GARYFEST, Gravitation, Solitons and Symmetries
Tours, March 22-24, 2017

AdS_2 Holographic Dictionary: An application to the subtracted geometry of non-extremal black holes

Mirjam Cvetič

Penn
University of Pennsylvania

CAMTP
University of Maribor

Univerza v Ljubljani
Fakulteta za matematiko in fiziko

Honored to be able to participate in the celebration of Gary's numerous contributions to gravitational physics, including classical and quantum aspects of black holes, topological defects and the role of (super)symmetry.

Cherish him as a friend and a colleague, since we met in the 90-ies. During my many visits at Cambridge U., and his visits at Penn, which also led to his association with Penn, we had the opportunity to not only collaborate but also to explore Philadelphia's dynamic cultural as well as wining and dining scenes.

His curiosity and interest in every aspect of human endeavor are boundless, which led to many animated discussions and arguments in different settings, and not only on topics pertinent to science…

He is a generous and a patient collaborator, with encyclopedic knowledge of numerous aspects of physics and mathematics. Our association resulted in a productive collaboration with over 30 papers.

Our collaboration covered many aspects of gravitational physics, with applications to supergravity and string theory as a common thread: special holonomy spaces, non-linear Kaluza-Klein reduction in supergravity theories, and extensive work on black holes there; topics that fit well into celebrating Gary's contributions to science.

↓

The topic of my talk

Outline:

I. Motivate Subtracted Geometry of general asymptotically flat black holes –
prototype STU black holes

II. Stepping stone toward holography:
Variational Principle for Subtracted Geometry
conserved charges and thermodynamics

III. Dual Field Theory of Subtracted Geometry via
Holography of 2D Einstein-Maxwell-Dilaton gravity
holographic dictionary & new insights & beyond

IV. Outlook

Background:

Initial work on subtracted geometry

M.C., Finn Larsen 1106.3341, 1112.4846, 1406.4536
M.C., Gary Gibbons 1201.0601
M.C., Monica Guica, Zain Saleem 1301.7032
…

Recent:

Toward holography of subtracted geometry:

Variational principle; conserved charges & thermodynamics
Ok Song An, M.C., Ioannis Papadimitriou, 1602.0150

Subtracted geometry and AdS_2 holography

M.C., Ioannis Papadimtiriou,1608.07018

I. General non-extremal, asymptotically flat black holes in effective string theory in D=4

specified by

M - mass, Q_i, P_i - multi-charges, J - angular momentum

w/ $M > \Sigma_i |Q_i| + \Sigma_i |P_i|$

Prototype solutions of a sector of maximally supersymmetric D=4 Supergravity

[sector of toroidally compactified effective string theory] → so-called STU model

Prototype: Black holes of STU Model

Lagrangian [A sector of toroidally compactified effective string theory]

$$2\kappa_4^2 \mathcal{L}_4 = R \star 1 - \frac{1}{2} \star d\eta_a \wedge d\eta_a - \frac{1}{2} e^{2\eta_a} \star d\chi^a \wedge d\chi^a$$
$$- \frac{1}{2} e^{-\eta_0} \star F^0 \wedge F^0 - \frac{1}{2} e^{2\eta_a - \eta_0} \star (F^a + \chi^a F^0) \wedge (F^a + \chi^a F^0)$$
$$+ \frac{1}{2} C_{abc} \chi^a F^b \wedge F^c + \frac{1}{2} C_{abc} \chi^a \chi^b F^0 \wedge F^c + \frac{1}{6} C_{abc} \chi^a \chi^b \chi^c F^0 \wedge F^0$$

(a=1,2,3; C_{abc}-anti-symmetric tensor)

w/ A^0 & three gauge fields A^a, the three dilatons η^a and the three axions χ^a.

Black holes: explicit solutions of equations of motion for the above Lagrangian w/ metric, four gauge potentials and three axio-dilatons

Prototype, four-charge rotating black hole, originally obtained via solution generating techniques

M.C., Youm 9603147
Chong, M.C., Lü, Pope 0411045

Four- SO(1,1) transfs.
time-reduced Kerr BH

$$H = \begin{pmatrix} \cosh\delta_i & \sinh\delta_i \\ \sinh\delta_i & \cosh\delta_i \end{pmatrix}$$

Full four-electric and four-magnetic charge solution only recently obtained

Chow, Compère 1310.1295;1404.2602

Compact form of the metric for rotating four-charge black holes

M.C. & Youm 9603147

Chong, M.C., Lü & Pope 0411045

$$ds_4^2 = -\Delta_0^{-1/2} G (dt + \mathcal{A})^2 + \Delta_0^{1/2} \left(\frac{dr^2}{X} + d\theta^2 + \frac{X}{G} \sin^2\theta d\phi^2 \right)$$

Compact form of the metric for rotating four-charge black holes

M.C. & Youm 9603147
Chong, M.C., Lü & Pope 0411045

$$ds_4^2 = -\Delta_0^{-1/2} G (dt + \mathcal{A})^2 + \Delta_0^{1/2} \left(\frac{dr^2}{X} + d\theta^2 + \frac{X}{G} \sin^2\theta d\phi^2 \right)$$

$X = r^2 - 2mr + a^2 \quad = 0$ outer & inner horizon

$$\Pi_c \equiv \prod_{I=0}^{3} \cosh\delta_I \ , \qquad \Pi_s \equiv \prod_{I=0}^{3} \sinh\delta_I$$

$$G = r^2 - 2mr + a^2\cos^2\theta$$

$$\mathcal{A} = \frac{2ma\sin^2\theta}{G} \left[(\Pi_c - \Pi_s) r + 2m\Pi_s \right] d\phi$$

$$\Delta_0 = \prod_{I=0}^{3} (r + 2m\sinh^2\delta_I) + 2a^2\cos^2\theta [r^2 + mr \sum_{I=0}^{3} \sinh^2\delta_I + 4m^2(\Pi_c - \Pi_s)\Pi_s - 2m^2 \sum_{I<J<K} \sinh^2\delta_I \sinh^2\delta_J \sinh^2\delta_K] + a^4\cos^4\theta \ .$$

$$G_4 M = \frac{1}{4} m \sum_{I=0}^{3} \cosh 2\delta_I \ ,$$ Mass

$$G_4 Q_I = \frac{1}{4} m \sinh 2\delta_I \ , \ (I = 0, 1, 2, 3)$$ Four charges

$$G_4 J = ma(\Pi_c - \Pi_s) \ ,$$ Angular momentum

Special cases:

- $\delta_I = \delta$ Kerr-Newman
- & $a = 0$ Reisner-Nordström
- $\delta_I = 0$ Kerr
- & $a = 0$ Schwarzschild

$\delta_I \to \infty$ $m \to 0$ w/m $\exp(2\delta_I)$-finite
extremal (BPS) black hole

Or equivalently: m, a, δ_I (I=0,1,2,3)

Thermodynamics of outer & inner horizons

suggestive of weakly interacting 2-dim. CFT
w/ ``left-'' & ``right-moving'' excitations

M.C., Youm '96
M.C., Larsen '97

Area of outer horizon $S_+ = S_L + S_R$

[Area of inner horizon $S_- = S_L - S_R$]

$$S_L = \pi\, m^2\,(\Pi_c + \Pi_s)$$

$$S_R = \pi\, m\sqrt{m^2 - a^2}\,(\Pi_c - \Pi_s)$$

Surface gravity (inverse temperature) of

outer horizon $\beta_H = ½\,(\beta_L + \beta_R)$

[inner horizon $\beta_- = ½\,(\beta_L - \beta_R)$]

$$\beta_L = 2\pi\, m\,(\Pi_c - \Pi_s)$$

$$\beta_R = \frac{2\pi\, m^2}{\sqrt{m^2 - a^2}}\,(\Pi_c + \Pi_s)$$

Similar structure for angular velocities Ω_+, Ω_- and momenta J_+, J_-.

Depend only on four parameters: m, a, $\Pi_c \equiv \prod_{I=0}^{3} \cosh\delta_I$, $\Pi_s \equiv \prod_{I=0}^{3} \sinh\delta_I$

Shown more recently, all independent of the warp factor Δ_o !

M.C., Larsen '11

M.C., Larsen 1112.4856

Motivation for Subtracted Geometry

Focus on the black hole "by itself" → enclose the black hole in a box (à la Gibbons Hawking) → an equilibrium system w/ conformal symmetry manifest *

The box chosen to lead to a ``mildly'' modified geometry changing only the warp factor $\Delta_0 \rightarrow \Delta$
[maintains the same horizon thermodynamic quantities]

*Determination of new warp factor $\Delta_0 \rightarrow \Delta$

Via scalar field wave eq.: separable & radial part solved by hypergeometric functions w/ $SL(2,R)^2$ → unique Δ

M.C., Larsen 1112.4856

Subtracted geometry for rotating four-charge black holes

$$ds_4^2 = -\Delta_0^{-1/2} G(dt + \mathcal{A})^2 + \Delta_0^{1/2} \left(\frac{dr^2}{X} + d\theta^2 + \frac{X}{G} \sin^2\theta d\phi^2 \right)$$

$$X = r^2 - 2mr + a^2 \ ,$$

$$G = r^2 - 2mr + a^2 \cos^2\theta \ ,$$

$$\mathcal{A} = \frac{2ma\sin^2\theta}{G} \left[(\Pi_c - \Pi_s) r + 2m\Pi_s \right] d\phi \ ,$$

$$\Delta_0 = \prod_{I=0}^{3} (r + 2m\sinh^2\delta_I) + 2a^2\cos^2\theta [r^2 + mr \sum_{I=0}^{3} \sinh^2\delta_I + 4m^2(\Pi_c - \Pi_s)\Pi_s - 2m^2 \sum_{I<J<K} \sinh^2\delta_I \sinh^2\delta_J \sinh^2\delta_K] + a^4\cos^4\theta \ .$$

$$\Delta_0 \to \Delta = (2m)^3 r(\Pi_c^2 - \Pi_s^2) + (2m)^4 \Pi_s^2 - (2m)^2 (\Pi_c - \Pi_s)^2 a^2 \cos^2\theta$$

Comments: while $\Delta_0 \sim r^4$, $\Delta \sim r$ (not asymptotically flat!)
subtracted geometry depends only on four parameters:

m, a, $\Pi_c \equiv \prod_{I=0}^{3} \cosh\delta_I \ , \quad \Pi_s \equiv \prod_{I=0}^{3} \sinh\delta_I$

Matter fields (gauge potentials and scalars)

M.C., Gibbons 1201.0601

Scalars: $\eta_1 = \eta_2 = \eta_3 \equiv \eta,\ \chi_1 = \chi_2 = \chi_3 \equiv \chi,$

Running dilaton: $e^\eta = \frac{(2m)^2}{\sqrt{\Delta}}, \qquad \chi = \frac{a\,(\Pi_c - \Pi_s)}{2m} \cos\theta$

Gauge potentials: $A^1 = A^2 = A^3 \equiv A.$

$$A^0 = \frac{(2m)^4 a\,(\Pi_c - \Pi_s)}{\Delta} \sin^2\theta d\phi + \frac{(2ma)^2 \cos^2\theta\,(\Pi_c - \Pi_s)^2 + (2m)^4 \Pi_c \Pi_s}{(\Pi_c^2 - \Pi_s^2)\,\Delta} dt,$$

$$A = \frac{2m\cos\theta}{\Delta} \left(\left[\Delta - (2ma)^2 (\Pi_c - \Pi_s)^2 \sin^2\theta\right] d\phi - 2ma\,(2m\Pi_s + r(\Pi_c - \Pi_s))\,dt \right),$$

Magnetic frame

Non-extremal black hole immersed in constant magnetic field

w/ $\Delta = (2m)^3 (\Pi_c^2 - \Pi_s^2) r + (2m)^4 \Pi_s^2 - (2ma)^2 (\Pi_c - \Pi_s)^2 \cos^2\theta$

Brief Remarks:

Asymptotic geometry of subtracted geometry is of Lifshitz-type w/ a deficit angle:

$$ds^2 = -(\frac{R}{R_0})^{2p} dt^2 + B^2 dR^2 + R^2 \big(d\theta^2 + \sin^2\theta^2 d\phi^2)\big)$$

p=3, B=4

→ black hole in an ``asymptotically conical box"

M.C., Gibbons 1201.0601

→ the box conformal to AdS_2 x S^2
(confining, but softer than AdS)

→ lift on a circle: locally AdS_3 x S^2 M.C., Larsen 1112.4856

Conformal symmetry of AdS_3 promoted to Virasoro algebra of dual CFT_2 , à la Brown-Hennaux →
reproduces entropy of 4D black holes à la Cardy

Origin of subtracted geometry

i. Subtracted geometry – as a scaling limit of near-horizon black hole w/ three-large charges Q, (mapped on m, a, Π_c, Π_s)

M.C., Gibbons 1201.0601

$$\epsilon \to 0 \qquad \tilde{r} = r\epsilon, \quad \tilde{t} = t\epsilon^{-1}, \quad \tilde{m} = m\epsilon\,, \quad \tilde{a} = a\epsilon$$

$$2\tilde{m}\sinh^2\tilde{\delta} \equiv Q = 2m\epsilon^{-1/3}(\Pi_c^2 - \Pi_s^2)^{1/3}, \quad \sinh^2\tilde{\delta}_0 = \frac{\Pi_s^2}{\Pi_c^2 - \Pi_s^2}$$

ii. Subtracted geometry - as an infinite boost Harrison transformations on the original BH

M.C., Gibbons 1201.0601
Virmani 1203.5088
Sahay, Virmani 1305.2800
M.C., Guica, Saleem 1302.7032..

SO(1,1): $H \sim \begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix} \qquad \beta \to 1$

iii. Subtracted geometry – as turning off certain integration constants in harmonic functions of asymptotically flat black holes

Baggio, de Boer, Jottar, Mayerson 1210.7695
An, M.C., Papardimitriou 1602.0150

→ non-extremal black hole microscopic properties associated with its horizon are captured by a dual field theory of subtracted geometry

M.C., Larsen 1406.4536

Subtracted geometry [$\Delta_0 \rightarrow \Delta = A\,r + B\cos^2\theta + C$; A,B,C-horrendous]

also works for most general black holes of the STU Model

(specified by mass, four electric and four magnetics charges and angular momentum)

Chow, Compère 1310.1295;1404.2602

M.C., Larsen 1106.3341

All also works in parallel for subtracted geometry of most general five-dimensional black holes

(specified by mass, three charges and two angular momenta)

M.C., Youm 9603100

Further developments

Quantum aspects of subtracted geometries:

i) Quasi-normal modes - exact results for scalar fields
two damped branches → no black hole bomb

M.C., Gibbons 1312.2250, M.C., Gibbons, Saleem 1401.0544

ii) Entanglement entropy –minimally coupled scalar
M.C., Satz, Saleem 1407.0310

No time

iii) Vacuum polarization <φ^2> analytic expressions
at the horizon: static M.C., Gibbons, Saleem, Satz 1411.4658
rotating M.C., Satz, Saleem 1506.07189
outside & inside horizon: rotating M.C., Satz 1612.06766

iv) Thermodynamics of subtracted geometry

via Komar integral: M.C., Gibbons, Saleem 1412.5996

→ Systematic approach via variational principle

highligts

II. Thermodynamics via variational principle

An, M.C., Papadimitriou 1602.0150

Following lessons from AdS geometries

Heningson, Skenderis '98; Balasubramanian ,Kraus '99; deBoer,Verlinde[2] '99; Skenderis, Solodukhin '99…

achieved through an algorithmic procedure for subtracted geometry:

- Integration constants, parameterizing solutions of the eqs. of motion, separated into `normalizable' - free to vary & 'non-normalizable' modes – fixed
- Non-normalizable modes – fixed only up to transformations induced by local symmetries of the bulk theory (radial diffeomorphisms & gauge transf.)
- Covariant boundary term, S_{ct}, to the bulk action - determined by solving asymptotically the radial Hamilton-Jacobi eqn. →

Skenderis,Papadimitriou '04, Papadimitriou '05

- Total action $S+S_{ct}$ independent of the radial coordinate
- First class constraints of Hamiltonian formalism lead to conserved charges associated with Killing vectors.
- Conserved charges satisfy the first law of thermodynamics

- Identify normalizable and non-rormalizable modes

Introduce new coordinates:

Rescaled radial coord.: $\ell^4 r \leftarrow (2m)^3(\Pi_c^2 - \Pi_s^2) r + (2m)^4 \Pi_s^2 - (2ma)^2 (\Pi_c - \Pi_s)^2,$

Rescaled time: $\frac{k}{\ell^3} t \leftarrow \frac{1}{(2m)^3(\Pi_c^2 - \Pi_s^2)} t,$

Trade four parameters m, a, Π_c, Π_s for:

$$\ell^4 r_\pm = (2m)^3 m(\Pi_c^2 + \Pi_s^2) - (2ma)^2(\Pi_c - \Pi_s)^2 \pm \sqrt{m^2 - a^2}(2m)^3(\Pi_c^2 - \Pi_s^2)$$

$$\ell^3 \omega = 2ma(\Pi_c - \Pi_s), \qquad B = 2m,$$

r_+, r_-, ω - normalizable modes

B - non-renormalizable mode
(fixed up to bulk diffeomorphisms & global gauge symmetries)

`Vacuum' solution

obtained by turning off r_+, r_-, ω – three normalizable modes:

Asymptotically conical box – conformal to $AdS_2 \times S^2$

Asymptotically conical box

$$ds^2 = \sqrt{r}\left(\ell^2 \frac{dr^2}{r^2} - rk^2 dt^2 + \ell^2 d\theta^2 + \ell^2 \sin^2\theta d\phi^2\right)$$

$$e^\eta = \frac{B^2/\ell^2}{\sqrt{r}}, \qquad \chi = 0, \qquad A^0 = 0, \qquad A = B\cos\theta d\phi$$

Non-normalizable (fourth) mode B, along with ℓ and k, fixed up to radial diffeomorphism:

$$r \to \lambda^{-4} r \qquad k \to \lambda^3 k, \quad \ell \to \lambda\ell, \quad B \to B$$

and global U(1) symmetry:

$$e^\eta \to \mu^2 e^\eta, \quad \chi \to \mu^{-2}\chi, \quad A^0 \to \mu^3 A^0, \quad A \to \mu A, \quad ds^2 \to ds^2$$

which keep kB^3/ℓ^3 - fixed

- # Radial Hamiltonian formalism

to determine S_{ct}, to the bulk action S

Suitable radial coordinate u, such that constant-u slices Σ_u

$\Sigma_u \to \partial\mathcal{M}$ as $u \to \infty$.

Decomposition of the metric and gauge fields:

$$ds^2 = (N^2 + N_i N^i)du^2 + 2N_i du dx^i + \gamma_{ij} dx^i dx^j$$
$$A^L = a^\Lambda du + A_i^\Lambda dx^i,$$

Decomposition leads to the radial Lagrangian L w/ canonical momenta:

$$\pi^{ij} = \frac{\delta L}{\delta \dot{\gamma}_{ij}}$$
$$\pi_I = \frac{\delta L}{\delta \dot{\varphi}^I}$$
$$\pi^i_\Lambda = \frac{\delta L}{\delta \dot{A}_i^\Lambda}$$

w/ momenta conjugate to N, N_i, and a^Λ vanish →

First class constraints: $\mathcal{H} = \mathcal{H}^i = \mathcal{F}_\Lambda = 0,$

Hamiltonian:

$$H = \int \mathrm{d}^3\mathbf{x} \left(\pi^{ij} \dot{\gamma}_{ij} + \pi_I \dot{\varphi}^I + \pi^i_\Lambda \dot{A}^\Lambda_i \right) - L = \int \mathrm{d}^3\mathbf{x} \left(N\mathcal{H} + N_i \mathcal{H}^i + a^\Lambda \mathcal{F}_\Lambda \right)$$

First class constraints $\mathcal{H} = \mathcal{H}^i = \mathcal{F}_\Lambda = 0,$ - Hamilton Jacobi eqs.:

& Momenta as gradients of Hamilton's principal function $\mathcal{S}(\gamma, A^{\wedge}, \varphi^I)$:

$$\pi^{ij} = \frac{\delta \mathcal{S}}{\delta \gamma_{ij}}, \quad \pi^i_\Lambda = \frac{\delta \mathcal{S}}{\delta A^\Lambda_i}, \quad \pi_I = \frac{\delta \mathcal{S}}{\delta \varphi^I}.$$

deBoer,Verlinde^2 '99,…Skenderis, Papadimitriou '04,…

Solve asymptotically (for `vacuum' asymptotic solutions) for

$\mathcal{S}(\gamma, A^{\wedge}, \varphi^I)$ $= - S_{ct}$!

$\mathcal{S}(\gamma, A^{\wedge}, \varphi^I)$ coincides with the on-shell action, up to terms that remain finite as $\Sigma_u \to \partial\mathcal{M}$. In particular, divergent part of $\mathcal{S}[\gamma, A^{\wedge}, \varphi^I]$ coincides with that of the on-shell action.

- Hamiltonian Formalism with ``Renormalized'' Action

Covariant S_{ct} calculated for vacuum asymptotic solutions
(conformal to AdS_2 x S^2 geometry)

$$S_{\rm ct} = -\frac{1}{\kappa_4^2}\int \mathrm{d}^3\mathbf{x}\sqrt{-\gamma}\,\frac{B}{4}e^{\eta/2}\left(\frac{4-\alpha}{B^2} + (\alpha-1)e^{-\eta}R[\gamma] - \frac{\alpha}{2}e^{-2\eta}F_{ij}F^{ij} + \frac{1}{4}e^{-4\eta}F^0_{ij}F^{0ij}\right)$$

$$S_{\rm reg} = S_4 + S_{\rm ct} \qquad S_{\rm ren} = \lim_{r\to\infty} S_{\rm reg}$$

Finite – Independent of r

Renormalized canonical momenta:

$$\Pi^{ij} = \pi^{ij} + \frac{\delta S_{\rm ct}}{\delta\gamma_{ij}}, \quad \Pi^i_\Lambda = \pi^i_\Lambda + \frac{\delta S_{\rm ct}}{\delta A_i^\Lambda}, \quad \Pi_I = \pi_I + \frac{\delta S_{\rm ct}}{\delta\varphi^I}$$

- # Conserved Charges

Conserved currents, a consequence of the first class constraints

$F_\Lambda = 0$ Conserved currents for gauge potentials: $D_i \Pi^i = 0, \quad D_i \Pi^{0i} = 0.$

Conserved charges:

$$Q_4^{(m)} = -\int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\, \Pi^t, \qquad Q_4^{0(e)} = -\int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\, \Pi^{0t}$$

$$= \frac{3B}{4G_4} \qquad\qquad = \frac{\ell^4}{4G_4 B^3}\left(\sqrt{r_+ r_-} + \omega^2 \ell^2\right)$$

$\mathcal{H}_i = 0$ Conserved currents: $-2D_j \Pi_i^j + \Pi_\eta \partial_i \eta + \Pi_\chi \partial_i \chi + F_{ij}^0 \Pi^{0j} + F_{ij} \Pi^j \approx 0$

Conserved ``charges'':

$$\mathcal{Q}[\zeta] = \int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\, \left(2\Pi_j^t + \Pi^{0t} A_j^0 + \Pi^t A_j\right) \zeta^j$$

Asymptotic Killing vector ζ_i

Mass: $M_4 = -\int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\, \left(2\Pi^t{}_t + \Pi_0^t A_t^0 + \Pi^t A_t\right) = \frac{\ell k}{8G_4}(r_+ + r_-)$

Angular Momentum: $J_4 = \int_{\partial\mathcal{M}\cap C} \mathrm{d}^2\mathbf{x}\, (2\Pi^t{}_\phi + \Pi_0^t A_\phi^0 + \Pi^t A_\phi) = -\frac{\omega \ell^3}{2G_4}$

• Thermodynamic relations and the first law

Free Energy: $I_4 = S^{\mathrm{E}}_{\mathrm{ren}} = -S_{\mathrm{ren}} = \beta_4 \mathcal{G}_4 = \frac{\beta_4 \ell k}{8G_4}\left((r_- - r_+) + 2\omega^2\ell^2\sqrt{\frac{r_-}{r_+}}\right)$

Quantum statistical relation: $\mathcal{G}_4 = M_4 - T_4 S_4 - \Omega_4 J_4 - \Phi^{0(e)} Q^{0(e)}$

First law: $\mathrm{d}M_4 - T_4 \mathrm{d}S_4 - \Omega_4 \mathrm{d}J_4 - \Phi_4^{0(e)} \mathrm{d}Q_4^{0(e)} - \Phi_4^{(m)} \mathrm{d}Q_4^{(m)} = 0$

Smarr's Formula: $M_4 = 2S_4 T_4 + 2\Omega_4 J_4 + Q_4^{0(e)} \Phi_4^{0(e)} + Q_4^{(m)} \Phi_4^{(m)}$

Varying parameters: r_+, r_-, ω, and B, k, ℓ subject to $\mathrm{k}B^3/\ell^3$ –fixed

⟺ original parameters m, a, Π_c, Π_s & a scaling parameter

III. Holography via 2D Einstein-Maxwell-Dilaton

M.C., Papadimitriou 1608.07018

4D STU fields can be consistently Kaluza-Klein reduced on S^2 by one-parameter family of Ansätze:

$$e^{-2\eta} = e^{-2\psi} + \lambda^2 B^2 \sin^2\theta, \qquad \chi = \lambda B \cos\theta$$

$$e^{-2\eta} A^0 = e^{-2\psi} A^{(2)} + \lambda B^2 \sin^2\theta d\phi, \quad A + \chi A^0 = B\cos\theta d\phi$$

$$e^{\eta} ds_4^2 = ds_2^2 + B^2 \left(d\theta^2 + \frac{\sin^2\theta}{1+\lambda^2 B^2 e^{2\psi}\sin^2\theta} (d\phi - \lambda A^{(2)})^2 \right)$$

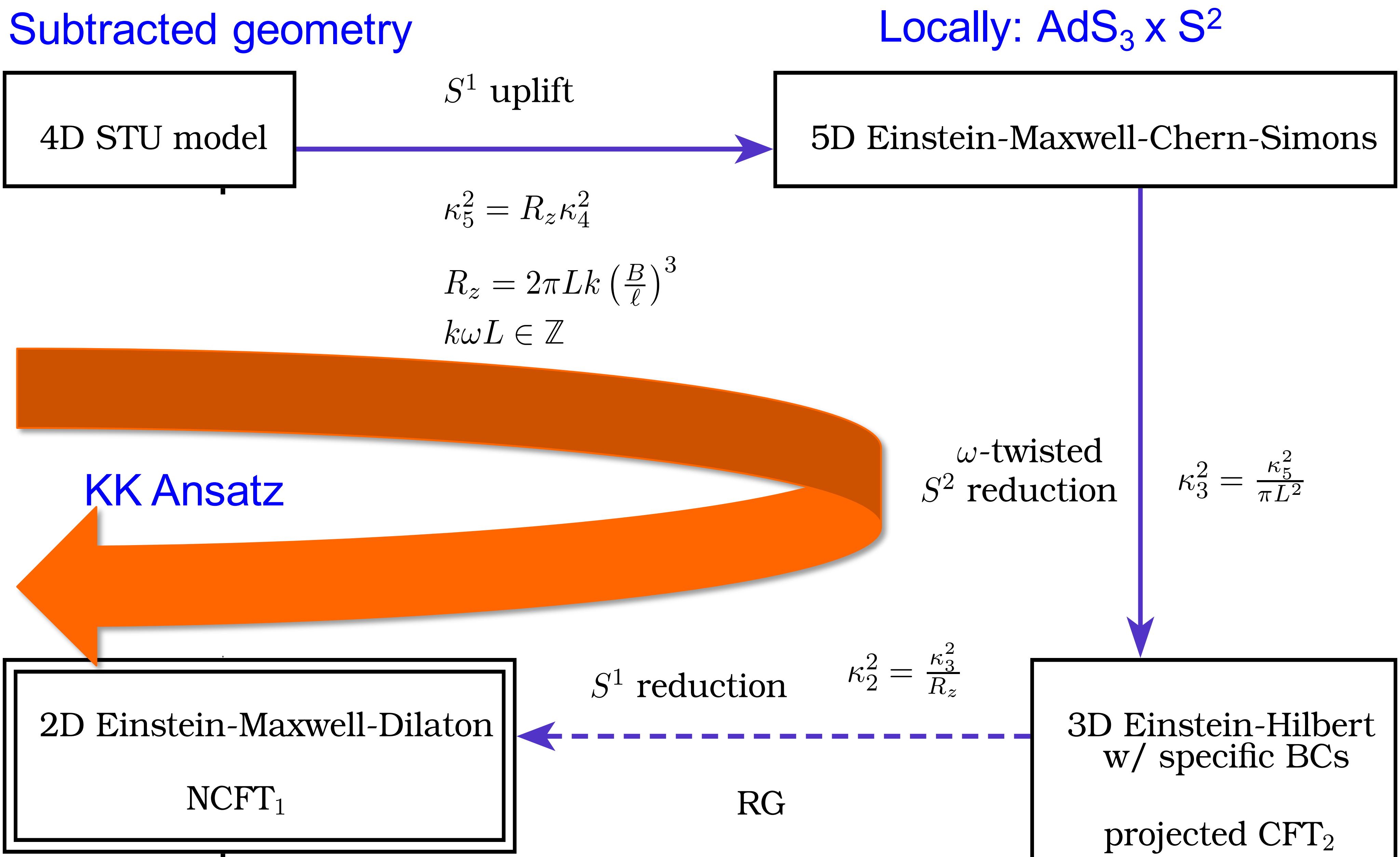
$ds_2{}^2$, ψ, $A^{(2)}$ -fields of 2D Einstein-Maxwell-Dilaton Gravity:

$$S_{2\mathrm{D}} = \frac{1}{2\kappa_2^2} \left(\int \mathrm{d}^2\mathbf{x}\sqrt{-g}\, e^{-\psi} \left(R[g] + \frac{2}{L^2} - \frac{1}{4} e^{-2\psi} F_{ab}F^{ab} \right) + \int \mathrm{d}t\sqrt{-\gamma}\, e^{-\psi} 2K \right)$$

B = 2L; λ-independent

$\lambda = \omega\ell^3/B^3$ rotational parameter of subtracted geometry

Web of Theories



General solution of 2D EMD Gravity – running dilaton

Feffeman-Graham gauge: $ds^2 = du^2 + \gamma_{tt}(u,t)dt^2, \quad A_u = 0$

Anaytic general solution:

$$e^{-\psi} = \beta(t)e^{u/L}\sqrt{\left(1 + \frac{m - \beta'^2(t)/\alpha^2(t)}{4\beta^2(t)}L^2 e^{-2u/L}\right)^2 - \frac{Q^2L^2}{4\beta^4(t)}e^{-4u/L}}$$

$$\sqrt{-\gamma} = \frac{\alpha(t)}{\beta'(t)}\partial_t e^{-\psi}$$

$$A_t = \mu(t) + \frac{\alpha(t)}{2\beta'(t)}\partial_t \log\left(\frac{4L^{-2}e^{2u/L}\beta^2(t) + m - \beta'^2(t)/\alpha^2(t) - 2Q/L}{4L^{-2}e^{2u/L}\beta^2(t) + m - \beta'^2(t)/\alpha^2(t) + 2Q/L}\right)$$

Leading asymptotic behavior:

$$\gamma_{tt} = -\alpha^2(t)e^{2u/L} + \mathcal{O}(1), \quad e^{-\psi} \sim \beta(t)e^{u/L} + \mathcal{O}(e^{-u/L}), \quad A_t = \mu(t) + \mathcal{O}(e^{-2u/L})$$

running dilaton

- Arbitrary functions *α(t), β(t)* and *μ(t)* identified with the sources of the corresponding dual operators
- 4D uplift results in asymptotically conformally $AdS_2 \times S^2$ subtracted geometries, generalized to include arbitrary time-dependent sources

Repeat Radial Hamiltonian Formalism in 2D

Radial ADM decomposition: $ds^2 = (N^2 + N_t N^t)du^2 + 2N_t du dt + \gamma_{tt} dt^2$

Countertern Action: $S_{\rm ct} = -\frac{1}{\kappa_2^2} \int \mathrm{d}t \sqrt{-\gamma}\, L^{-1} \left(1 - u_o L \Box_t\right) e^{-\psi}$

Renormalized one-point functions: $\mathcal{T} = 2\widehat{\pi}^t_t, \quad \mathcal{O}_\psi = -\widehat{\pi}_\psi, \quad \mathcal{J}^t = -\widehat{\pi}^t$

$$\widehat{\pi}^t_t = \frac{1}{2\kappa_2^2} \lim_{u\to\infty} e^{u/L} \left(\partial_u e^{-\psi} - e^{-\psi} L^{-1}\right)$$

$$\widehat{\pi}^t = \lim_{u\to\infty} \frac{e^{u/L}}{\sqrt{-\gamma}} \pi^t$$

$$\widehat{\pi}_\psi = -\frac{1}{\kappa_2^2} \lim_{u\to\infty} e^{u/L} e^{-\psi} \left(K - L^{-1}\right)$$

Explicit one-point functions:

$$\mathcal{T} = -\frac{L}{2\kappa_2^2}\left(\frac{m}{\beta} - \frac{\beta'^2}{\beta\alpha^2}\right), \quad \mathcal{J}^t = \frac{1}{\kappa_2^2}\frac{Q}{\alpha}, \quad \mathcal{O}_\psi = \frac{L}{2\kappa_2^2}\left(\frac{m}{\beta} - \frac{\beta'^2}{\beta\alpha^2} - 2\frac{\beta'\alpha'}{\alpha^3} + 2\frac{\beta''}{\alpha^2}\right)$$

Ward Identities: $\partial_t\mathcal{T} - \mathcal{O}_\psi\partial_t \log\beta = 0, \qquad \mathcal{D}_t\mathcal{J}^t = 0$

Conformal anomaly: $\mathcal{T} + \mathcal{O}_\psi = \frac{L}{\kappa_2^2}\left(\frac{\beta''}{\alpha^2} - \frac{\beta'\alpha'}{\alpha^3}\right) = \frac{L}{\kappa_2^2\alpha}\partial_t\left(\frac{\beta'}{\alpha}\right) \equiv \mathcal{A}$

Exact generating function ($\mathcal{T} = \frac{\delta S_{\rm ren}}{\delta\alpha}, \quad \mathcal{O}_\psi = \frac{\beta}{\alpha}\frac{\delta S_{\rm ren}}{\delta\beta}, \quad \mathcal{J}^t = -\frac{1}{\alpha}\frac{\delta S_{\rm ren}}{\delta\mu}$):

$$S_{\rm ren}[\alpha,\beta,\mu] = -\frac{L}{2\kappa_2^2}\int \mathrm{d}t\left(\frac{m\alpha}{\beta} + \frac{\beta'^2}{\beta\alpha} + \frac{2\mu Q}{L}\right) \quad + \quad S_{\rm global}$$

Asymptotic symmetries and conserved charges

Asymptotic symmetries: subset of Penrose-Brown-Henneaux (PBH) transformations, diffeomorphisms and gauge transformations, preserving the Fefferman-Graham gauge, that preserve boundary conditions of the solution:

$$\delta_{\rm PBH}\alpha = \partial_t(\varepsilon\alpha) + \alpha\sigma/L, \qquad \delta_{\rm PBH}\beta = \varepsilon\beta' + \beta\sigma/L, \qquad \delta_{\rm PBH}\mu = \partial_t(\varepsilon\mu + \varphi)$$

$\delta_{\rm PBH}\,(\text{sources}) = 0$ → constrain functions $\varepsilon(t)$, $\sigma(t)$ and $\varphi(t)$ in term of two constants $\xi_{1,2}$

Conserved Charges: boundary terms obtained by varying the action with respect to the asymptotic symmetries (and Ward identities) →

U(1)xU(1): $$\mathcal{Q}_1 = -\left(\beta\mathcal{T} - \frac{L}{2\kappa_2^2}\frac{\beta'^2}{\alpha^2}\right) = \frac{mL}{2\kappa_2^2}, \qquad \mathcal{Q}_2 = \alpha\mathcal{J}^t = \frac{Q}{\kappa_2^2}.$$

3D perspective: two copies of the Virasoro algebra with the Brown-Henneaux central charge. Only $L^{\pm}_0$ are realized non-trivially in 2D.

Effective action as Schwarzian derivative

Under PBH transformations the sources:

$$\alpha = e^{\sigma}(1+\varepsilon'+\varepsilon\sigma')+\mathcal{O}(\varepsilon^2), \quad \beta = e^{\sigma}(1+\varepsilon\sigma')+\mathcal{O}(\varepsilon^2), \quad \mu = \varphi'+\varepsilon'\varphi'+\varepsilon\varphi''+\mathcal{O}(\varepsilon^2),$$

prime - derivative with respect to t

Inserting these expressions in the renormalized action

$$S_{\text{ren}}[\alpha,\beta,\mu] = -\frac{L}{2\kappa_2^2}\int \mathrm{d}t \left(\frac{m\alpha}{\beta}+\frac{\beta'^2}{\beta\alpha}+\frac{2\mu Q}{L}\right)$$

and absorbing total derivative terms in S_{global} one obtains:

$$S_{\text{ren}} = \frac{L}{\kappa_2^2}\int \mathrm{d}t\,(\{\tau,t\}-m/2)+S_{\text{global}}, \qquad \sigma = \log\tau'$$

$$\{\tau,t\} = \frac{\tau'''}{\tau'}-\frac{3}{2}\frac{\tau''^2}{\tau'^2}$$

Schwarzian derivative

c.f., Sadchev, Ye, Kitaev '93,… Almeheiri, Polochinski '14;
Maldacena, Stanford, Yang '16, Engelsoy, Merens, Verlinde '16,…

Constant dilaton solutions and AdS_2 holography

c.f., Strominger '98, …Castro, Grumiller, Larsen, McNees '08,…
Compère, Song, Strominger '13,…Castro, Song'14,…

Holography depends on the structure of non-extremal constant dilaton solutions and choice of boundary conditions →

Provided systematic holographic dictionary for each choice
M.C., Papadimitriou 1608.07018
no time

Note: non-extremal running dilaton solution → (Q=mL/2)
extremal running-dilaton solution
with RG flow to IR fixed point
extremal constant dilaton solution → (VEV of irrelevant scalar op.)
non-extremal constant dilaton branch (`Coulomb phase')
(does not lift into subtracted geometry)

Summary/Outlook with focus on AdS_2 Holography

- Provided consistent KK Ansätze that allow us to uplift any solution of 2D EMD gravity to 4D STU solutions, which are non-extremal 4D black holes, asymptotically (conformally) $AdS_2 \times S^2$ – subtracted geometry.
  [Works also for 5D solutions asymptotically (conformally) $AdS_2 x S^3$.]

- 2D EMD gravity has a well defined UV fixed point, described by a sector of 2D CFT.

- Constructed holographic dictionary of 2D EMD gravity theory obtained by an S^2 reduction of 4D STU subtracted geometry – runing dilaton solution as well as constant dilaton solutions.

- Many aspects of the holographic description are generic and should apply to generic 2D dilaton gravity theories.

Thank you!

&

Congratulations Gary, and to many more productive, scientific contributions!