A two-phase model for magma flowing in a volcanic conduit

Céline Acary-Robert [1], Didier Bresch [1], Alain Burgisser [2], Marielle Collombet[2], Gladys Narbona-Reina[3]

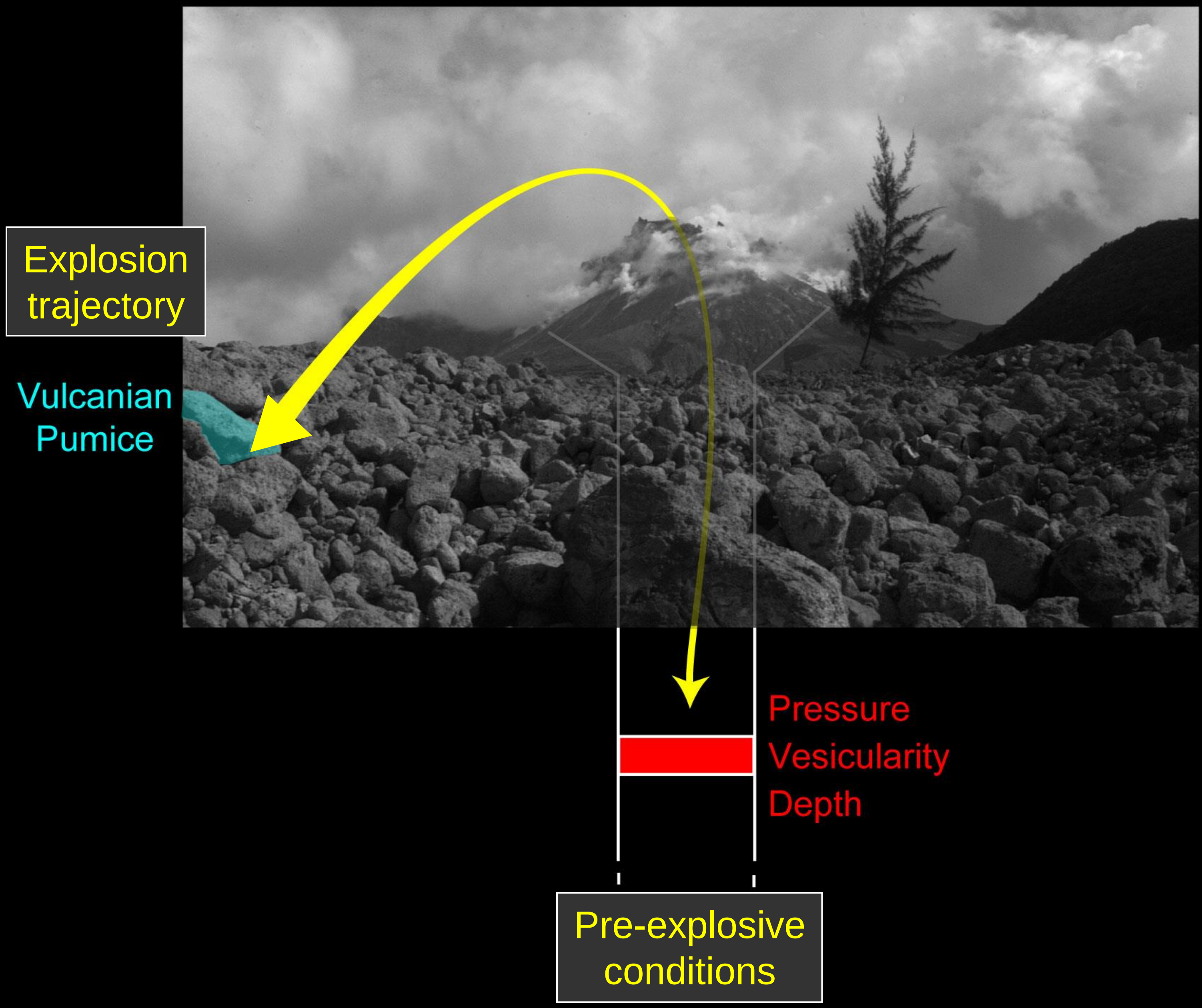
[1] LAMA, Université Savoie Mont Blanc - CNRS, Chambéry, France
[2] ISTerre, Université Savoie Mont Blanc - CNRS, Chambéry, France
[3] Dpto. Matemática Aplicada I, Universidad de Sevilla

VULCANIAN EXPLOSIONS

ANAK KRAKATAU VOLCANO, INDONESIA

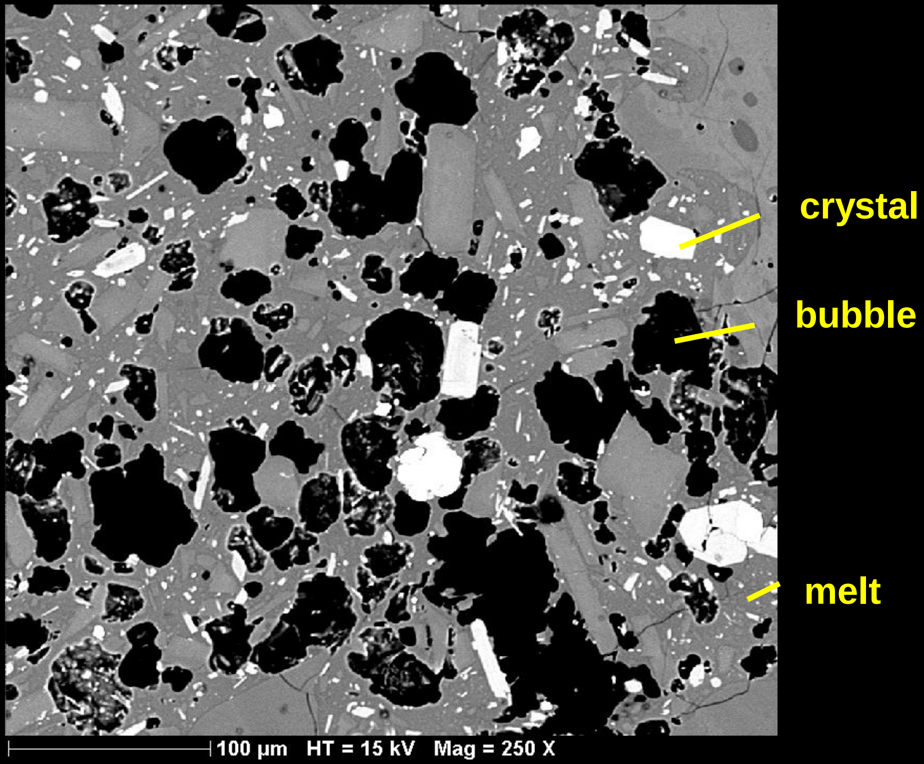
PRINCIPLE

Nature



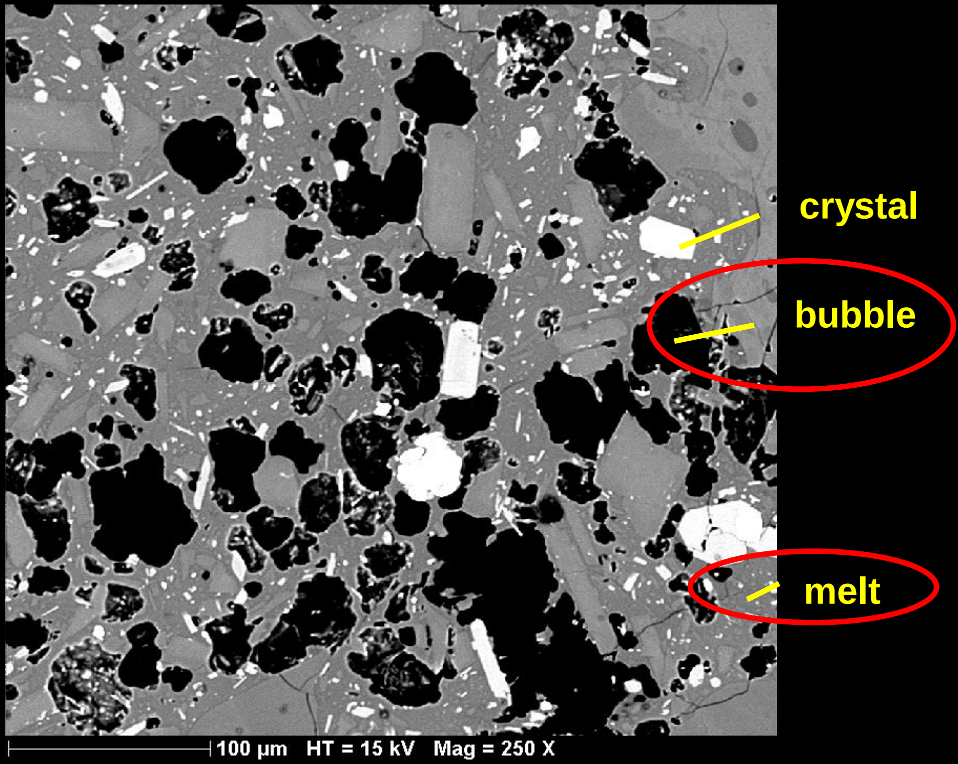
INSIDE A PIECE OF MAGMA

Quenched piece of magma



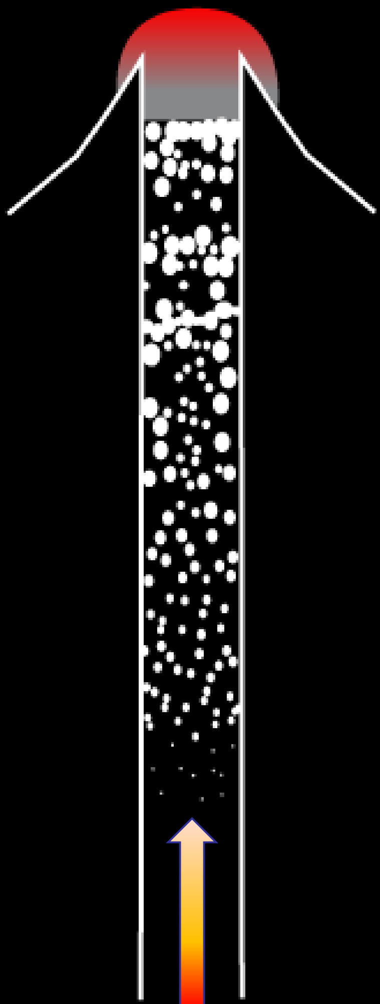
INSIDE A PIECE OF MAGMA

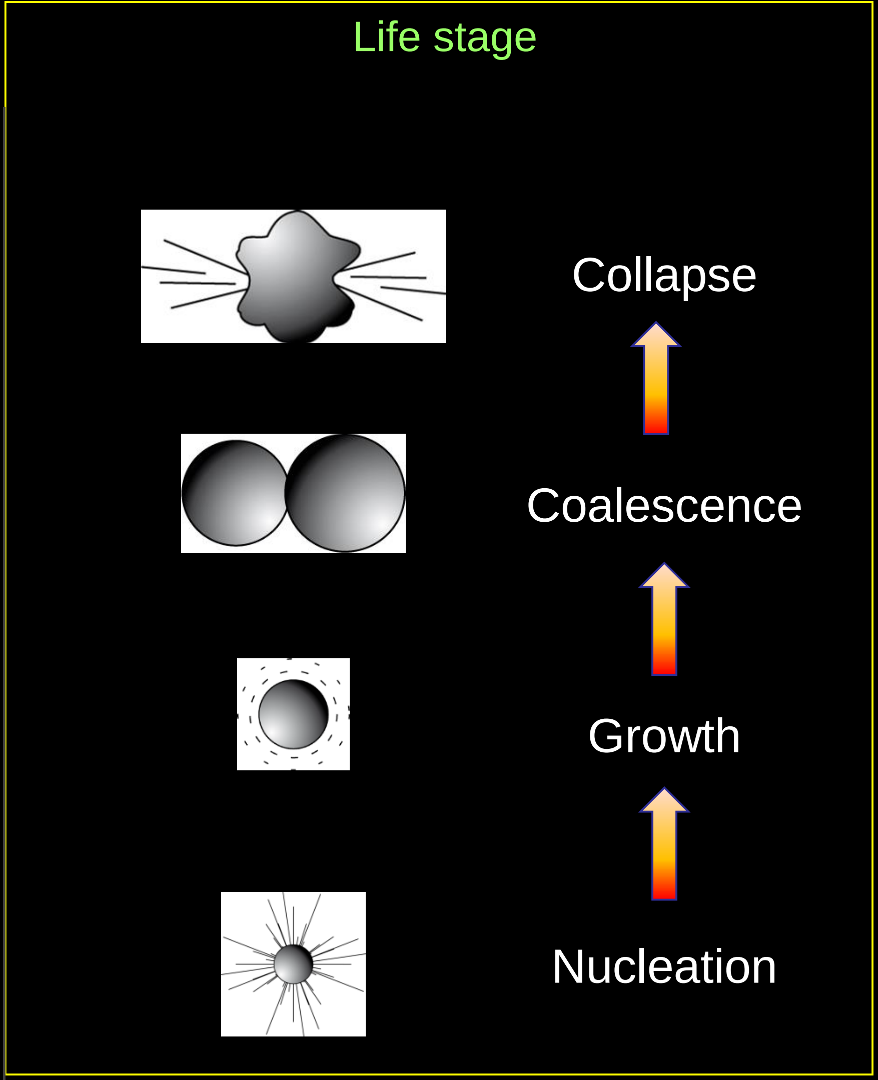
Quenched piece of magma

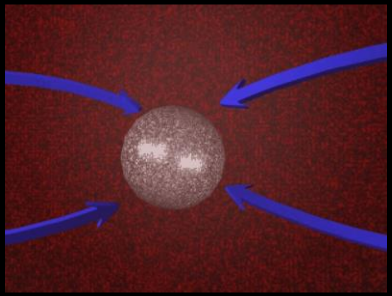


Model with ***fluid dynamics*** and ***two phases***

LIFE CYCLE OF BUBBLES IN MAGMAS



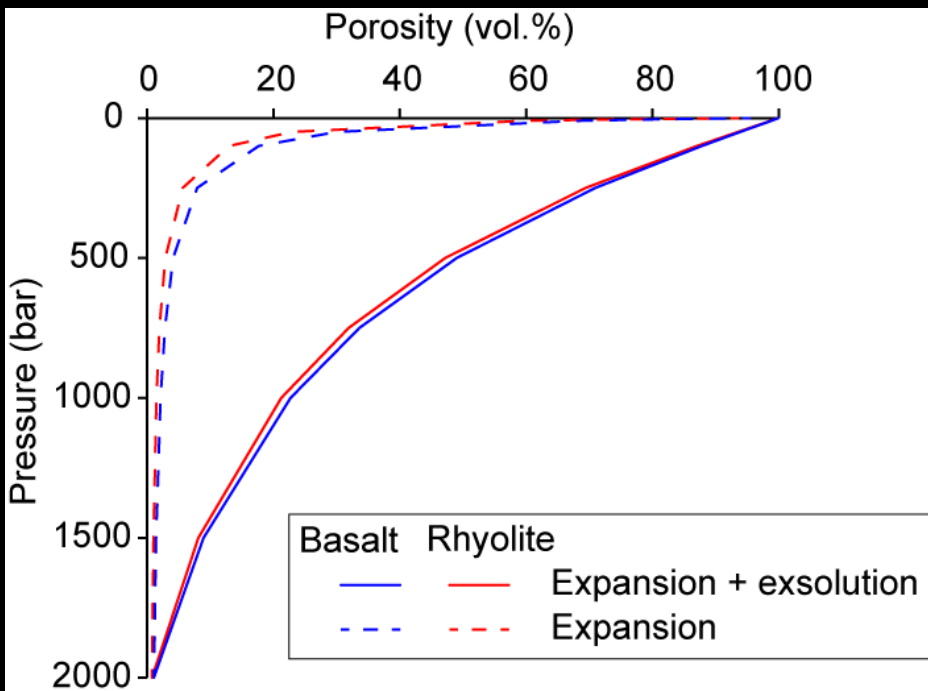




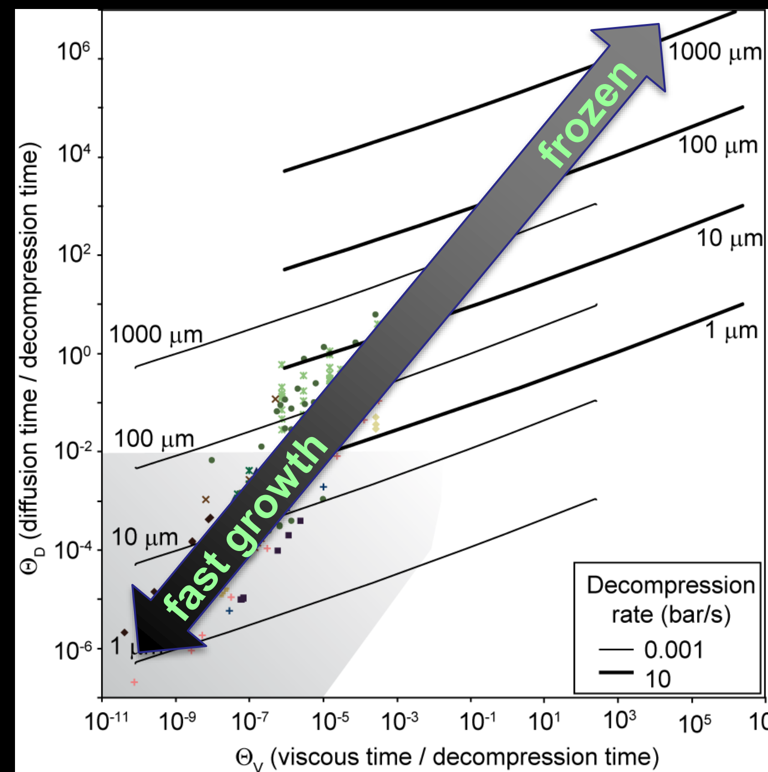
BUBBLE GROWTH

Mechanisms:

gas expansion + water diffusion



- water diffusion in melt +



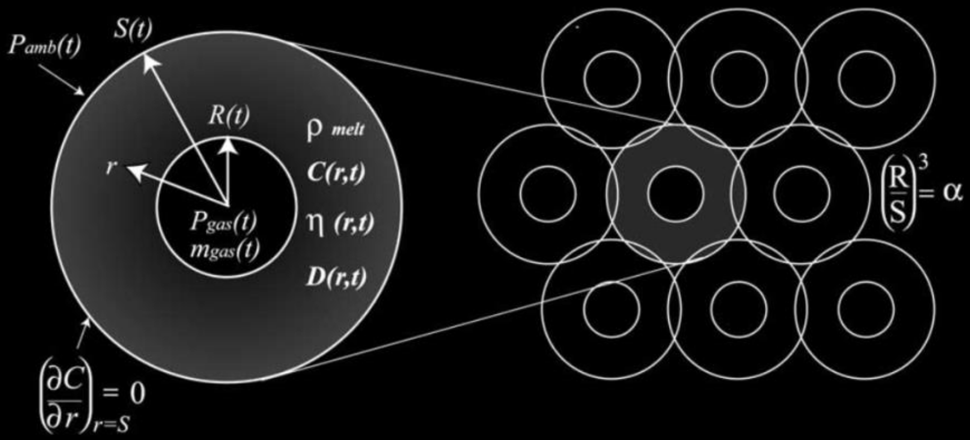
- melt viscosity +

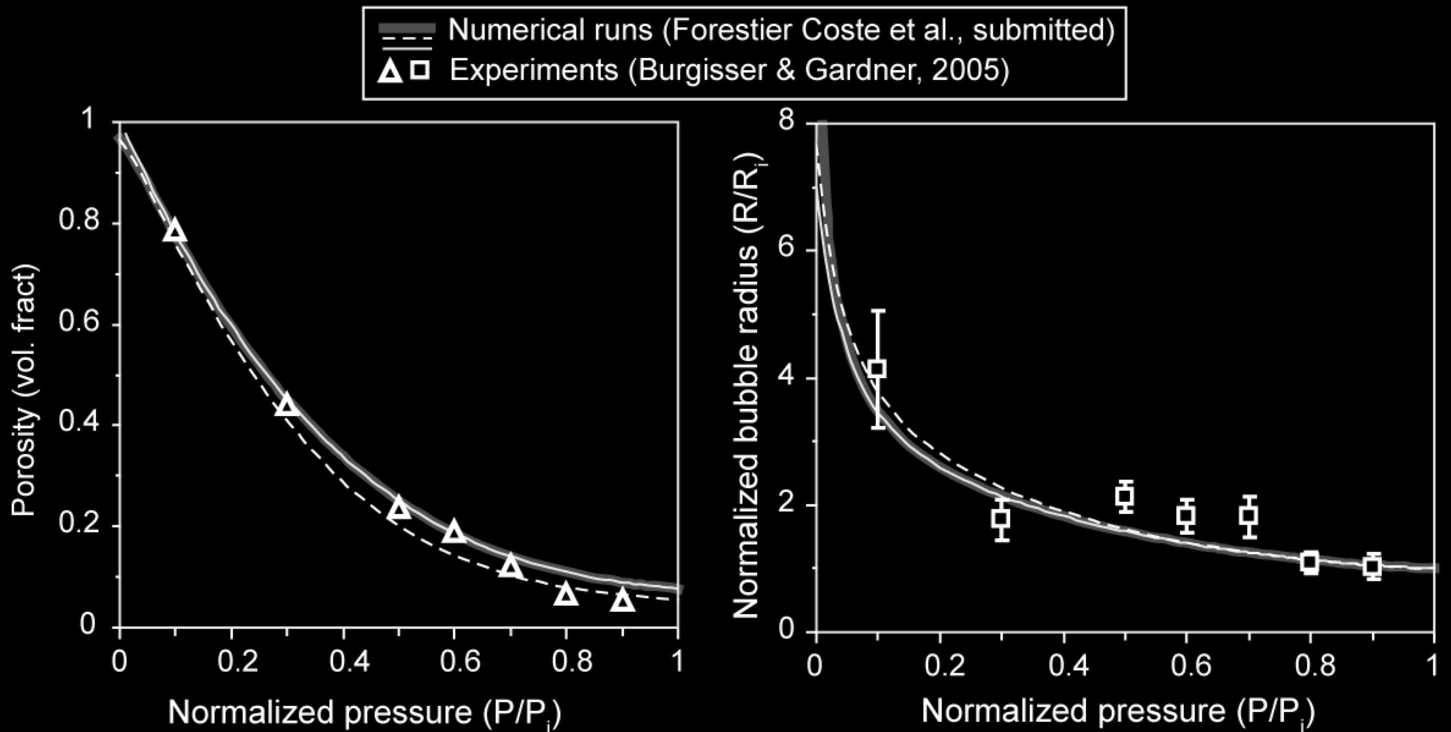
LAGRANGIAN BUBBLE GROWTH

Experiment

Model

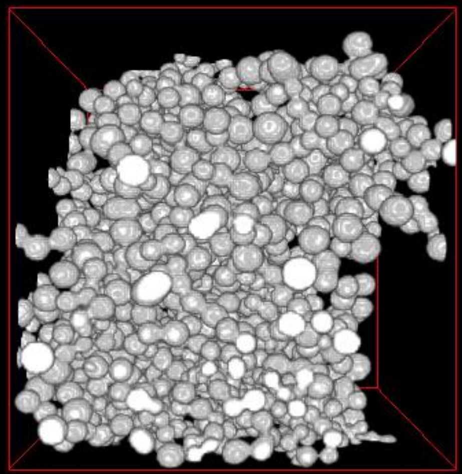
Lots of Lagrangian models
Few Eulerian models



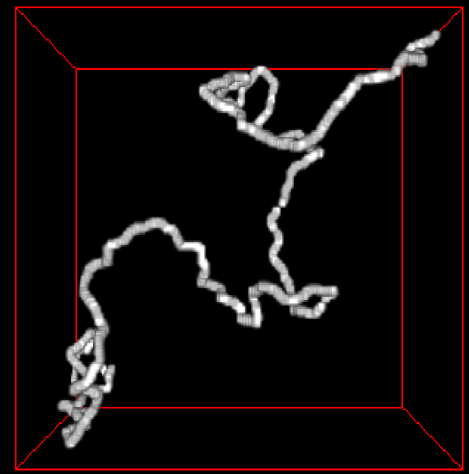


Forestier Coste et al. (2012)

BUBBLE COALECENCE & COLLAPSE



Connected volume

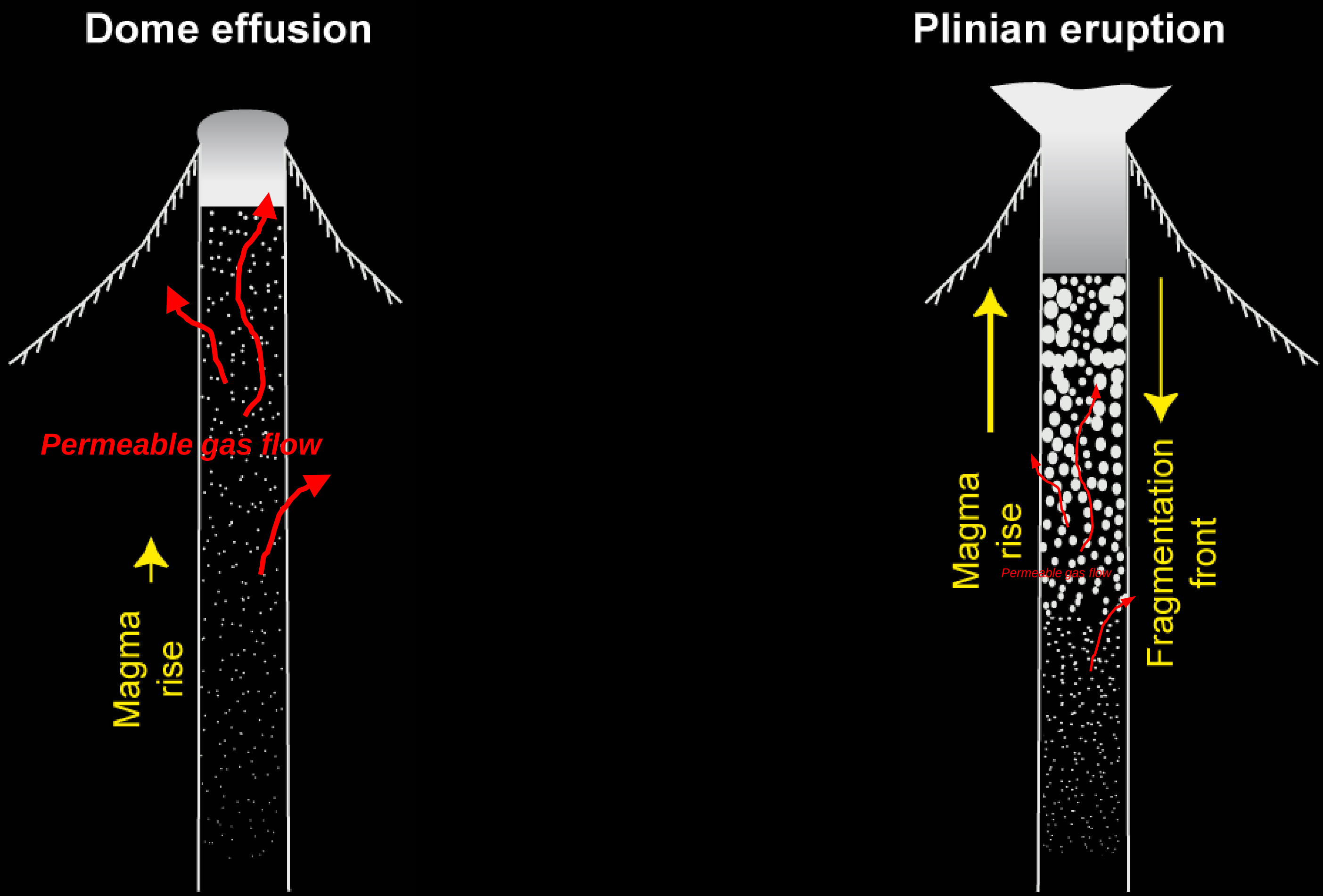


1 path connecting top & base

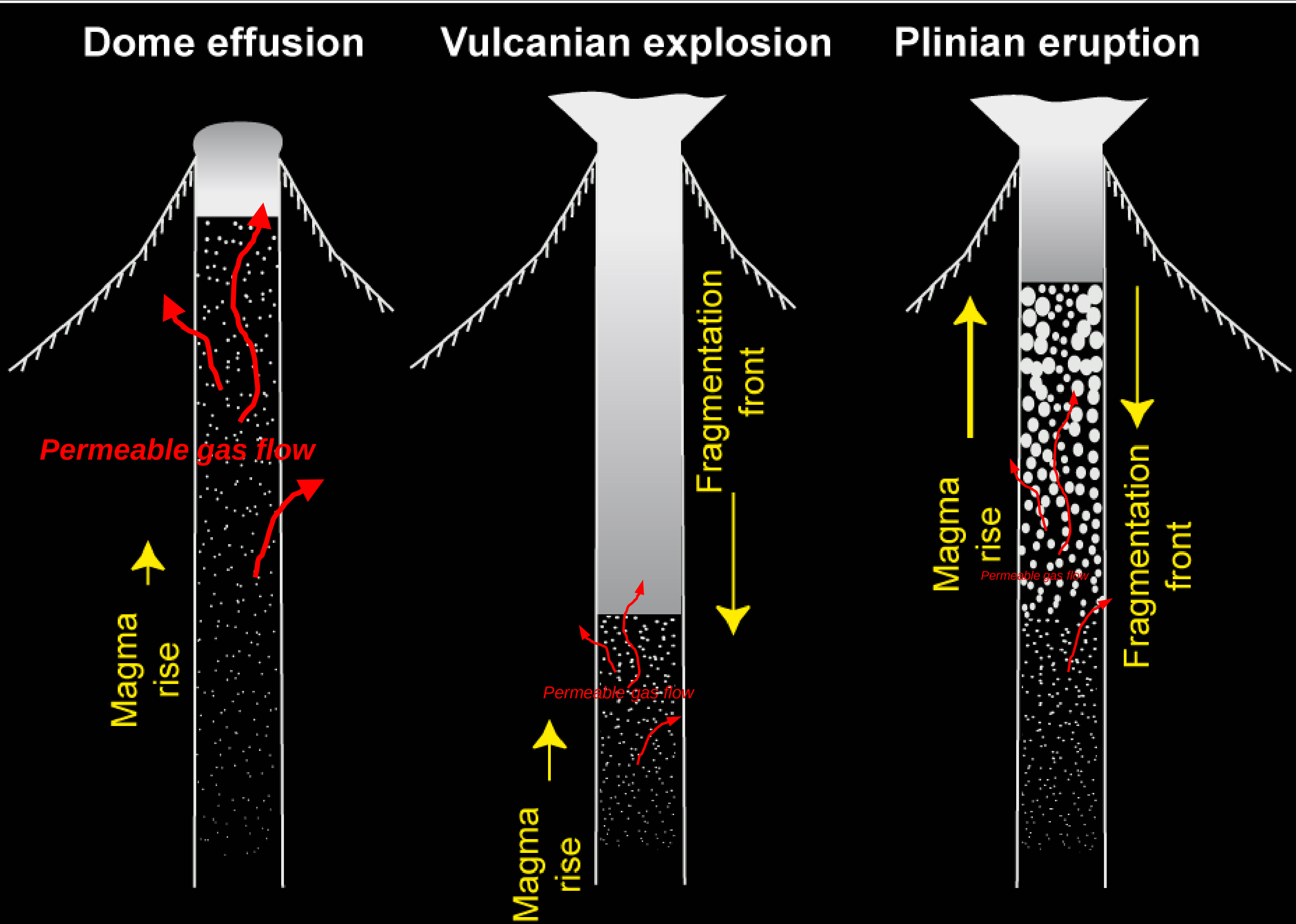
3D images of experimental magmas (1 mm across)

Many bubbles connect with each other → gas velocity ≠ melt velocity

EULERIAN BUBBLE GROWTH:
TRANSITION BETWEEN ERUPTIVE REGIMES



EULERIAN BUBBLE GROWTH:
TRANSITION BETWEEN ERUPTIVE REGIMES



NAVIER – STOKES EQUATIONS

Incompressible

$$\begin{cases} \partial_t \vec{u} + \nabla \cdot (\vec{u} \otimes \vec{u}) + \nabla p + \mathcal{D} = 0 & \textit{momentum conservation} \\ \nabla \cdot \vec{u} = 0 & \textit{mass conservation} \end{cases}$$

$\vec{u}, p$

$$\mathcal{D} = -\nabla \cdot [\eta\, (\nabla \vec{u} + \nabla^t \vec{u})]$$

NAVIER – STOKES EQUATIONS

Incompressible

$$\begin{cases} \partial_t \vec{u} + \nabla \cdot (\vec{u} \otimes \vec{u}) + \nabla p + \mathcal{D} = 0 \\ \nabla \cdot \vec{u} = 0 \end{cases}$$

$\vec{u}, p$

$$\mathcal{D} = -\nabla \cdot [\eta\, (\nabla \vec{u} + \nabla^t \vec{u})]$$

Compressible

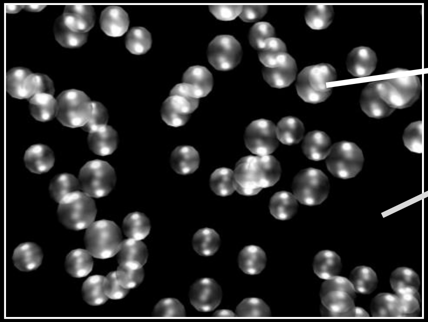
$$\begin{cases} \partial_t (\rho \vec{u}) + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) + \nabla p + \mathcal{D} = 0 \\ \partial_t \rho \ + \nabla \cdot (\rho \vec{u}) = 0 \end{cases}$$

$\vec{u}, p$

$$\mathcal{D} = -\nabla(\lambda\, \nabla \cdot \vec{u}) - \nabla \cdot [\eta\, (\nabla \vec{u} + \nabla^t \vec{u})]$$

$p = c_p \rho$ $\qquad$ $c_p \in (0, \infty)$ given

TWO PHASE FLOWS



gas = phase +

liquid = phase -

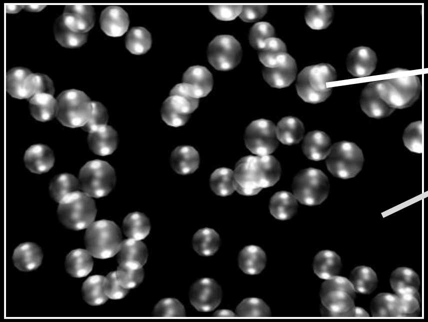
$$\varphi_+ + \varphi_- = 1$$

volume fractions sum to one

Two-phase compressible system

$$\vec{u}_\pm$$
$$\varphi_\pm$$
$$\rho_\pm$$
$$p_\pm$$

TWO PHASE FLOWS



gas = phase +

liquid = phase -

Two-phase compressible system

$\vec{u}_\pm$

$\varphi_\pm$

$\rho_\pm$

$p_\pm$

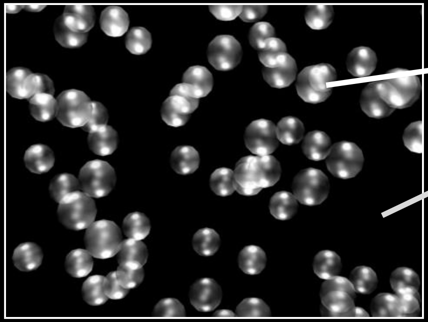
volume fractions sum to one

$$\left\{\begin{array}{l} \varphi_+ + \varphi_- = 1 \\ p_\pm = c_\pm \rho_\pm \qquad c_\pm \in (0,\infty) \text{ given} \\ \partial_t(\varphi_\pm\rho_\pm\vec{u}_\pm) + \nabla\cdot(\varphi_\pm\rho_\pm\vec{u}_\pm \otimes \vec{u}_\pm) + \varphi_\pm \nabla p_\pm + \mathcal{D}_\pm = F_\pm \\ \partial_t(\varphi_\pm\rho_\pm) + \nabla\cdot(\varphi_\pm\rho_\pm\vec{u}_\pm) = 0 \end{array}\right.$$

$$\mathcal{D}_\pm = -\nabla(\lambda_\pm \nabla\cdot\vec{u}_\pm) - \nabla\cdot[\eta_\pm(\nabla\vec{u}_\pm + \nabla^t\vec{u}_\pm)]$$

$F_\pm = c_F(\vec{u}_+ - \vec{u}_-) + I_\pm$ $\qquad c_F \in (0,\infty)$ given, $I_\pm$ are interface terms

TWO PHASE FLOWS



gas = phase +

liquid = phase -

Two-phase compressible system

$\vec{u}_\pm$

$\varphi_\pm$

$\rho_\pm$

$p_\pm$

8 unknowns

volume fractions sum to one

7 equ.

$$\begin{cases} \varphi_+ + \varphi_- = 1 \\ p_\pm = c_\pm \rho_\pm \qquad c_\pm \in (0,\infty) \text{ given} \\ \partial_t(\varphi_\pm \rho_\pm \vec{u}_\pm) + \nabla\cdot(\varphi_\pm\rho_\pm\vec{u}_\pm \otimes \vec{u}_\pm) + \varphi_\pm \nabla p_\pm + \mathcal{D}_\pm = F_\pm \\ \partial_t(\varphi_\pm\rho_\pm) + \nabla\cdot(\varphi_\pm\rho_\pm\vec{u}_\pm) = 0 \end{cases}$$

$$\mathcal{D}_\pm = -\nabla(\lambda_\pm \nabla\cdot\vec{u}_\pm) - \nabla\cdot[\eta_\pm(\nabla\vec{u}_\pm + \nabla^t\vec{u}_\pm)]$$

$$F_\pm = c_F(\vec{u}_+ - \vec{u}_-) + I_\pm$$

$c_F \in (0,\infty)$ given, $I_\pm$ are interface terms

TWO PHASE FLOWS

Two-phase compressible system

7 equu.

$$\begin{cases} \partial_t(\varphi_\pm \rho_\pm \vec{u}_\pm) + \nabla \cdot (\varphi_\pm \rho_\pm \vec{u}_\pm \otimes \vec{u}_\pm) + \varphi_\pm \nabla p_\pm + \mathcal{D}_\pm = F_\pm \\ \partial_t(\varphi_\pm \rho_\pm) + \nabla \cdot (\varphi_\pm \rho_\pm \vec{u}_\pm) = 0 \\ \varphi_+ + \varphi_- = 1 \\ p_\pm = c_\pm \rho_\pm \end{cases}$$

$\vec{u}_\pm$
$\varphi_\pm$
$\rho_\pm$
$p_\pm$

8 unknowns

1) Algebraic closure

$$p_+ = p_-$$

M. Ishii (1975), D.A. Drew & S.L. Passman (1998),
H.B. Stewart & B. Wendroff, *Two-phase flow: models and methods*, J. Comput.Phys. **56** (1984).

TWO PHASE FLOWS

Two-phase compressible system

7 equu.

$$\begin{cases} \partial_t(\varphi_\pm \rho_\pm \vec{u}_\pm) + \nabla \cdot (\varphi_\pm \rho_\pm \vec{u}_\pm \otimes \vec{u}_\pm) + \varphi_\pm \nabla p_\pm + \mathcal{D}_\pm = F_\pm \\ \partial_t(\varphi_\pm \rho_\pm) + \nabla \cdot (\varphi_\pm \rho_\pm \vec{u}_\pm) = 0 \\ \varphi_+ + \varphi_- = 1 \\ p_\pm = c_\pm \rho_\pm \end{cases}$$

$\vec{u}_\pm$
$\varphi_\pm$
$\rho_\pm$
$p_\pm$

8 unknowns

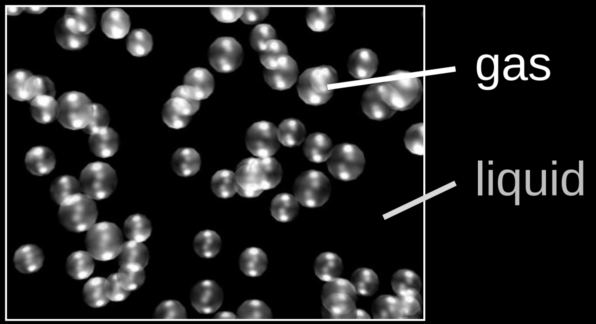
2) PDE closure

$$\partial_t \varphi_\pm + \vec{u}_\pm \nabla \varphi_\pm = \frac{p_+ - p_-}{\varepsilon}$$

Well-posed in specific cases such as constant η

Bresch, D, Hillairet, M. A compressible multifluid system with new physical relaxation terms. 2016. <hal-01262617>

TWO PHASE FLOWS IN VOLCANOLOGY



$$\varphi_g + \varphi_l = 1$$

volume fractions sum to one

Two-phase system in magma

$\overrightarrow{u_g}, \overrightarrow{u_l}$

φ_g, φ_l

ρ_g, ρ_l

p_g, p_l

TWO PHASE FLOWS IN VOLCANOLOGY

Two-phase compressible/incompressible system

5 equu.

$$\begin{cases} \partial_t(\varphi\rho_g\vec{u}_g) + \nabla\cdot\left(\varphi\rho_g\vec{u}_g\otimes\vec{u}_g\right) + \nabla(\varphi p_g) + \mathcal{D}_g = F \\ \partial_t((1-\varphi)\rho_l\vec{u}_l) + \nabla\cdot\left((1-\varphi)\rho_l\vec{u}_l\otimes\vec{u}_l\right) + \nabla((1-\varphi)p_l) + \mathcal{D}_l = -F \\ \partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l) = 0 \\ \partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g) = 0 \\ p_g = c_g\rho_g \qquad \rho_l \text{ given} \end{cases}$$

6 unknowns

$\vec{u_g}, \vec{u_l}$

φ

ρ_g

p_g, p_l

$$F = c_F\left(\vec{u}_g - \vec{u}_l\right) + I_\pm$$

gas/liquid drag

2) PDE closure

$$\partial_t\varphi_\pm + \vec{u}_\pm\nabla\varphi_\pm = \frac{p_+ - p_-}{\varepsilon}$$

TWO PHASE FLOWS IN VOLCANOLOGY

Two-phase compressible/incompressible system

5 equu.

work done by the interface pressure p_i

$$\begin{cases} \partial_t(\varphi\rho_g\vec{u}_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g\otimes\vec{u}_g) + \nabla(\varphi p_g) + \mathcal{D}_g = F - p_i\nabla\varphi \\ \partial_t((1-\varphi)\rho_l\vec{u}_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l\otimes\vec{u}_l) + \nabla((1-\varphi)p_l) + \mathcal{D}_l = -F + p_i\nabla(1-\varphi) \\ \partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l) = 0 \\ \partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g) = 0 \\ p_g = c_g\rho_g \qquad \rho_l \text{ given} \end{cases}$$

$F = c_F(\vec{u}_g - \vec{u}_l)$ gas/liquid drag

6 unknowns

$\overrightarrow{u_g}, \overrightarrow{u_l}$

φ

ρ_g

p_g, p_l

2) PDE closure

$$\partial_t\varphi_\pm + \vec{u}_\pm\nabla\varphi_\pm = \frac{p_+ - p_-}{\varepsilon}$$

TWO PHASE FLOWS IN VOLCANOLOGY

Two-phase compressible/incompressible system

5 equu.

work done by the interface pressure p_i

$$
\begin{cases}
\partial_t(\varphi\rho_g\vec{u}_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g\otimes\vec{u}_g) + \nabla(\varphi p_g) + \mathcal{D}_g = F - p_i\nabla\varphi \\
\partial_t((1-\varphi)\rho_l\vec{u}_l) + \nabla\cdot\big((1-\varphi)\rho_l\vec{u}_l\otimes\vec{u}_l\big) + \nabla\big((1-\varphi)p_l\big) + \mathcal{D}_l = -F + p_i\nabla(1-\varphi) \\
\partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l) = 0 \\
\partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g) = 0 \\
p_g = c_g\rho_g \qquad \rho_l \text{ given}
\end{cases}
$$

$F = c_F(\vec{u}_g - \vec{u}_l)$ gas/liquid drag

6 unknowns

$\overrightarrow{u_g}, \overrightarrow{u_l}$

φ

ρ_g

p_g, p_l

2) PDE closure

$p_i = p_g$?

$$\partial_t\varphi + \vec{u}_l\nabla\varphi = \frac{p_g - p_l}{\varepsilon}$$

?

$p_i = p_l$?

$$\partial_t\varphi + \vec{u}_g\nabla\varphi = \frac{p_g - p_l}{\varepsilon}$$

?

TWO PHASE FLOWS IN VOLCANOLOGY

Two-phase compressible/incompressible system

5 equu.

work done by the interface pressure p_i

$$\begin{cases} \partial_t(\varphi\rho_g\vec{u}_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g\otimes\vec{u}_g) + \nabla(\varphi p_g) + \mathcal{D}_g = F - p_i\nabla\varphi \\ \partial_t((1-\varphi)\rho_l\vec{u}_l) + \nabla\cdot\big((1-\varphi)\rho_l\vec{u}_l\otimes\vec{u}_l\big) + \nabla\big((1-\varphi)p_l\big) + \mathcal{D}_l = -F + p_i\nabla(1-\varphi) \\ \partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l) = 0 \\ \partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g) = 0 \\ p_g = c_g\rho_g \qquad \rho_l \text{ given} \end{cases}$$

$$F = c_F(\vec{u}_g - \vec{u}_l)$$

6 unknowns

$\vec{u}_g, \vec{u}_l$

φ

ρ_g

p_g, p_l

2) PDE closure

$$\partial_t\varphi + \vec{u}_l\nabla\varphi = \frac{p_g - p_l}{\varepsilon} \qquad ?$$

$\vec{u}_l \rightarrow p_i = p_g$

$$\partial_t\varphi + \vec{u}_g\nabla\varphi = \frac{p_g - p_l}{\varepsilon} \qquad ?$$

Guillemaud (2007),
Gallouët, Hérard & N. Seguin (2004) Numerical modelling of two-phase flows using the two-fluid two-pressure approach. *Mathematical Models and Methods in Applied Sciences*, 14:663–700.

TWO PHASE FLOWS IN VOLCANOLOGY

Two-phase compressible/incompressible system

5 equ.

$$\begin{cases} \partial_t(\varphi\rho_g\vec{u}_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g\otimes\vec{u}_g) + \varphi\nabla p_g + \mathcal{D}_g = F \\ \partial_t((1-\varphi)\rho_l\vec{u}_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l\otimes\vec{u}_l) + \nabla\big((1-\varphi)p_l\big) + p_g\nabla\varphi + \mathcal{D}_l = -F \\ \partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l) = 0 \\ \partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g) = 0 \\ p_g = c_g\rho_g \qquad \rho_l \text{ given} \end{cases}$$

$$F = c_F\big(\vec{u}_g - \vec{u}_l\big)$$

6 unknowns

$\overrightarrow{u_g}, \overrightarrow{u_l}$

φ

ρ_g

p_g, p_l

2) PDE closure

$$\partial_t\varphi + \vec{u}_l\nabla\varphi = \frac{p_g - p_l}{\varepsilon} \qquad ?$$

$\vec{u}_l \rightarrow p_i = p_g$

$$\partial_t\varphi + \vec{u}_g\nabla\varphi = \frac{p_g - p_l}{\varepsilon} \qquad ?$$

Guillemaud (2007),
Gallouët, Hérard & N. Seguin (2004) Numerical modelling of two-phase flows using the two-fluid two-pressure approach. *Mathematical Models and Methods in Applied Sciences*, 14:663–700.

TWO PHASE FLOWS IN VOLCANOLOGY

Two-phase compressible/incompressible system

5 equ.

$$\begin{cases} \partial_t(\varphi\rho_g\vec{u}_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g\otimes\vec{u}_g) + \varphi\nabla p_g + \mathcal{D}_g = F \\ \partial_t((1-\varphi)\rho_l\vec{u}_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l\otimes\vec{u}_l) + \nabla((1-\varphi)p_l) + p_g\nabla\varphi + \mathcal{D}_l = -F \\ \partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l) = 0 \\ \partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g) = 0 \\ p_g = c_g\rho_g \qquad \rho_l \text{ given} \end{cases}$$

$$F = c_F(\vec{u}_g - \vec{u}_l)$$

6 unknowns

$\overrightarrow{u_g}, \overrightarrow{u_l}$

φ

ρ_g

p_g, p_l

2) PDE closure

$$\partial_t\varphi + \vec{u}_l\nabla\varphi = \frac{p_g - p_l}{\varepsilon}$$

$$\vec{u}_l \rightarrow p_i = p_g$$

$$\partial_t\varphi + \nabla\cdot(\varphi\vec{u}_l) = \frac{p_g - p_l}{\varepsilon}$$

Two bubble growth mechanisms:

1. by gas expansion

Lensky et al. (2004) J. Volcanol. Geotherm. Res.

TWO PHASE FLOWS IN VOLCANOLOGY

Two-phase compressible/incompressible system

6 equ.

$$\begin{cases} \partial_t(\varphi\rho_g\vec{u}_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g\otimes\vec{u}_g) + \varphi\nabla p_g + \mathcal{D}_g = F \\ \partial_t((1-\varphi)\rho_l\vec{u}_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l\otimes\vec{u}_l) + \nabla((1-\varphi)p_l) + p_g\nabla\varphi + \mathcal{D}_l = -F \\ \partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l) = 0 \\ \partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g) = 0 \\ \partial_t\varphi + \nabla\cdot(\varphi\vec{u}_l) = \dfrac{p_g - p_l}{\varepsilon} \end{cases}$$

6 unknowns

$\overrightarrow{u_g}, \overrightarrow{u_l}$

φ

ρ_g

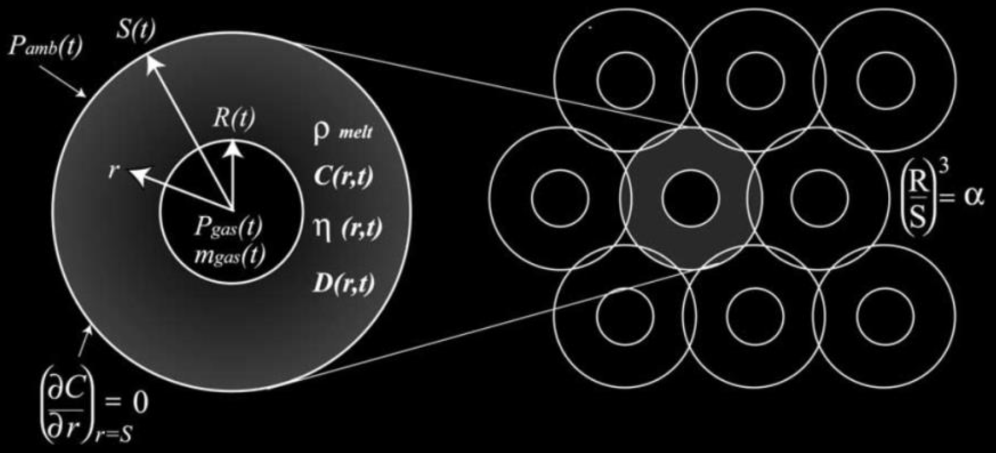
p_g, p_l

$$p_g = c_g\rho_g \qquad \rho_l \text{ given}$$

$$F = c_F(\vec{u}_g - \vec{u}_l)$$

Two bubble growth mechanisms:

1. by gas expansion
2. by mass addition



Lensky et al. (2004) J. Volcanol. Geotherm. Res.

TWO PHASE FLOWS IN VOLCANOLOGY

Two-phase compressible/incompressible system

6 equ.

$$\begin{cases} \partial_t(\varphi\rho_g\vec{u}_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g\otimes\vec{u}_g) + \varphi\nabla p_g + \mathcal{D}_g - \vec{u}_l R = F \\ \partial_t((1-\varphi)\rho_l\vec{u}_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l\otimes\vec{u}_l) + \nabla((1-\varphi)p_l) + p_g\nabla\varphi + \mathcal{D}_l + \vec{u}_l R = -F \\ \partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l) = -R \\ \partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g) = R \\ \partial_t\varphi + \nabla\cdot(\varphi\vec{u}_l) = \dfrac{p_g - p_l}{\varepsilon} \end{cases}$$

6 unknowns

$\overrightarrow{u_g}, \overrightarrow{u_l}$

φ

ρ_g

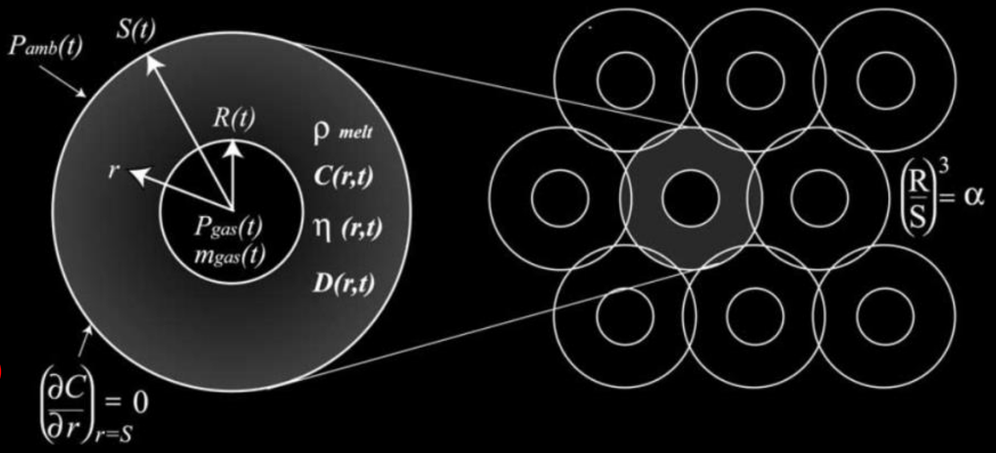
p_g, p_l

$$p_g = c_g\rho_g \qquad \rho_l \text{ given}$$

$$F = c_F(\vec{u}_g - \vec{u}_l)$$

Two bubble growth mechanisms:

1. by gas expansion
2. by mass addition



Mancini, Forestier-Coste, Burgisser, James, Castro (2016)
J. Volcanol. Geotherm. Res.

TWO PHASE FLOWS IN VOLCANOLOGY

Our system is well posed if it dissipates energy.

Calculation of the total energy of the system

$\vec{u}_g \cdot$ momentum conservation of g
$\vec{u}_l \cdot$ momentum conservation of l
} add

mass conservation of g
mass conservation of l

} combine to get:

$$\partial_t E + \nabla F = G$$

E = total energy (kinetic, potential, intern)
F = energy flux
G = source term; G<0 ↔ dissipative

$$\partial_t \left(\varphi \rho_g \frac{\left|\vec{u}_g\right|^2}{2} + \cdots \right) + \nabla \cdot \left(\varphi \rho_g \vec{u}_g \frac{\left|\vec{u}_g\right|^2}{2} + \cdots \right) = -(1-\varphi)\left(p_g - p_l\right)\frac{R}{\rho_l} - \cdots$$

TWO PHASE FLOWS IN VOLCANOLOGY

Our system is well posed if it dissipates energy.

Calculation of the total energy of the system

$\vec{u}_g \cdot$ momentum conservation of g
$\vec{u}_l \cdot$ momentum conservation of l
} add

mass conservation of g
mass conservation of l

} combine to get:

$$\partial_t E + \nabla F = G$$

E = total energy (kinetic, potential, intern)
F = energy flux
G = source term; G<0 ↔ dissipative

$$\partial_t\left(\varphi\rho_g\frac{|\vec{u}_g|^2}{2}+\cdots\right)+\nabla\cdot\left(\varphi\rho_g\vec{u}_g\frac{|\vec{u}_g|^2}{2}+\cdots\right)=-(1-\varphi)\left(p_g-p_l\right)\frac{R}{\rho_l}-\cdots$$

$$R = A(1-\varphi)\rho_l(C_l - k_h\sqrt{p_l})$$

A, k_h given
C_l new variable, the mass of which is conserved

TWO PHASE FLOWS IN VOLCANOLOGY

Our system is well posed if it dissipates energy.

Calculation of the total energy of the system

$\vec{u}_g \cdot$ momentum conservation of g
$\vec{u}_l \cdot$ momentum conservation of l
} add

mass conservation of g
mass conservation of l

} combine to get:

$$\partial_t E + \nabla F = G$$

E = total energy (kinetic, potential, intern)
F = energy flux
G = source term; G<0 ↔ dissipative

$$\partial_t \left(\varphi \rho_g \frac{\left|\vec{u}_g\right|^2}{2} + \cdots \right) + \nabla \cdot \left(\varphi \rho_g \vec{u}_g \frac{\left|\vec{u}_g\right|^2}{2} + \cdots \right) = -(1-\varphi)\left(p_g - p_l\right)\frac{R}{\rho_l} - \cdots$$

Gives us dissipations conditions such as if $R > 0$, then $p_g - p_l > 0$

TWO PHASE FLOWS IN VOLCANOLOGY

Our system is self-consistent if it dissipates energy.

Calculation of the total energy of the system

$\vec{u}_g \cdot$ momentum conservation of g
$\vec{u}_l \cdot$ momentum conservation of l
} add

mass conservation of g
mass conservation of l

} combine to get:

$$\partial_t E + \nabla F = G$$

E = total energy (kinetic, potential, intern)
F = energy flux
G = source term; G<0 ↔ dissipative

Gives us dissipations conditions such as if $R > 0$, then $p_l > x$

$$R = A(1-\varphi)\rho_l(C_l - k_h\sqrt{p_l})$$

A, k_h	given
C_l	new variable, the mass of which is conserved

TWO PHASE FLOWS IN VOLCANOLOGY

Work in progress:

- Asymptotic cases thanks to a drift flux formulation
- 1D case
- Equilibrium states
- Numerical resolution

TWO PHASE FLOWS IN VOLCANOLOGY

Work in progress:

- Asymptotic cases thanks to a drift flux formulation
- 1D case
- Equilibrium states
- Numerical resolution

Current conclusion:

very few published 1D systems to study conduit flow seem to be dissipative

END

$$\partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l u_l) = -R^{H_2O}$$

$$\partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g u_g) = R^{H_2O}$$

$$\partial_t((1-\varphi)\rho_l C_l) + \nabla\cdot((1-\varphi)\rho_l C_l u_l) = -R^{H_2O}$$

$$\partial_t((1-\varphi)\rho_l u_l) + \nabla\cdot((1-\varphi)\rho_l u_l\otimes u_l) + \nabla((1-\varphi)p_l) - p_g\nabla(1-\varphi) + \mathcal{D}_l - K_d\,\varphi(1-\varphi)(u_g-u_l) + (1-\varphi)\rho_l g + u_l R^{H_2O} = 0$$

$$\partial_t(\varphi\rho_g u_g) + \nabla\cdot(\varphi\rho_g u_g\otimes u_g) + \nabla(\varphi p_g) - p_g\nabla\varphi + \mathcal{D}_g + K_d\,\varphi(1-\varphi)(u_g-u_l) + \varphi\rho_g g - u_l R^{H_2O} = 0$$

$$\partial_t n + \nabla\cdot(n u_l) = R^c$$

$$\partial_t\varphi + \nabla\cdot(\varphi u_l) = \frac{3}{4\eta_l}\varphi(p_g-p_l) + \frac{\varphi R^c}{n}$$

$$\rho_g = c_0 p_g$$

$$M_g = \frac{4}{3}\pi R^3\rho_g$$

$$\varphi = \frac{4}{3}\pi R^3 n$$

$\nabla\eta_l = 0$, or $\eta_l = a_l\bar{p}^{b_l}$ with $a_l = 10^{10}$ and $b_l = -1.6$.

$$R^{H_2O} = A(1-\varphi)\rho_l(C_l - k_h\sqrt{p_l})$$

with $A = 10^{10}D$ for a diffusion coefficient $D\sim 10^{-10}, 10^{-12}$ and k_h is the water solubility constant.

TWO PHASE FLOWS IN VOLCANOLOGY

Two-phase compressible/incompressible system

6 equ.

$$\begin{cases} \partial_t(\varphi\rho_g\vec{u}_g) + \nabla\cdot\left(\varphi\rho_g\vec{u}_g\otimes\vec{u}_g\right) + \nabla(\varphi p_g) + \mathcal{D}_g = F \\ \partial_t((1-\varphi)\rho_l\vec{u}_l) + \nabla\cdot\left((1-\varphi)\rho_l\vec{u}_l\otimes\vec{u}_l\right) + \nabla((1-\varphi)p_l) + \mathcal{D}_l = -F \\ \partial_t((1-\varphi)\rho_l) + \nabla\cdot((1-\varphi)\rho_l\vec{u}_l) = 0 \\ \partial_t(\varphi\rho_g) + \nabla\cdot(\varphi\rho_g\vec{u}_g) = 0 \end{cases}$$

$$p_g = c_g\rho_g \qquad \rho_l \text{ given}$$

$$F = c_F\left(\vec{u}_g - \vec{u}_l\right) + \cdots$$

7 unknowns

$\overrightarrow{u_g}, \overrightarrow{u_l}$

φ

ρ_g

p_g, p_l

2) PDE closure

$$\partial_t\varphi_\pm + \vec{u}_\pm\nabla\varphi_\pm = \frac{p_+ - p_-}{\varepsilon}$$

TRANSITION BETWEEN ERUPTIVE REGIMES

$$\partial_t[\varphi\rho_g\overrightarrow{u_g}] + \nabla[\varphi\rho_g\overrightarrow{u_g}\otimes\overrightarrow{u_g}] + \varphi\nabla p_g + \mathcal{D}_g + f(Darcy) + \varphi\rho_g\vec{g} = 0$$

$$\begin{cases} \partial_t\vec{u} + \nabla\cdot(\vec{u}\otimes\vec{u}) + \nabla p - \nabla\cdot[\eta\,(\nabla\vec{u} + \nabla^t\vec{u})] = 0 \\ \nabla\cdot\vec{u} = 0 \end{cases}$$

$\vec{u}, p$

$$\mathcal{D}_g = -\nabla\cdot[\eta\,(\nabla\vec{u} + \nabla^t\vec{u})]$$

$$\begin{cases} \partial_t\vec{u} + \nabla\cdot(\vec{u}\otimes\vec{u}) + \nabla p + \mathcal{D}_g = 0 \\ \nabla\cdot\vec{u} = 0 \end{cases}$$

$\vec{u}, p$