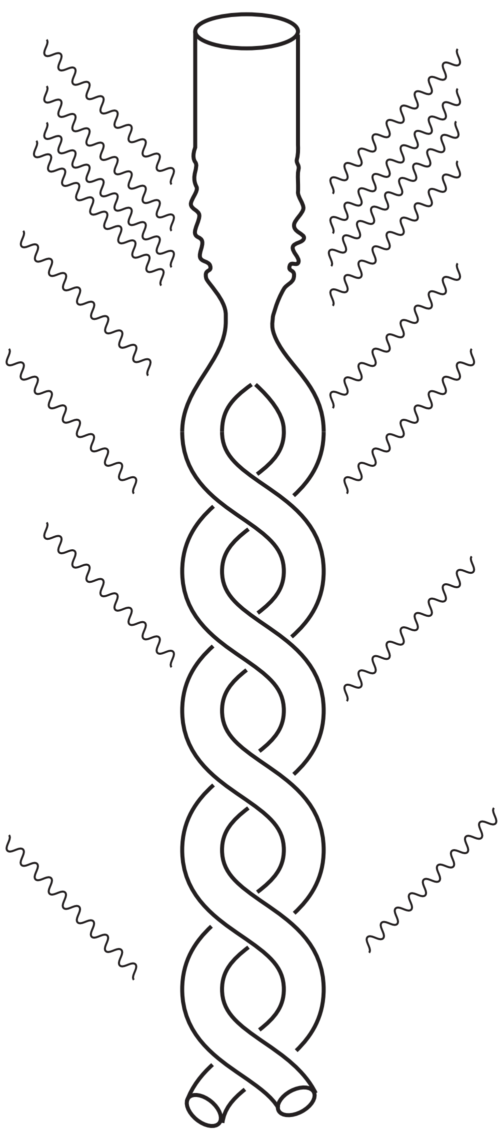




Excision + generalized harmonic coordinates (Friedrich, Garfinkle)

$$C_a \equiv g_{ab}\left(H^a - \Box x^a\right) = 0.$$

+ Constraint damping (Brodbeck et al., Gundlach et al., Pretorius, Lindblom et al.)



$$\begin{aligned} \frac{1}{2} g^{cd} g_{ab,cd} \quad &+ \\ g^{cd}{}_{(,a} g_{b)d,c} + H_{(a,b)} \quad &- \quad H_d \Gamma^d_{ab} + \Gamma^c_{bd} \Gamma^d_{ac} \\ +\kappa \left[n_{(a} C_{b)} \right. \quad &- \quad \left. \frac{1}{2} g_{ab} n^d C_d \right] \\ &= \quad -8\pi \left(T_{ab} - \frac{1}{2} g_{ab} T \right). \end{aligned}$$

$$\Box C^a = -R^a{}_b C^b + 2\kappa \nabla_b \left[n^{(b} C^{a)} \right],$$

The first EOB vs NR comparison

Buonanno-Cook-Pretorius 2007

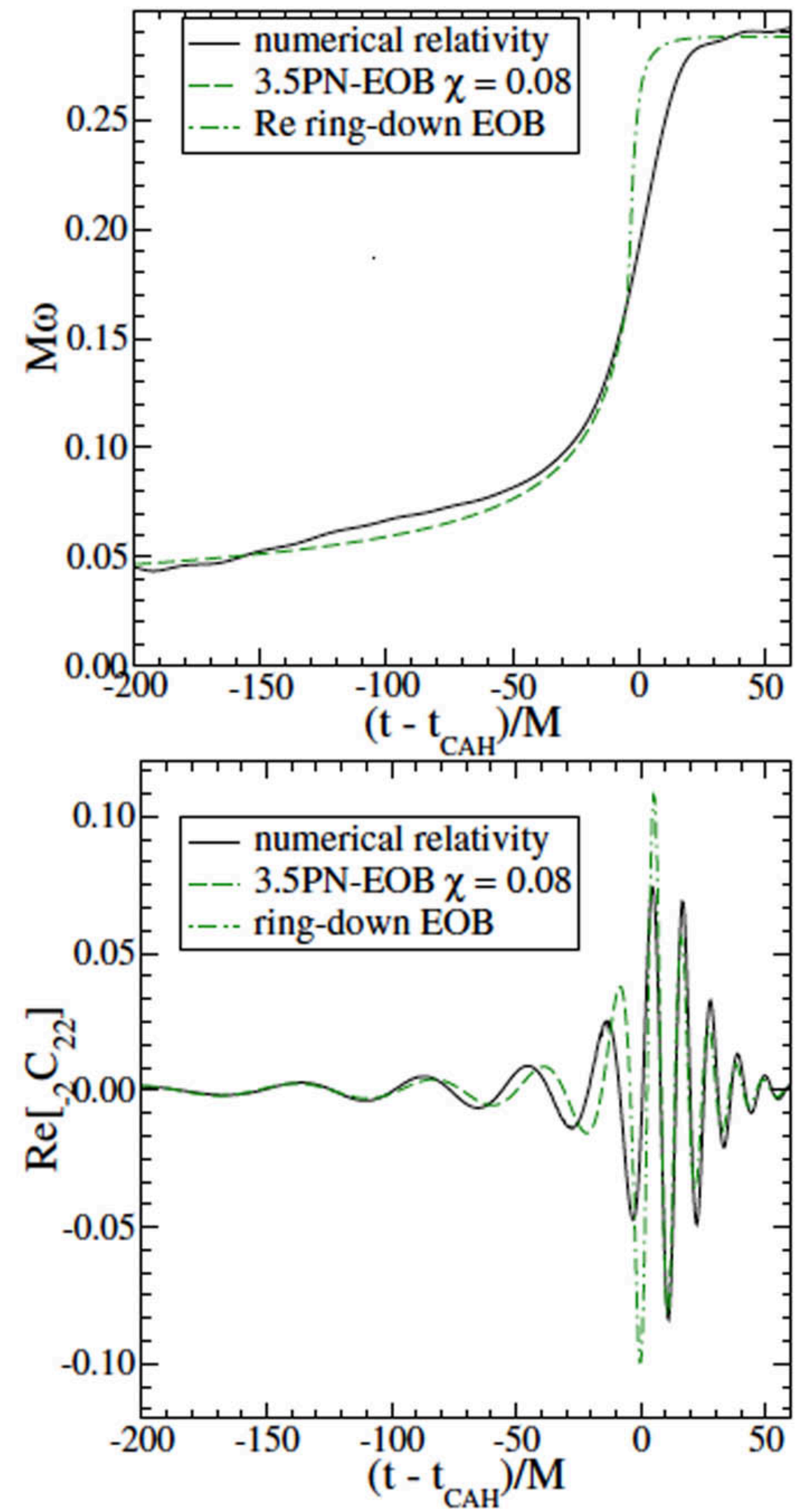
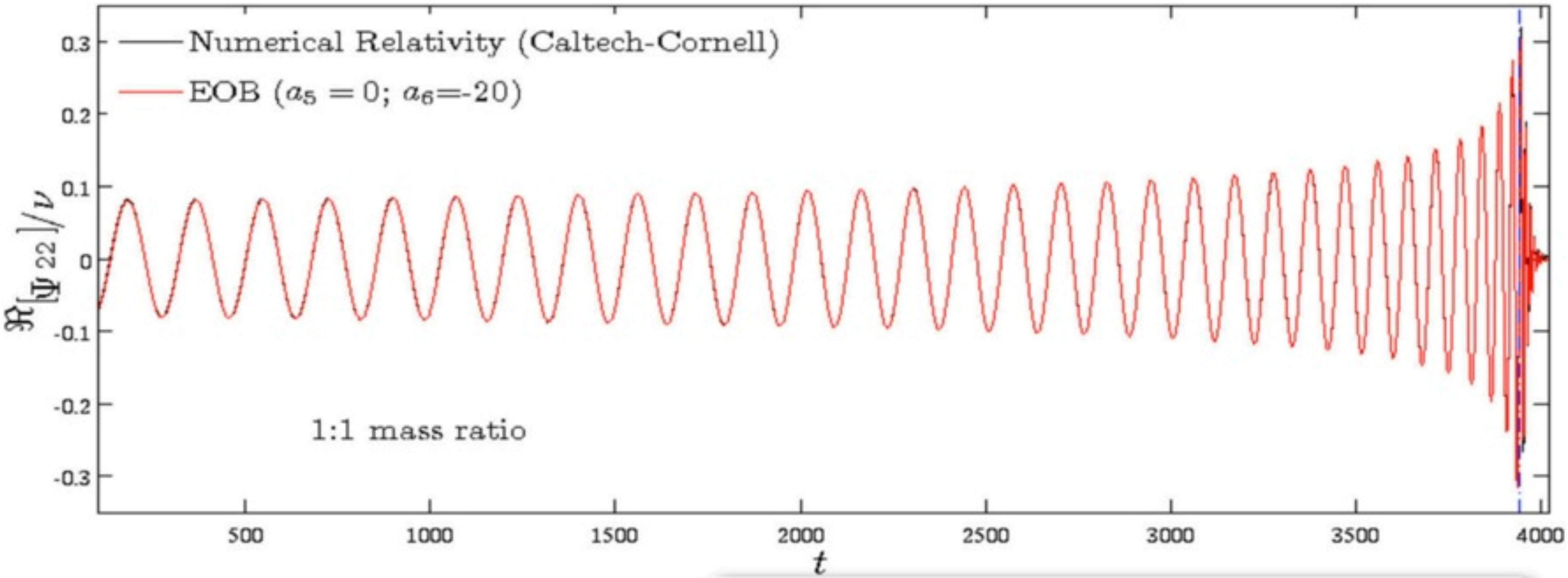
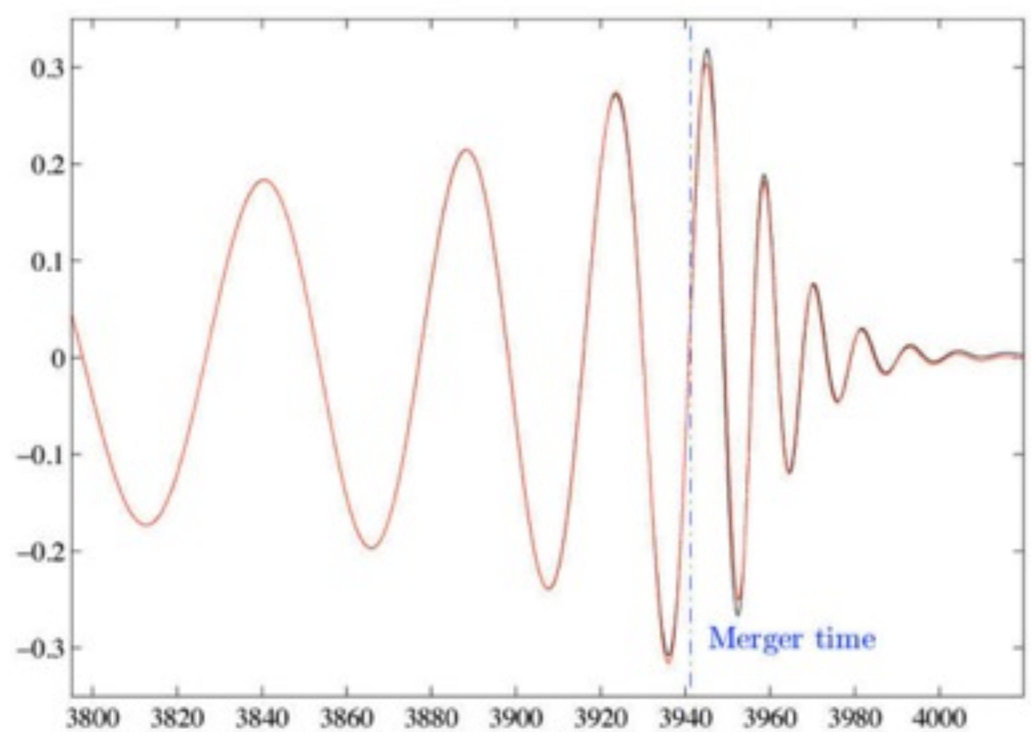
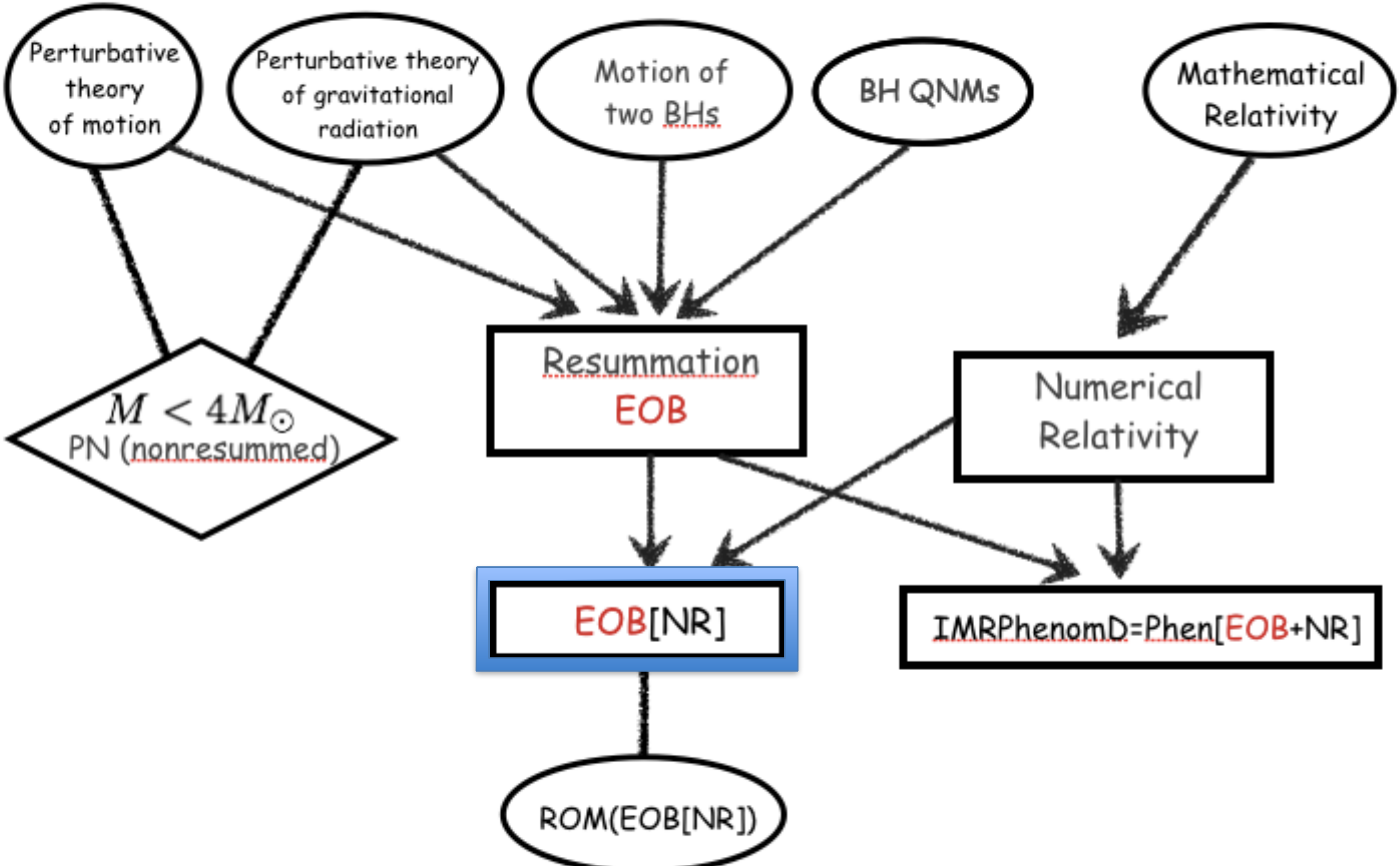


FIG. 21 (color online). We compare the NR and EOB frequency and Re$[_{-2}C_{22}]$ waveforms throughout the entire inspiral–merger–ring-down evolution. The data refers to the $d = 16$ run.

Numerical Relativity Waveform (Caltech-Cornell, SXS)







EOB[NR]: Damour-Gourgoulhon-Grandclement '02, Damour-Nagar '07-16, Buonnano-Pan-Taracchini-….'07-16

NR-completed resummed 5PN EOB radial A potential

(Damour-Nagar-Bernuzzi ’13 Nagar-Damour-Reisswig-Pollney ’16)

4PN analytically complete + 5 PN logarithmic term in the A(u, nu) function,
With u = GM/R and nu = m1 m2 / (m1 + m2)^2
[Damour 09, Blanchet et al 10, Barack-Damour-Sago 10, Le Tiec et al 11, Barausse et al 11, Akcay et al 12, Bini-Damour 13, Damour-Jaranowski-Schäfer 14, Nagar-Damour-Reisswig-Pollney 15]

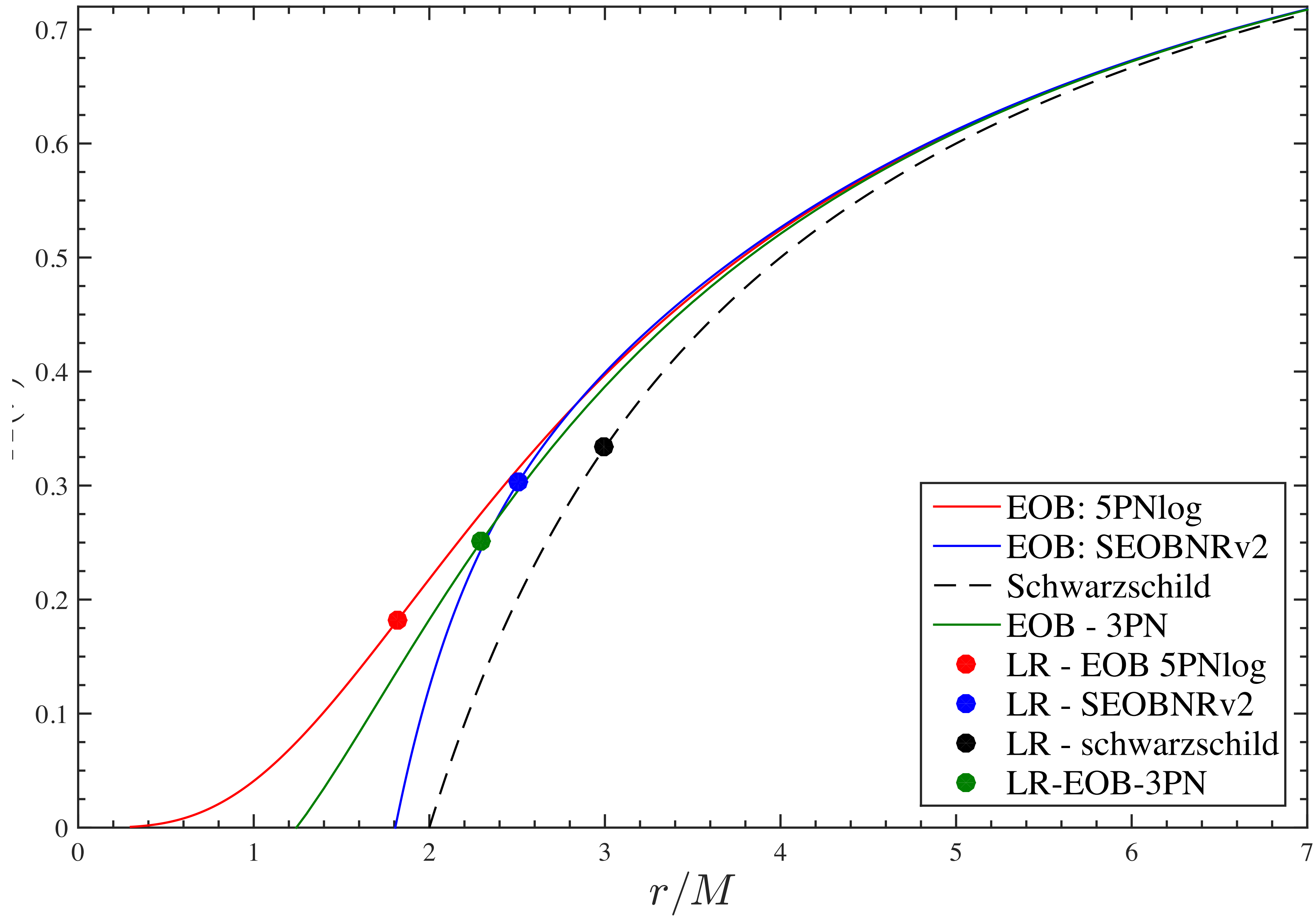
$$\begin{aligned} A(u;\nu,a_6^c) \;=\; & P_5^1\Big[1-2u+2\nu\,u^3+\nu\left(\frac{94}{3}-\frac{41}{32}\pi^2\right)u^4 \\ & +\;\nu\left[-\frac{4237}{60}+\frac{2275}{512}\pi^2+\left(-\frac{221}{6}+\frac{41}{32}\pi^2\right)\nu+\frac{64}{5}\ln(16e^{2\gamma}u)\right]u^5 \\ & +\;\nu\left[a_6^c(\nu)-\left(\frac{7004}{105}+\frac{144}{5}\nu\right)\ln u\right]u^6\Big] \end{aligned}$$

$$a_6^{c\,\text{NR-tuned}}(\nu) = 81.38 - 1330.6\,\nu + 3097.3\,\nu^2$$

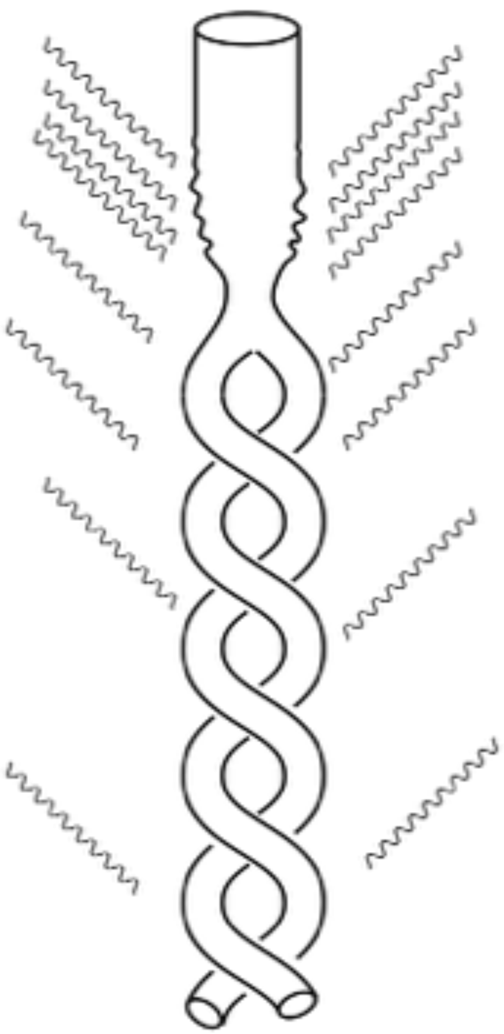
MAIN RADIAL RADIAL EOB POTENTIAL A(R)

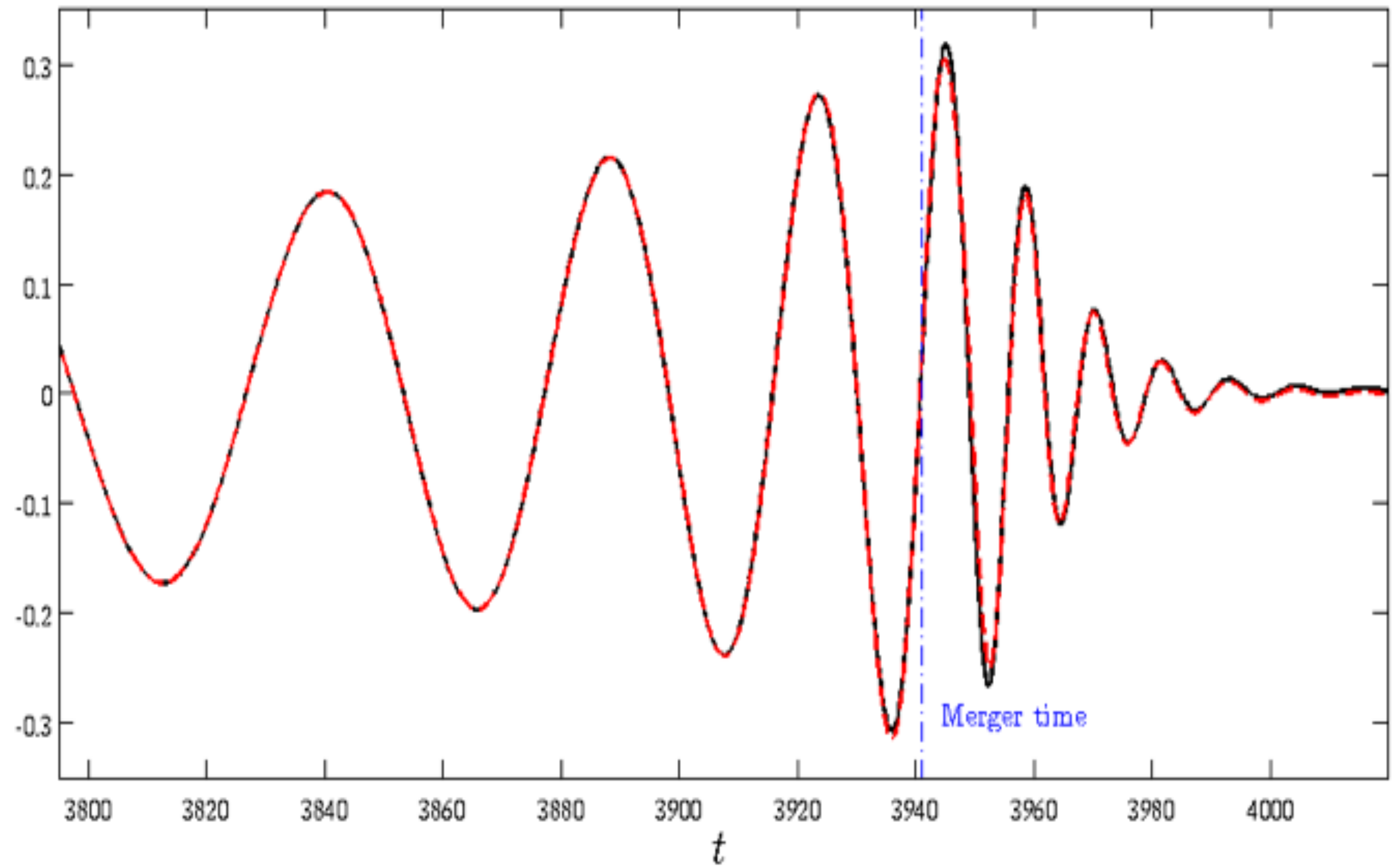
m1=m2 case

A(r)

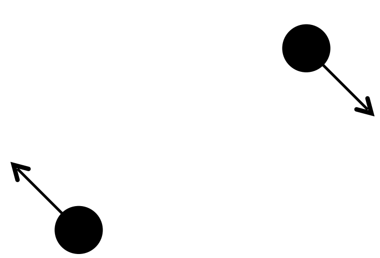


EOB / NR Comparison



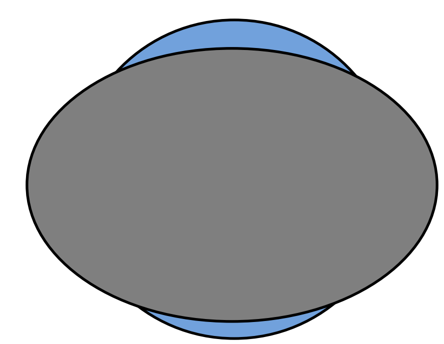


Inspiral + « plunge »

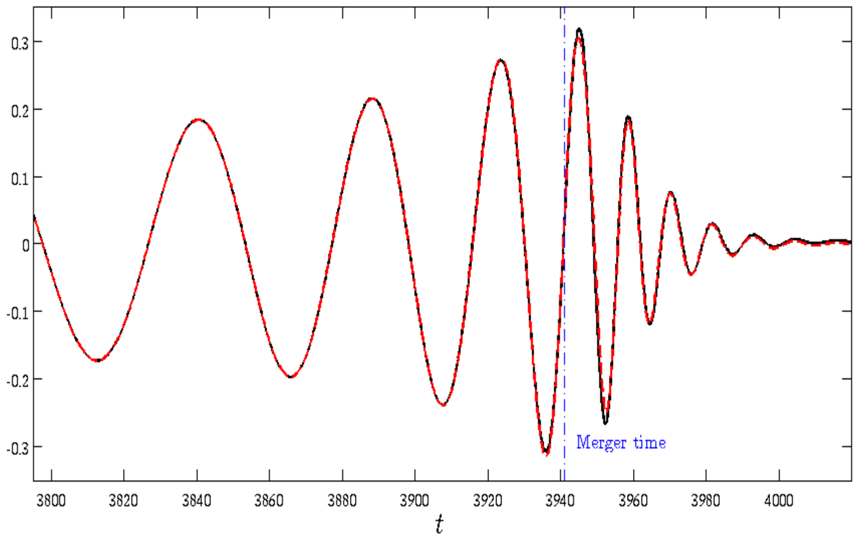


Two orbiting point-masses:
Resummed dynamics

Ringing BH

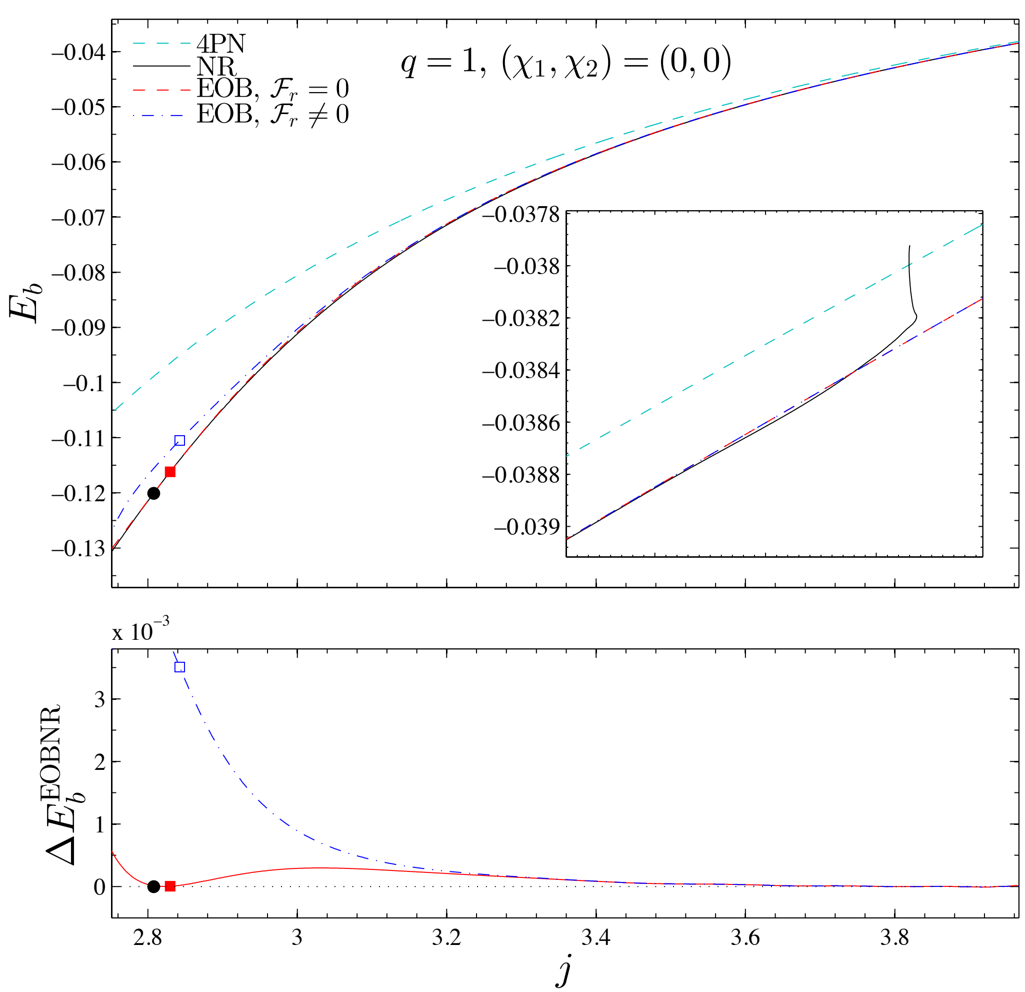


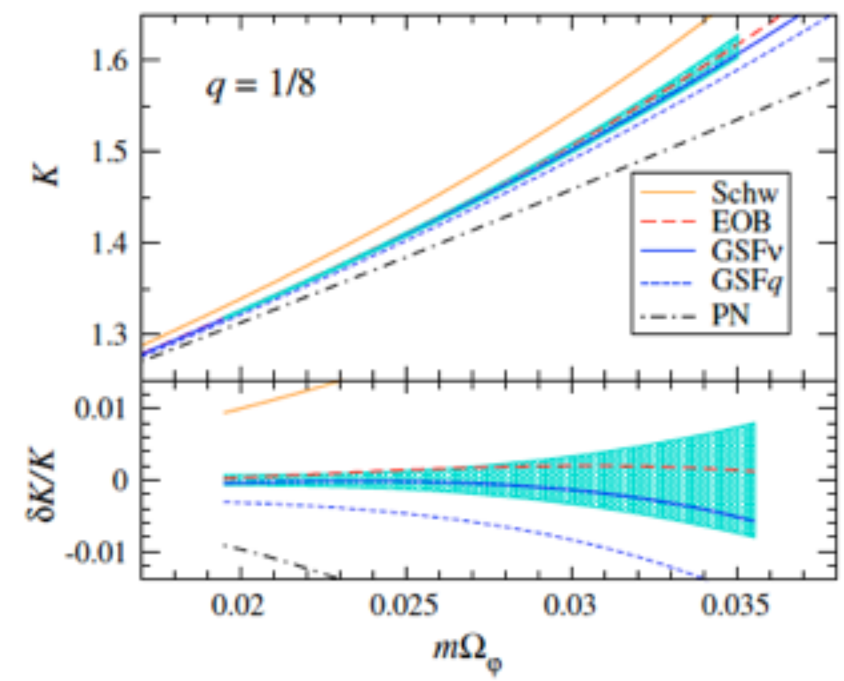
Peak emitted power ~ 3 x 10^56 erg/s ~ 0.001 c^5/G

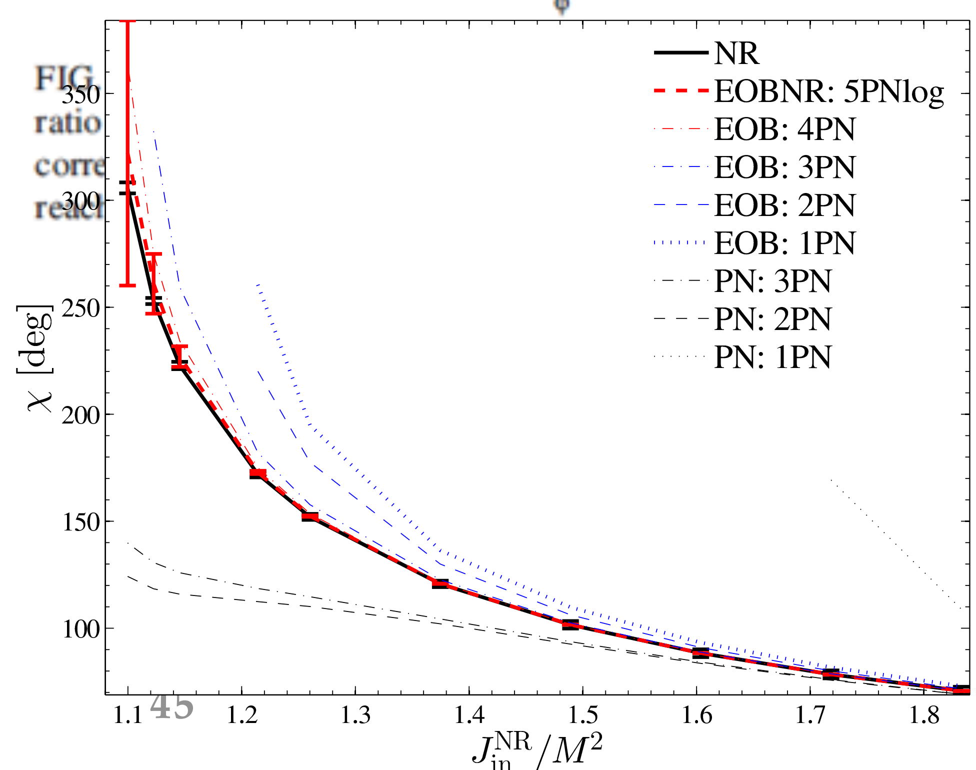


EOB VS NR

waveform (Damour-Nagar 09,Buonanno et al), energetics (Nagar-Damour-Reisswig-Pollney 16), periastron precession (LeTiec-Mroue-Barack-Buonanno-Pfeiffer-Sago-Tarachini 11, Hinderer et al 13); and scattering angle (Damour-Guercilena-Hinder-Hopper-Nagar-Rezzolla 14)





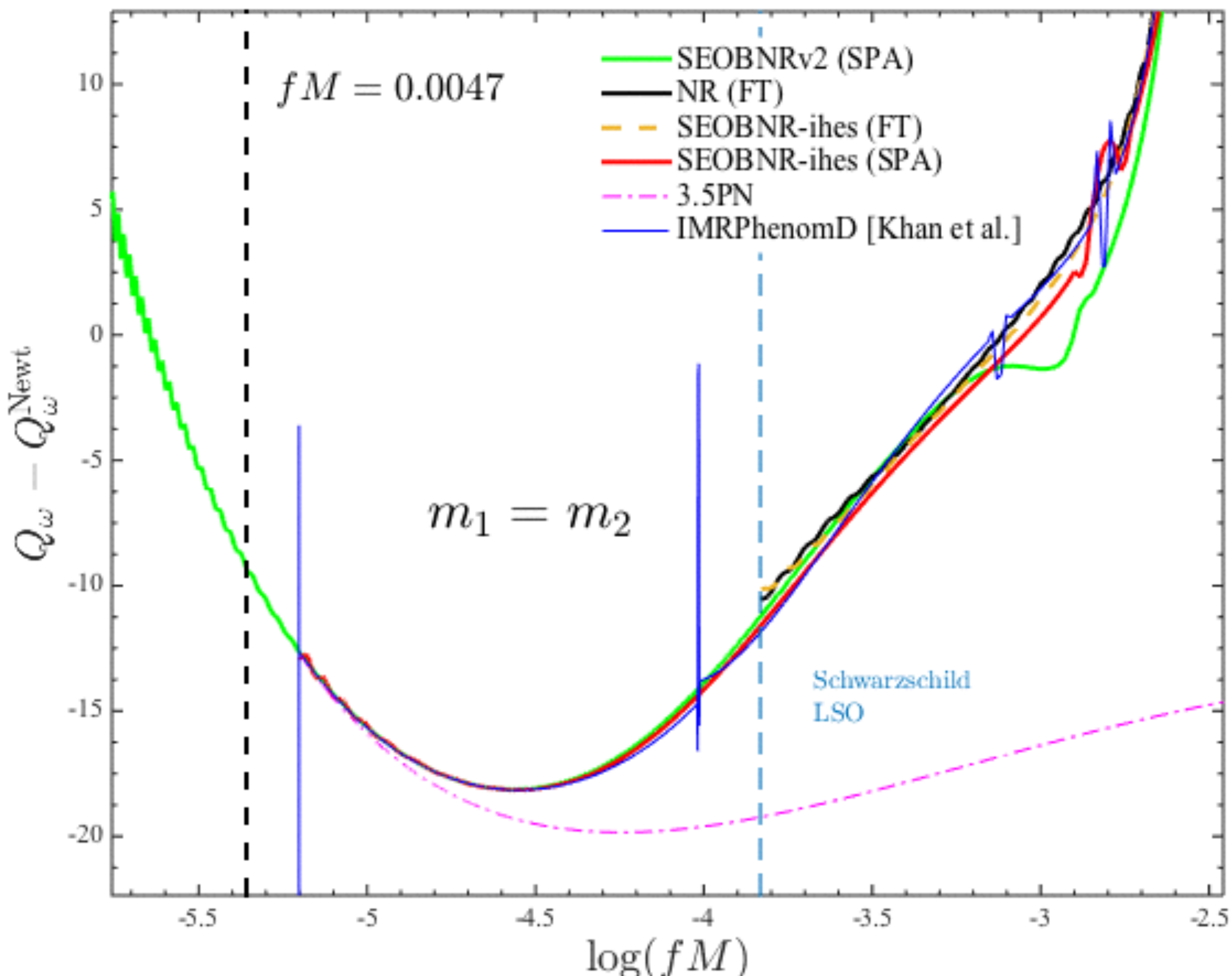


PN, EOB, NR, PHENOMD

PN accuracy loss during inspiral

Dimensionless « quality factor » of GW phase $Q_\omega = f^2 \frac{d^2\psi(f)}{df^2} \approx \frac{\omega^2}{\dot{\omega}}$

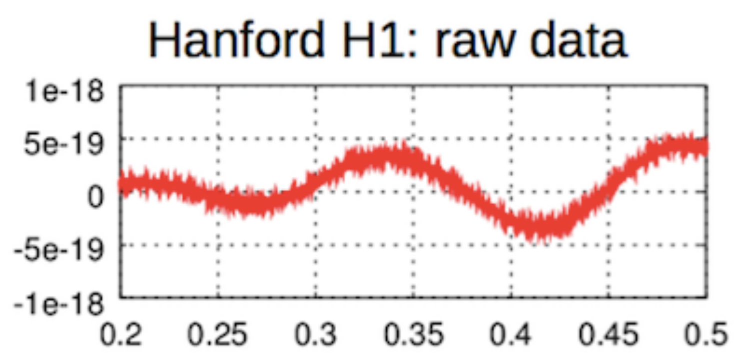
$Q_\omega - Q_\omega^N$

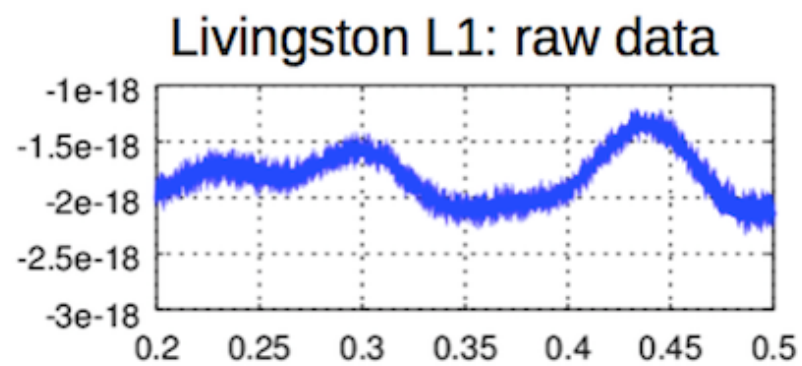


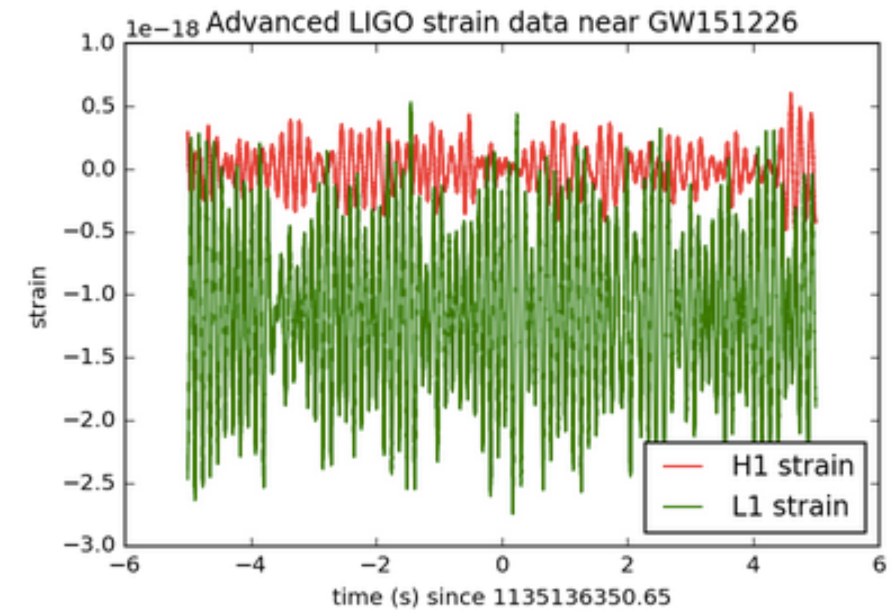
GW150914 and GW151226: incredibly small signals lost in the broad-band noise

GW151226 from LIGO open data

GW150914, from Chassande-Mottin 5 April 2016



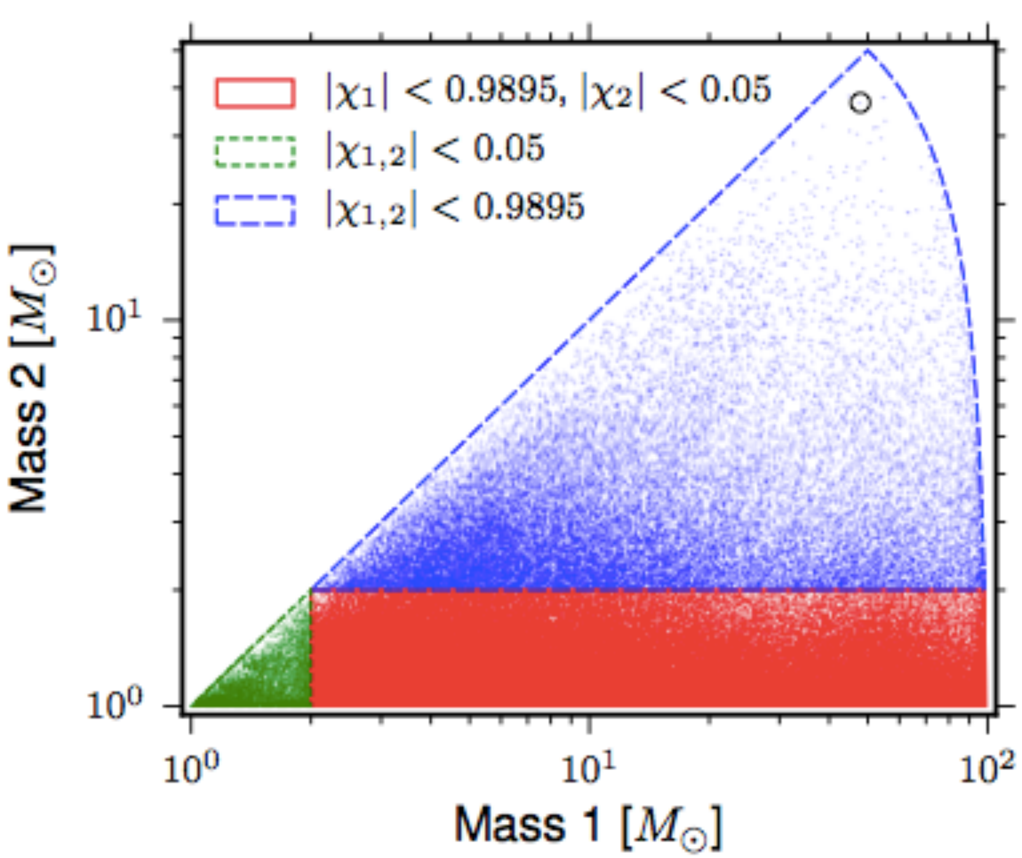


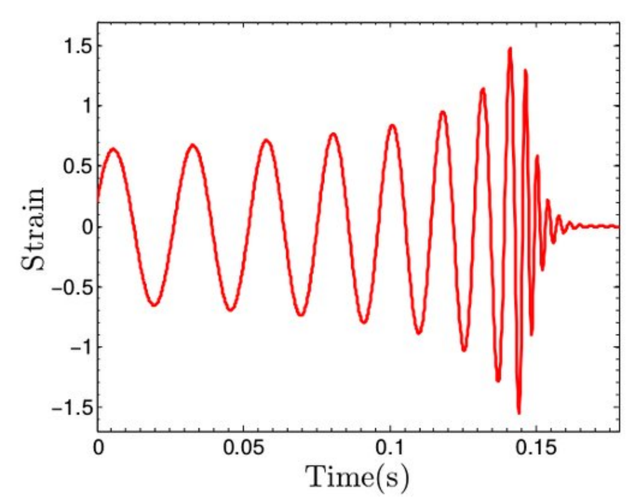


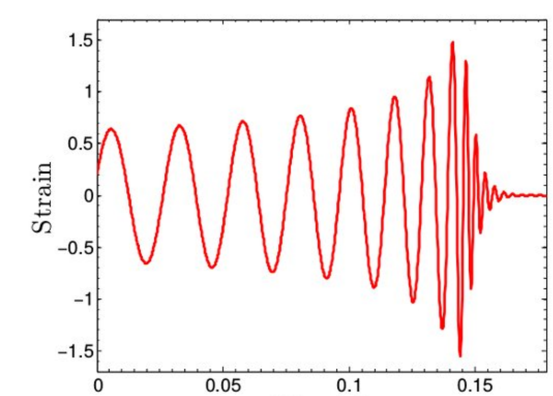
$$h_{GW}^{max} \sim 10^{-21} \sim 10^{-3}\, h_{LIGO}^{broadband}$$

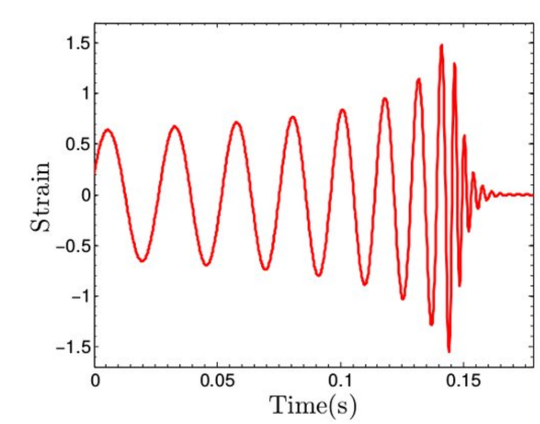
Matched Filtering

$$\langle output | h_{template} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{template}(f)$$





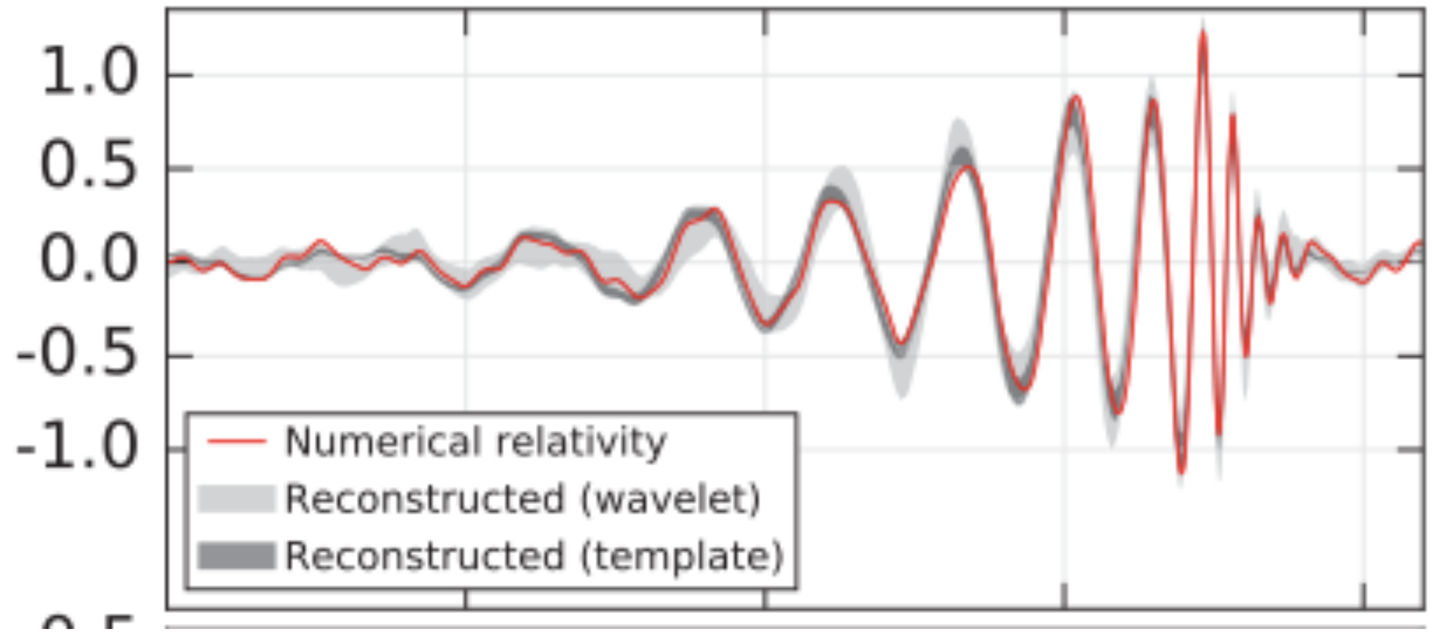




A POSTERIORI WAVEFORM CHECKS USING NR SIMULATIONS

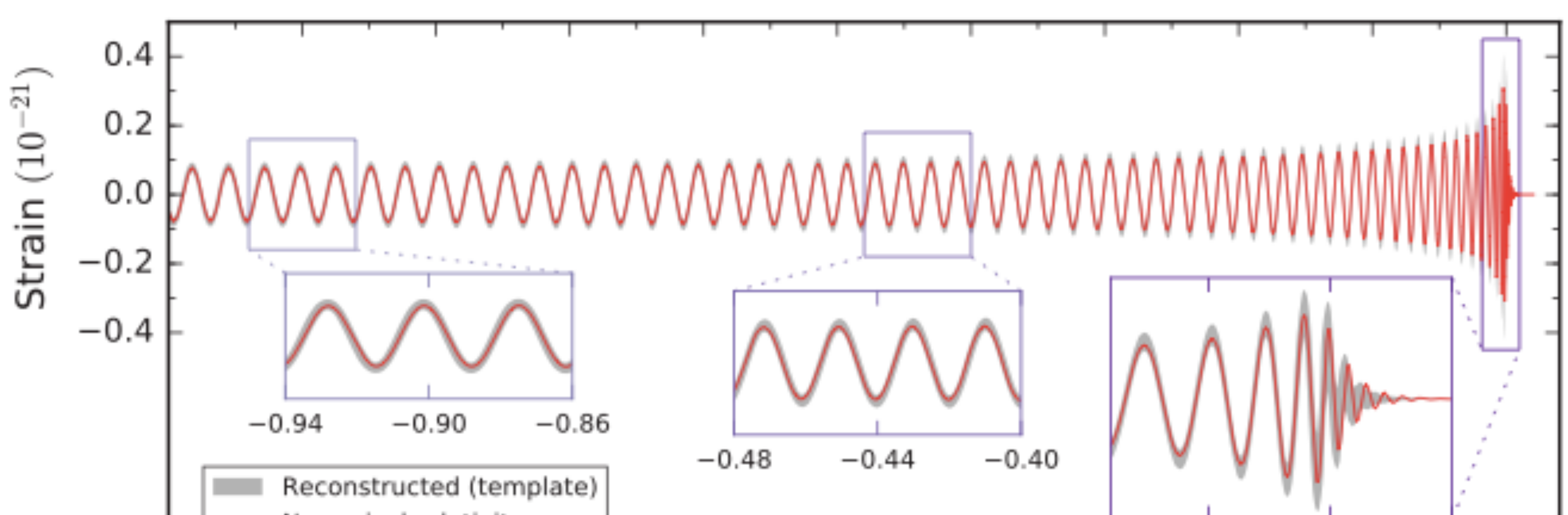
SXS simulation

GW150914
Abbott et al 16a

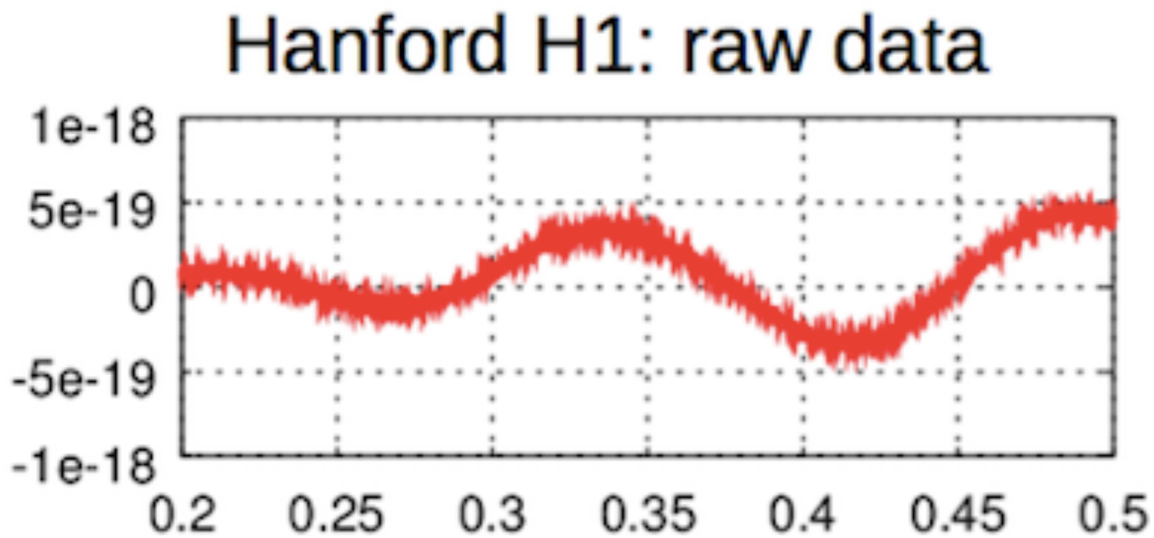


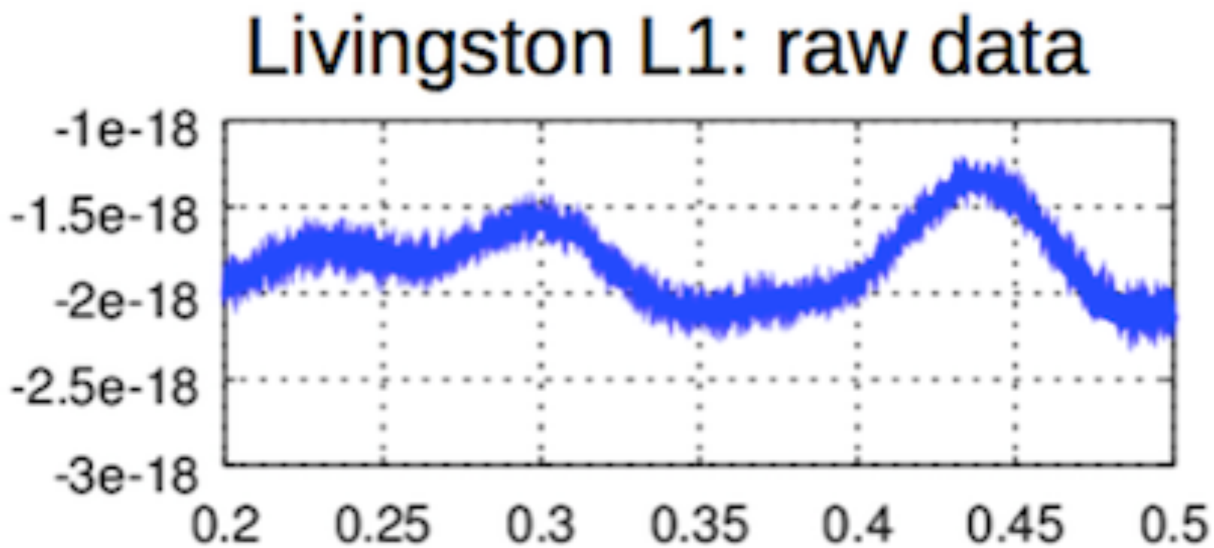
GW151226
Abbott et al 16b

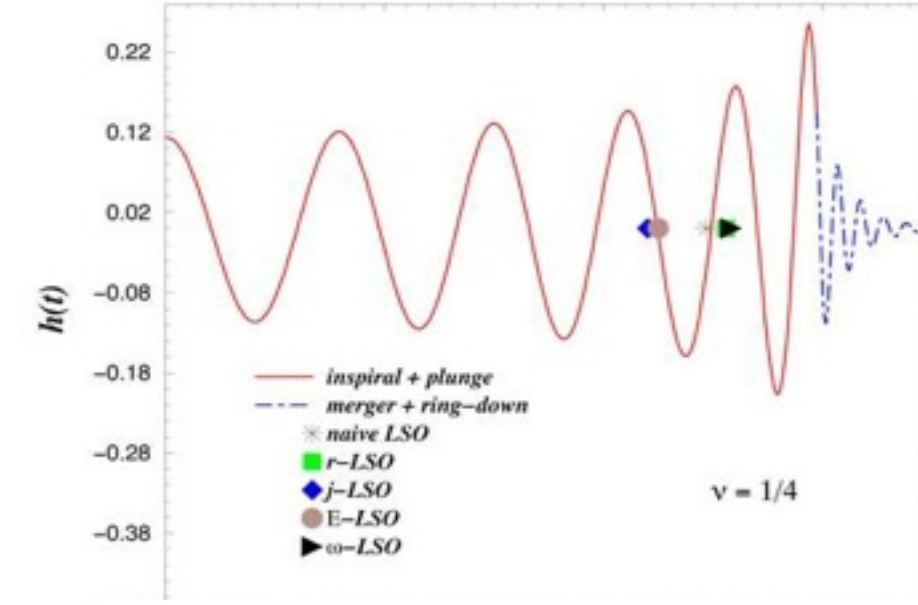
took three months and 70 000 CPU hours !



GW150914 vs EOB[NR]

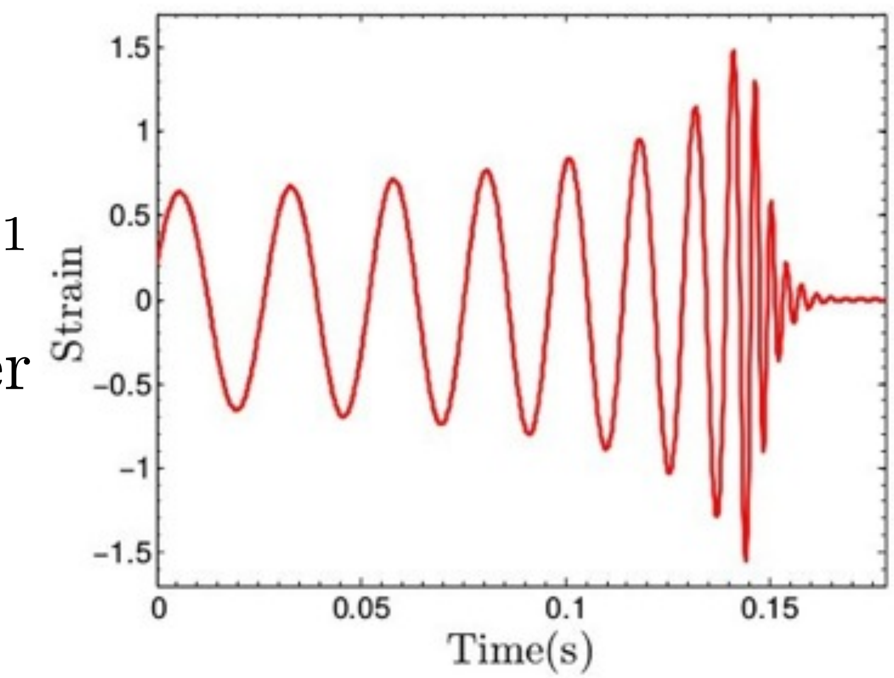




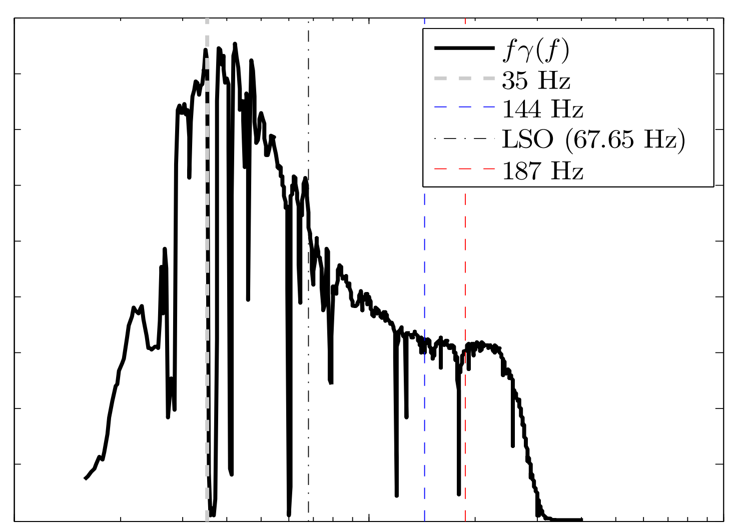


scale : 10^{-21}

$500 \times$ smaller



$$\frac{d\rho^2}{d\ln f} = \frac{f|\tilde{h}(f)|^2}{S_n(f)}$$



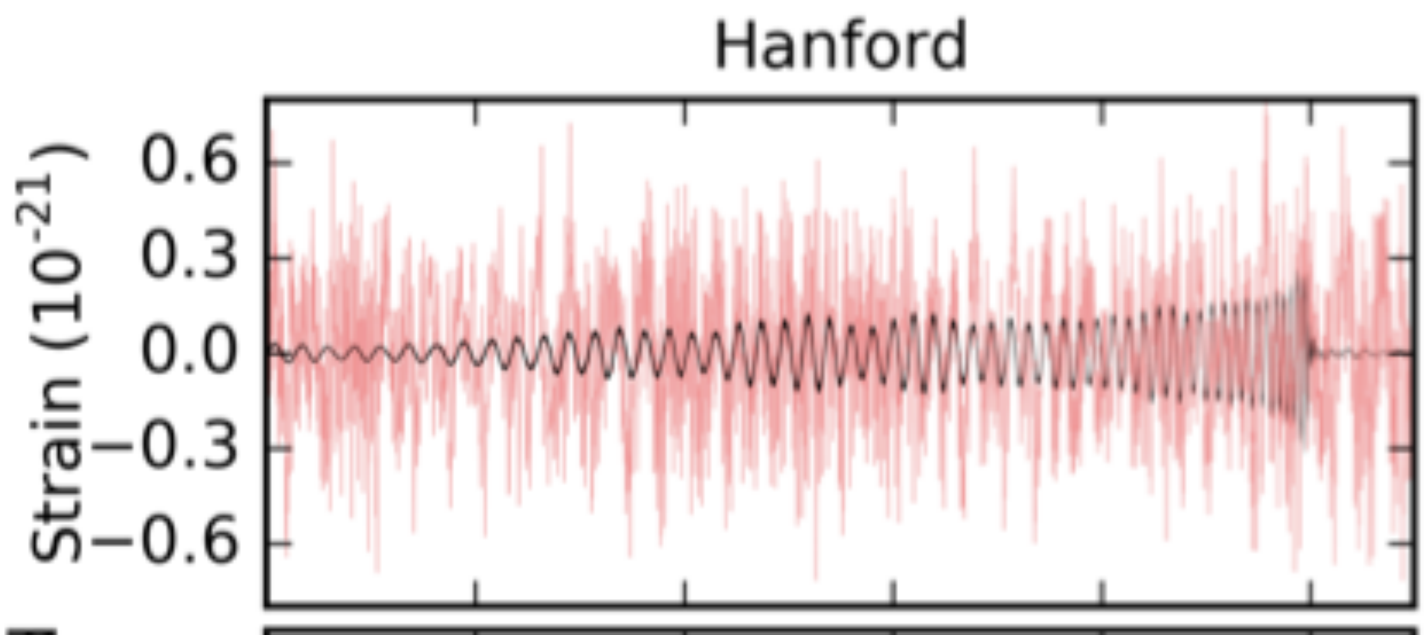
$$m_1 = 36^{+5}_{-4} M_\odot$$

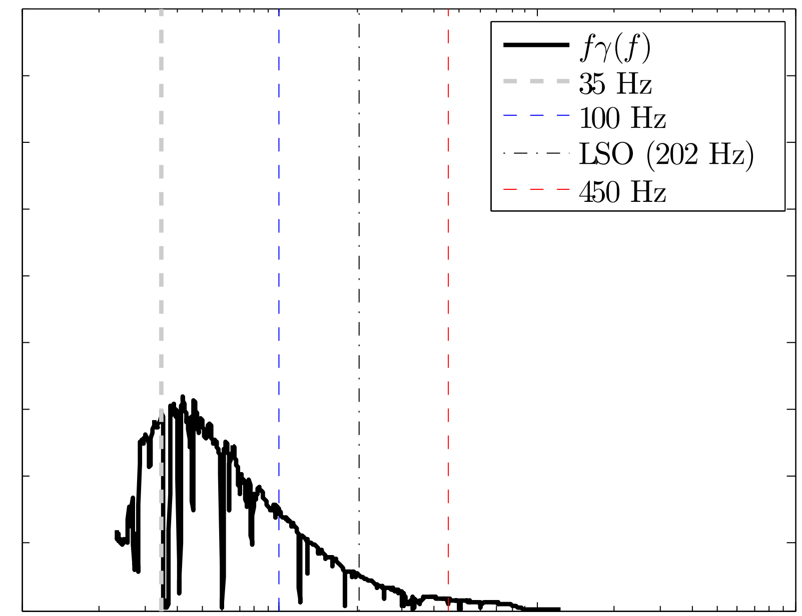
$$m_2 = 29^{+4}_{-4} M_\odot$$

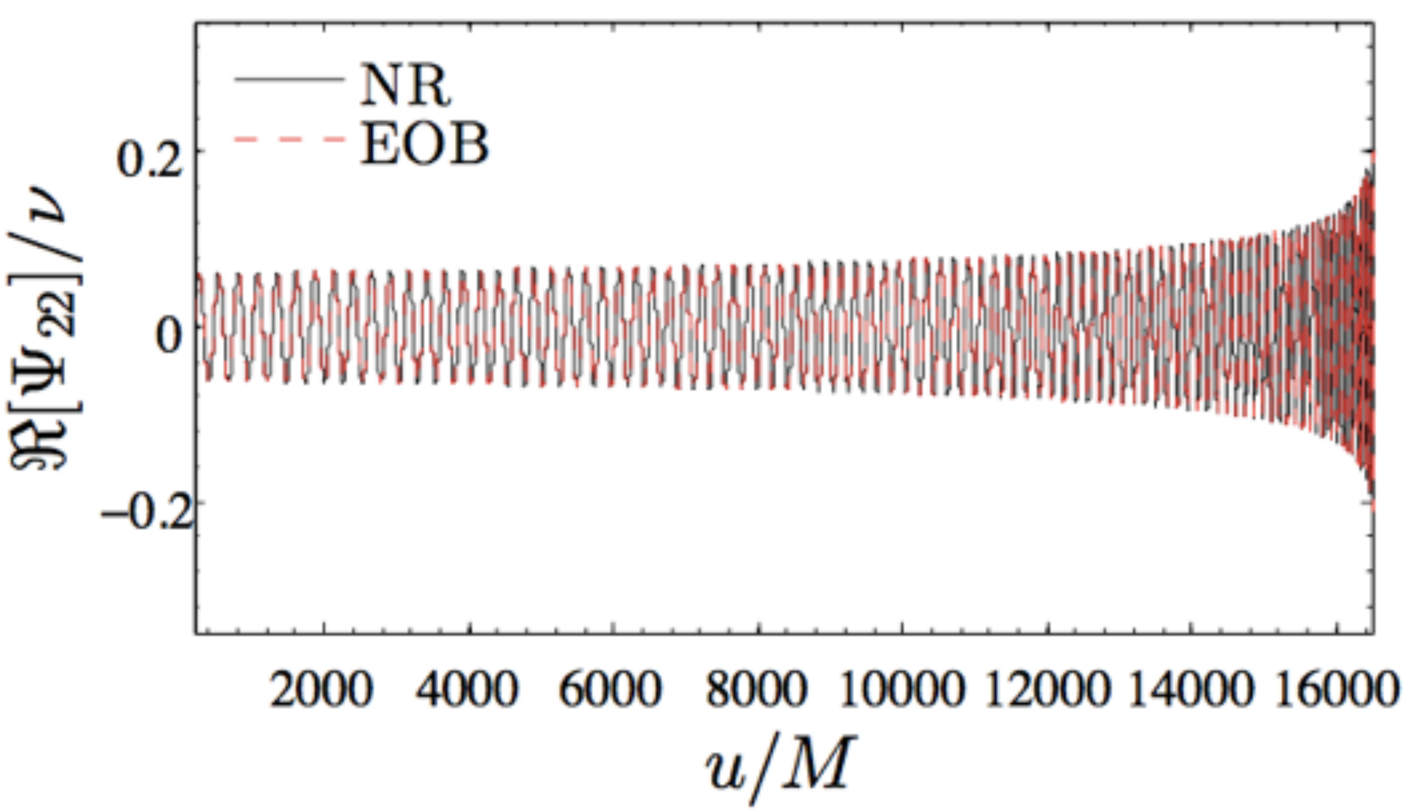
$$\chi_{\rm eff} = -0.06^{+0.17}_{-0.18}$$

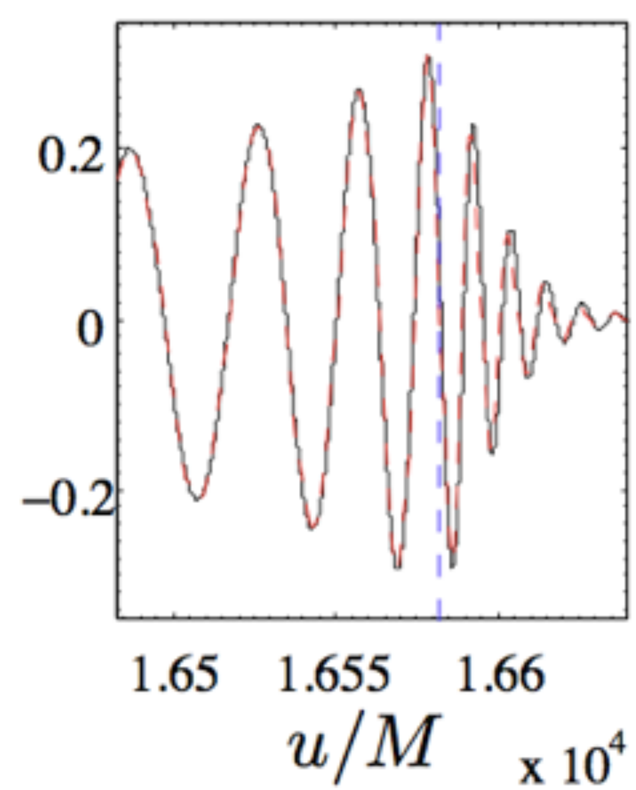
$$D_{\rm L} = 410^{+160}_{-180} \rm Mpc$$

GW151226: only detected via accurate matched filters









$$m_1 = 14.2^{+8.3}_{-3.7} M_\odot$$

$$m_2 = 7.5^{+2.3}_{-2.3} M_\odot$$

$$\chi_{\rm eff} = +0.21^{+0.20}_{-0.10}$$

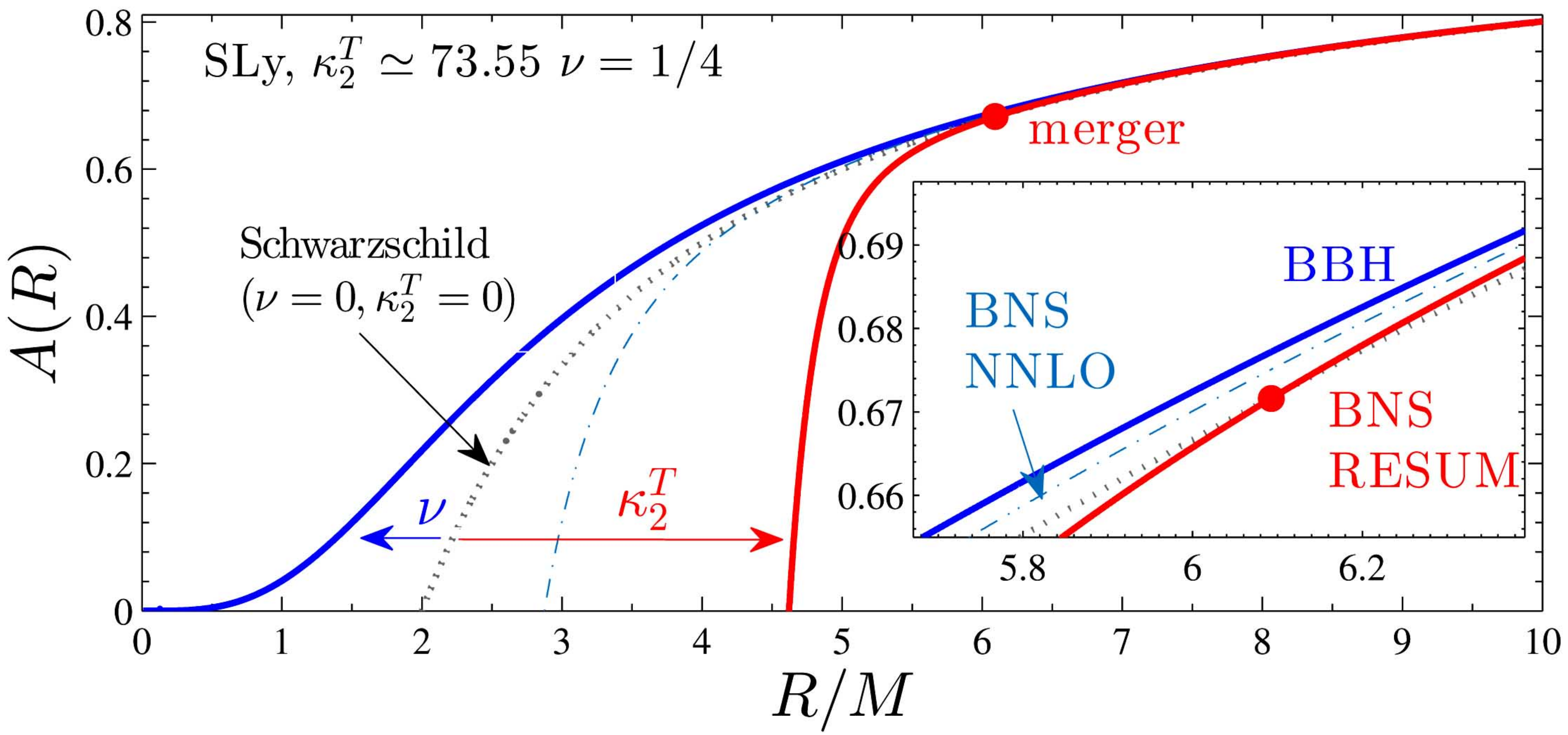
$$D_{\rm L} = 440^{+180}_{-190} {\rm Mpc}$$

FUTURE PROSPECT 1: NSNS AND BHNS GW: PROBING THE NUCLEAR EOS FROM LATE INSPIRAL TIDAL EFFECTS IN NSNS OR BHNS

Tidal extension of EOB (TEOB) [Damour-Nagar 09]

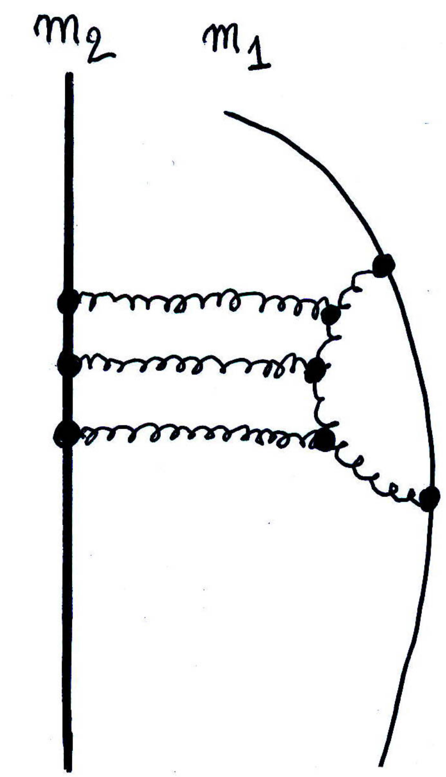
$$\begin{aligned} A(r) &= A_r^0 + A^{\rm tidal}(r) \\ A^{\rm tidal}(r) &= -\kappa_2^T u^6 \left(1 + \bar{\alpha}_1 u + \bar{\alpha}_2 u^2 + \ldots\right) + \ldots \end{aligned}$$

TEOB[NR] A(R) potential (Bernuzzi et al. 2015)



FUTURE PROSPECT: GW DETECTORS IN SPACE LISA

Here also analytical methods (completed by numerical ones) will be important: Gravitational Self-Force program: m1 << m2



- Analytical high-PN results : Blanchet-Detweiler-LeTiec-Whiting '10, Damour '10, Blanchet et al '10, LeTiec et al '12, Bini-Damour '13-15, Kavanagh-Ottewill-Wardell '15
- (gauge-invariant) Numerical results : Detweiler '08, Barack-Sago '09, Blanchet-Detweiler-LeTiec-Whiting '10, Barack-Damour-Sago '10, Shah-Friedman-Keidl '12, Dolan et al '14, Nolan et al '15, …
- Analytical PN results from high-precision (hundreds to thousands of digits !) numerical results : Shah-Friedman-Whiting '14, Johnson-McDaniel-Shah-Whiting '15

EOB[SF] program: import high PN-SF results in EOB (Bini-Damour '15, '16 Kavanagh et al '15,'16, Bini-Damour-Geralico '15,'16, Akcay, van de Meent, Hopper, ……)

$$a_{10}^{c} = \frac{186054788842060273}{7079830758000}\ln(2) - \frac{1619008}{405}\zeta(3) - \frac{213398732147280 97}{1011404394000}\gamma + \frac{27101981341}{100663296}\pi^6 - \frac{6236861670873}{125565440}\ln(3) + \frac{360126}{49}\ln(2)\ln(3) + \frac{180063}{49}\ln(3)^2 - \frac{121494974752}{9823275}\ln(2)^2 - \frac{24229836023352153}{549755813888}\pi^4 + \frac{1115369140625}{124540416}\ln(5) + \frac{96889010407}{277992000}\ln(7) + \frac{754370143706233186232 99}{186907532011200 00} - \frac{60648244288}{9823275}\ln(2)\gamma + \frac{200706848}{280665}\gamma^2 + \frac{11980569677139}{2306867200}\pi^2 + \frac{360126}{49}\gamma\ln(3),$$

$$a_{10}^{\ln} = -\frac{212751433335120 97}{2022808788000} + \frac{200706848}{280665}\gamma - \frac{30324122144}{9823275}\ln(2) + \frac{180063}{49}\ln(3),$$

$$a_{10}^{\ln^2} = \frac{50176712}{280665},$$

Conclusions

• Several aspects of Analytical Relativity have played a key role in the recent discovery, interpretation and parameter estimation of coalescing BBH: perturbative theory of motion, perturbative theory of GW generation, EOB formalism.

• The analytical EOB method had predicted in 2000 the complete GW signal emitted by the coalescence of two black holes. This was confirmed, and refined, in 2005 by Numerical Relativity. The Numerical-Relativity completion of Analytical Relativity (and particularly EOB[NR]) has been crucial for computing the ~ 200, 000 theoretical GW templates $h(t;m_1, m_2, S_1, S_2)$ which have been used for extracting the GW signals from the noise by matched filtering, for assessing their physical significance, and for measuring the source parameters. One expects most of the BBH (and BNS) signals to be detected only by means of accurate EOB[NR] templates (as was the case for GW151226).

• Analytical approaches will also be crucial for future GW detectors: space detectors, second generation ground-based detectors. In particular, the union of EOB and Self-Force methods promises to help computing accurate waveforms for LISA-type sources.

• Opening of a new window on the universe: GW astronomy: might be dominated by BBH (Belczynski et al 2010); waiting for BNS + EM signal (GRB ?), and for LIGO/Virgo/Kagra/Indigo network. The detailed study of coalescences involving NS will open a window on the EOS of nuclear matter (tidal polarizability).

• Window on cosmic-size strings especially via GW bursts above the Gaussian background

• Potentially new window on early universe via GW cosmological background