3He topological fermionic fluids and their relativistic analogs

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LE STUDIUM, June 13, 2016

1. from topological defects to topological matter

* monopoles, half quantum vortices (Alice strings), skyrmions, etc.
* topological confinement, hybridization of topological defects
* Topological materials as defects & skyrmions in momentum space

2. normal 3He: from Fermi surface to flat bands

* Fermi surface as vortex in p-space
* from Fermi surface to flat band: route to room T_c superconductivity

3. superfluid 3He-A: Weyl point - Berry phase monopole

* chiral superfluids, Weyl semimetals, vacuum of Standard Model in massless phase
* topological origin of physical laws: chiral fermions, gauge fields & gravity as emergent phenomena
* chiral anomaly, chiral magnetic & chiral vortical effects

4. polar phase of 3He: Dirac nodal line & surface flat band

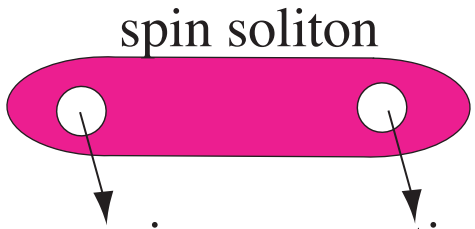
5. superfluid 3He-B as fully gapped topological superfluids: *p*-space skyrmion

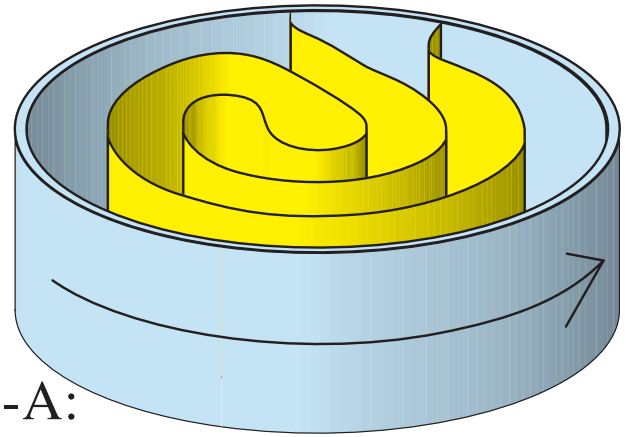
* vacuum of Standard Model in massive phase
* relativistic fermions on the surface
* fermion zero modes on vortices & strings
* Z_8 - Z_{16} problem & Standard Model

6. phases to be observed

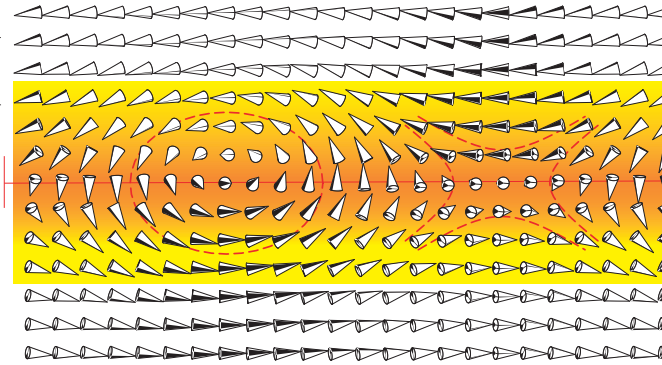
* phase with 8 Weyl points: analog of 4D graphene

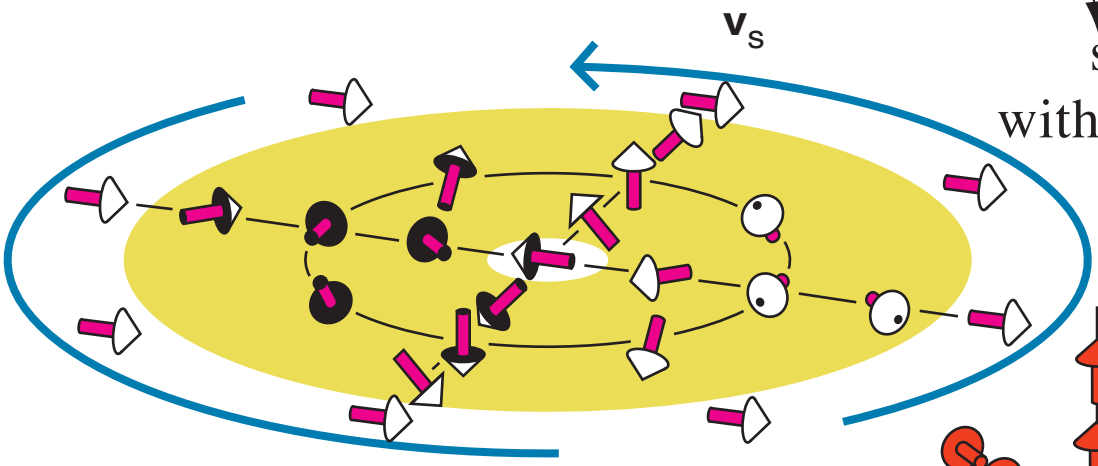
3He topological defects



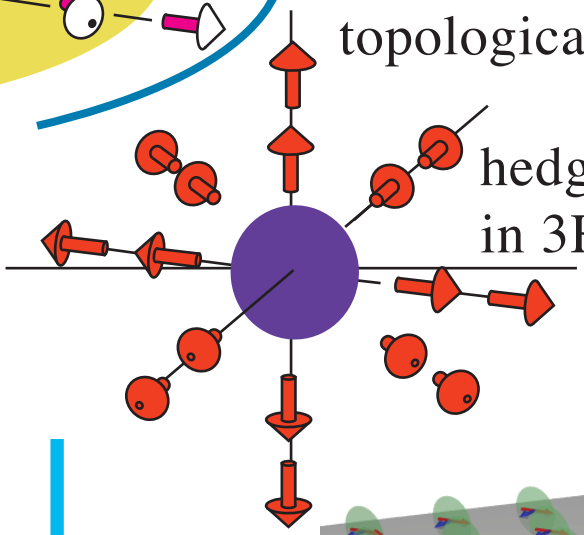


vortex sheet in 3He-A:
merons within
topological soliton

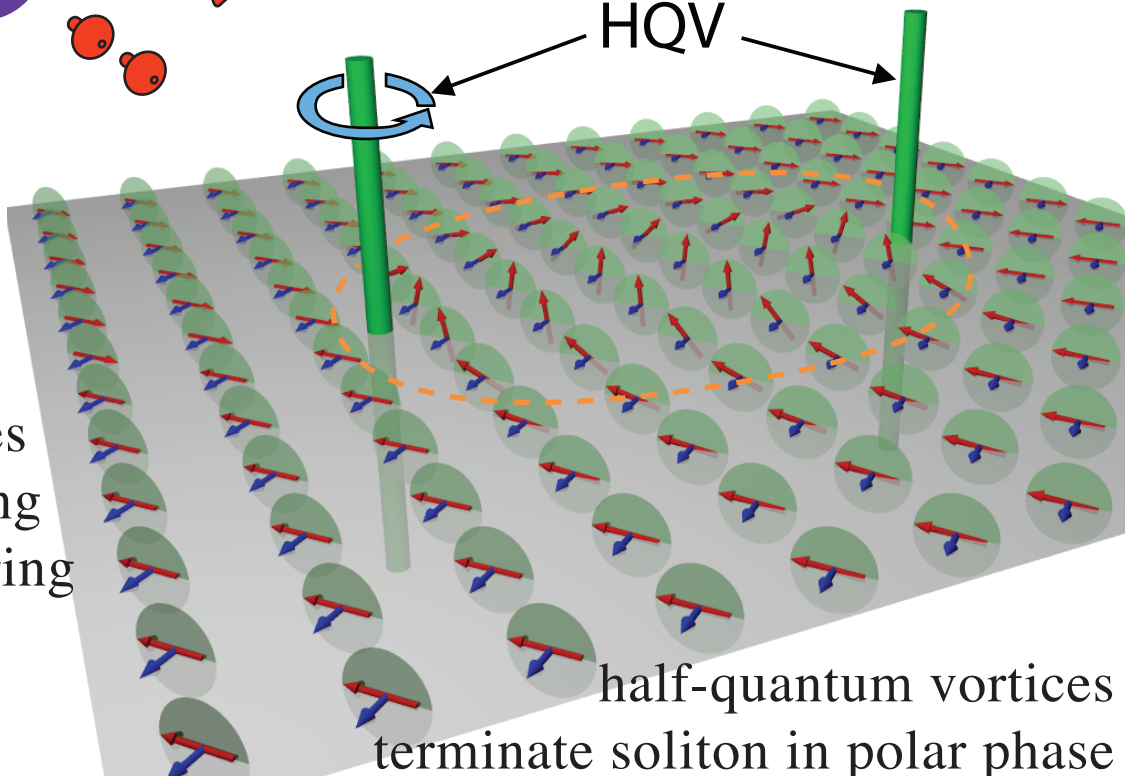


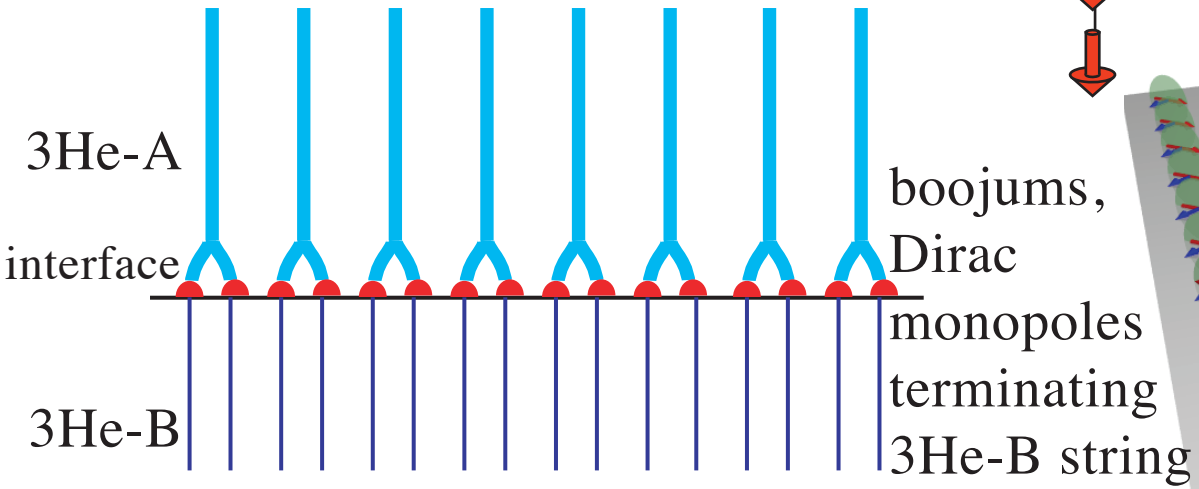


vortex-skyrmions
in chiral superfluid 3He-A
terminate on boojums



hedgehog
in 3He-A



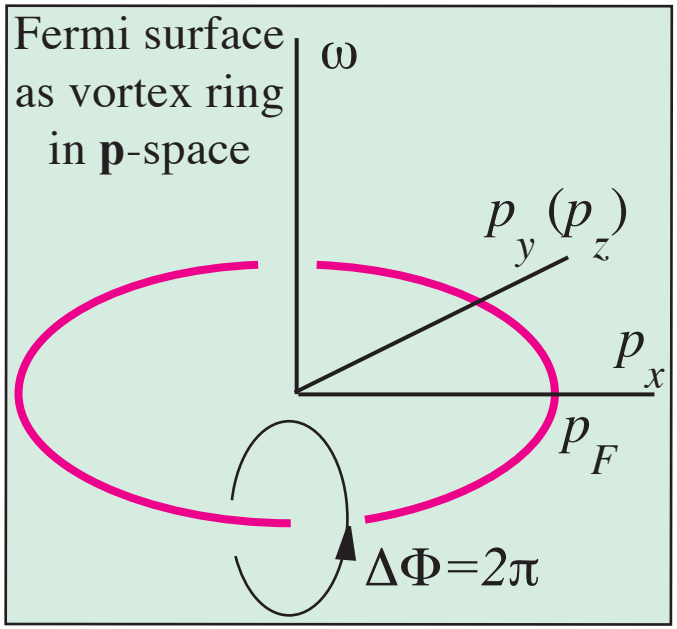


boojums,
Dirac
monopoles
terminating
3He-B string

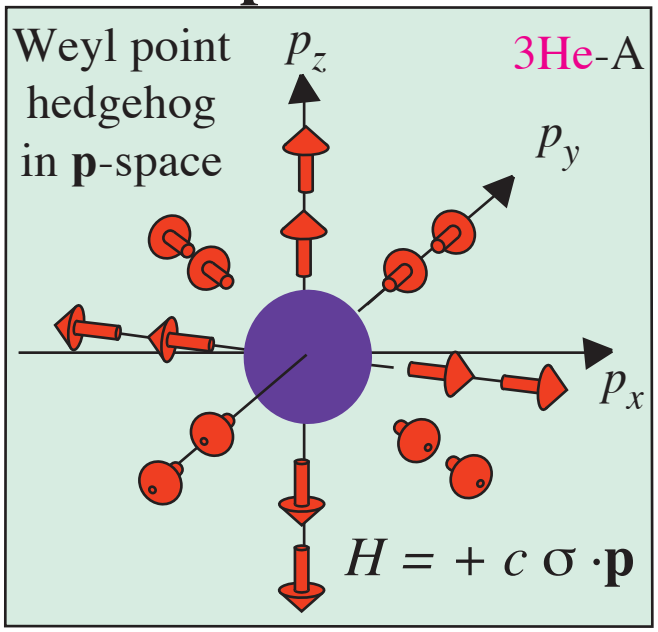
quantized vortices
in time reversal symmetric 3He-B

half-quantum vortices
terminate soliton in polar phase

gapless (massless) topological vacua as defects in momentum space

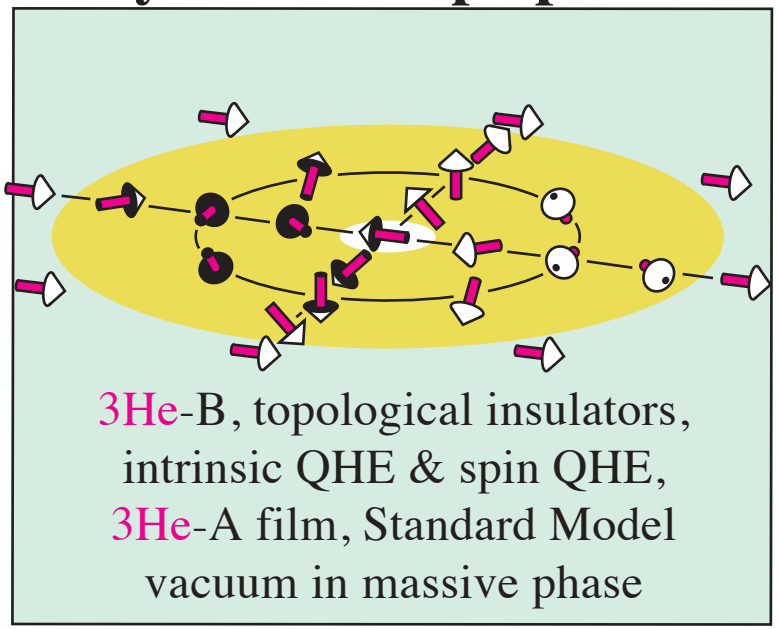


normal 3He, metals



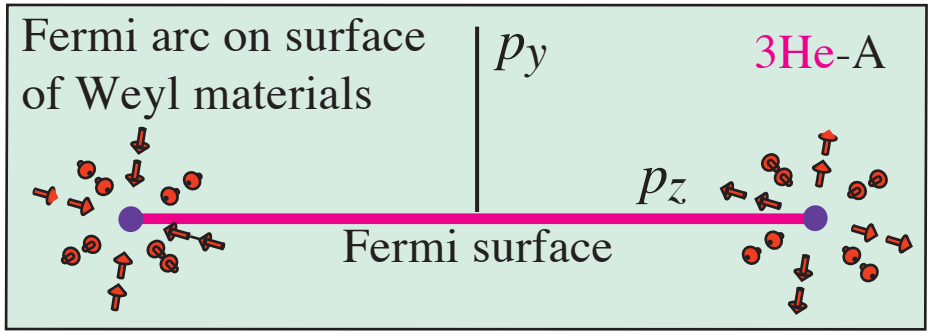
3He-A, vacuum of Standard Model,
topological Weyl semimetals

gapped topological matter as skyrmions in p-space

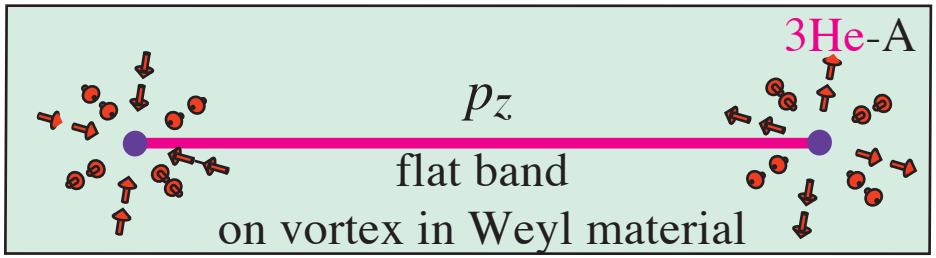


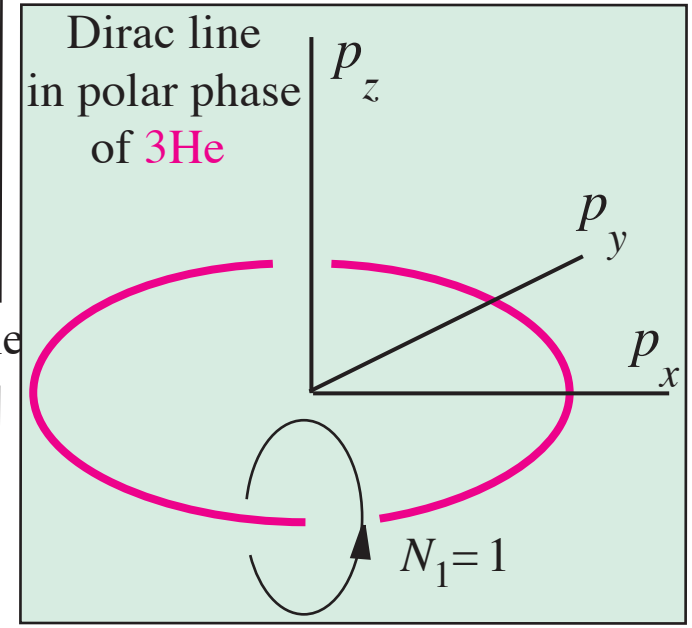
3He-B, topological insulators,
intrinsic QHE & spin QHE,
3He-A film, Standard Model
vacuum in massive phase

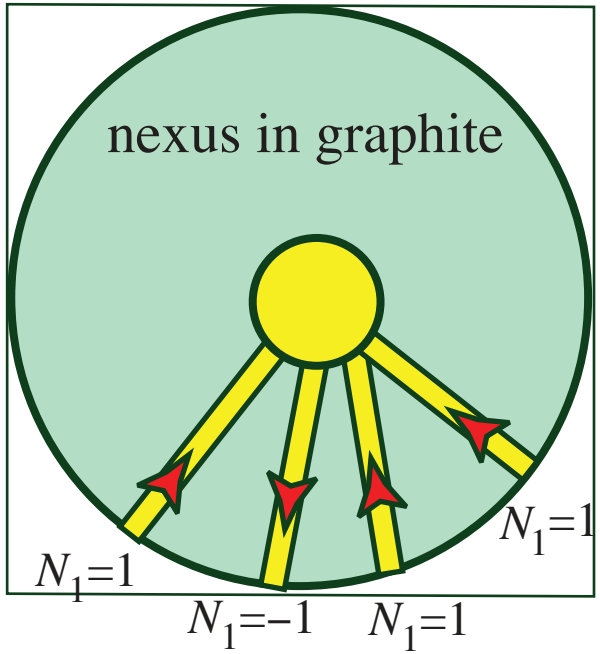
termination of Dirac lines vs monopole
terminating 3He-A half-quantum vortices



Dirac strings in **p**-space terminating on monopole







normal liquid 3He & metals as the most stable topological materials

Fermi surface topological class

Energy spectrum of
non-interacting gas of fermionic atoms

$$\varepsilon(p) = \frac{p^2}{2m} - \mu = \frac{p^2}{2m} - \frac{p_F^2}{2m}$$

$\varepsilon > 0$
empty levels

$\varepsilon < 0$
occupied
levels:
Fermi sea

Fermi surface
$\varepsilon = 0$

$p=p_F$

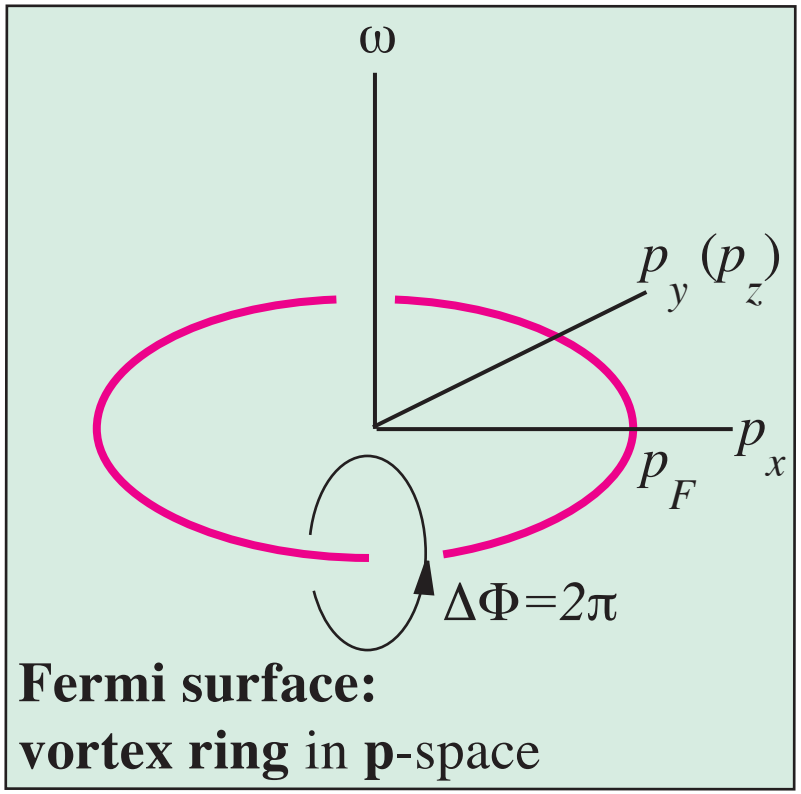
why metals are ubiquitous ?

because Fermi surface is topologically protected

Fermi surface = quantized vortex in p-space

Green's function

$$G^{-1} = i\omega - \varepsilon(p)$$



Fermi surface:
vortex ring in **p**-space

phase of Green's function

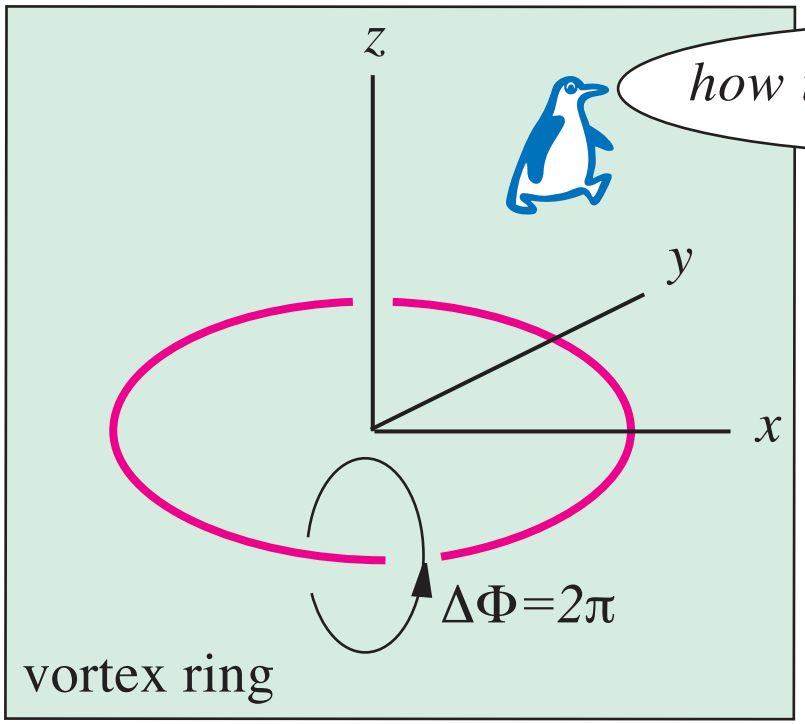
$$G(\omega,\mathbf{p}) = |G| e^{i\Phi}$$

has winding number $N = 1$

origin of topological stability of **vortex** in **r**-space & **Fermi surface** in **p**-space

homotopy group π_1

Topology in **r**-space



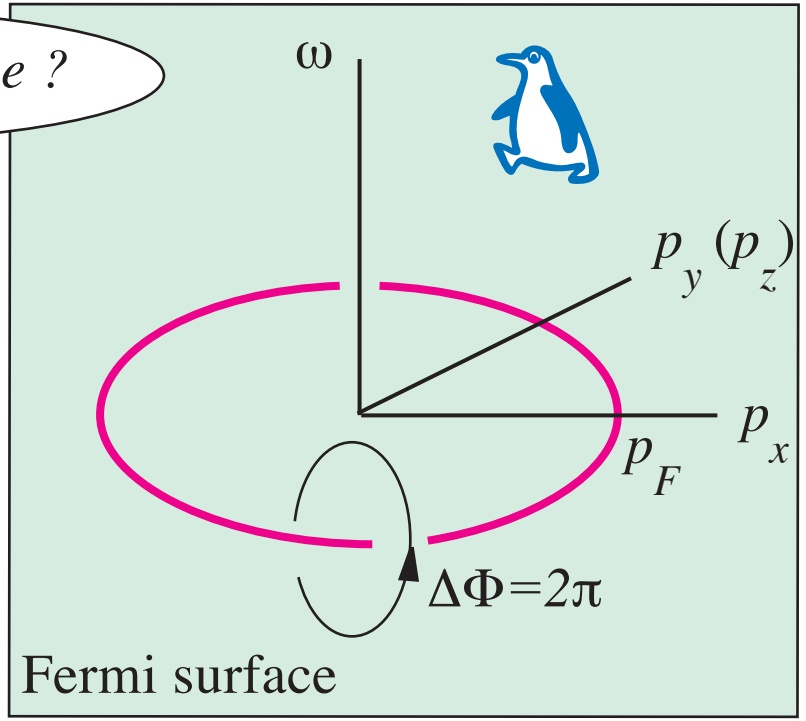
$\Psi(\mathbf{r})=|\Psi|\, e^{\,i\Phi}$

scalar order parameter
of superfluid & superconductor

how is it in **p**-*space ?*

winding
number
$N_1 = 1$

Topology in **p**-space



$G(\omega,\mathbf{p})=|G|e^{\,i\Phi}$

Green's function (propagator)

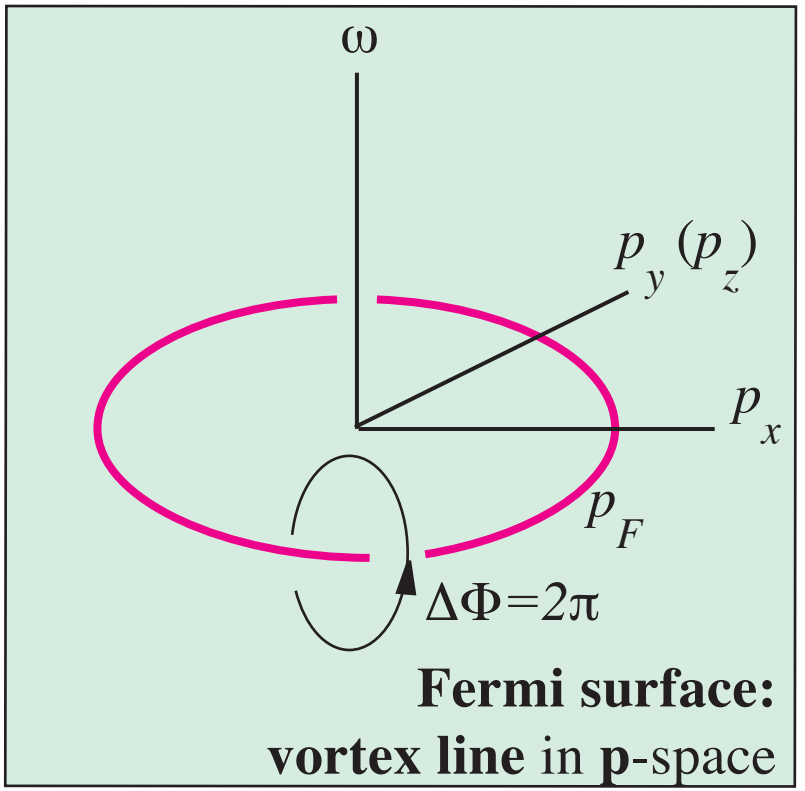
classes of mapping S^1 to $U(1)$
manifold of
broken symmetry vacuum states

classes of mapping S^1 to $GL(n,C)$
space of
non-degenerate complex matrices

Fermi liquids & non-Fermi liquids

Singularity at Fermi surface is robust to perturbations:

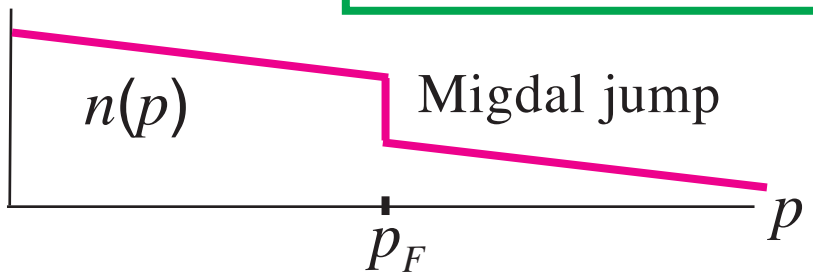
winding number $N=1$ *cannot change continuously,*
interaction (perturbative) cannot destroy singularity
but behavior near FS can be different



Fermi surface:
vortex line in **p**-space

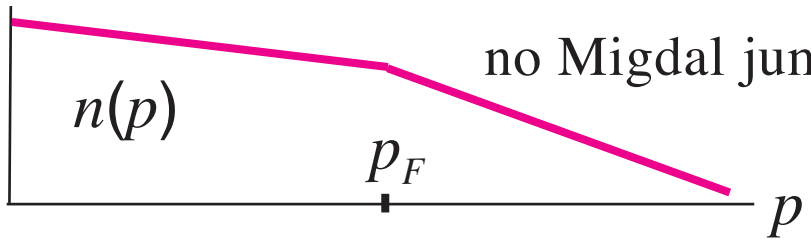
$G(\omega,\mathbf{p})=|G|e^{i\Phi}$

Landau Fermi-liquid



Fermi surface exists in superfluids/superconductors
examples: 3He-A in flow & Gubankova-Schmitt-Wilczek, PRB74 (2006) 064505, but no Luttinger theorem

Luttinger Fermi liquid, marginal Fermi liquid



$$G(\omega,\mathbf{p}) = \frac{Z(p,\omega)}{i\omega - \varepsilon(p)}$$

$$Z(p,\omega) = (\omega^2 + \varepsilon^2(p))^{\gamma}$$

zeroes in $G(\omega,\mathbf{p})$
for $\gamma > 1/2$

unparticles in particle physics

zeroes in $G(\omega,\mathbf{p})$ have the same $N=1$ as poles

from Fermi-surface to flat band

Khodel-Shaginyan flat band from Landau theory

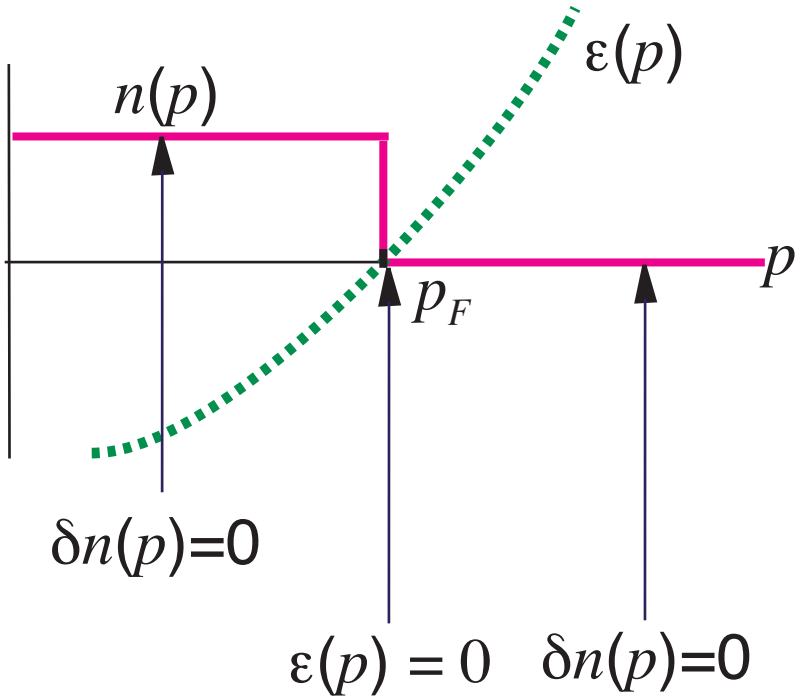
Khodel-Shaginyan, JETP Lett. **51**, 553 (1990)
GV, JETP Lett. **53**, 222 (1991)
Nozieres, J. Phys. (Fr.) **2**, 443 (1992)

$$E\{n(p)\}$$

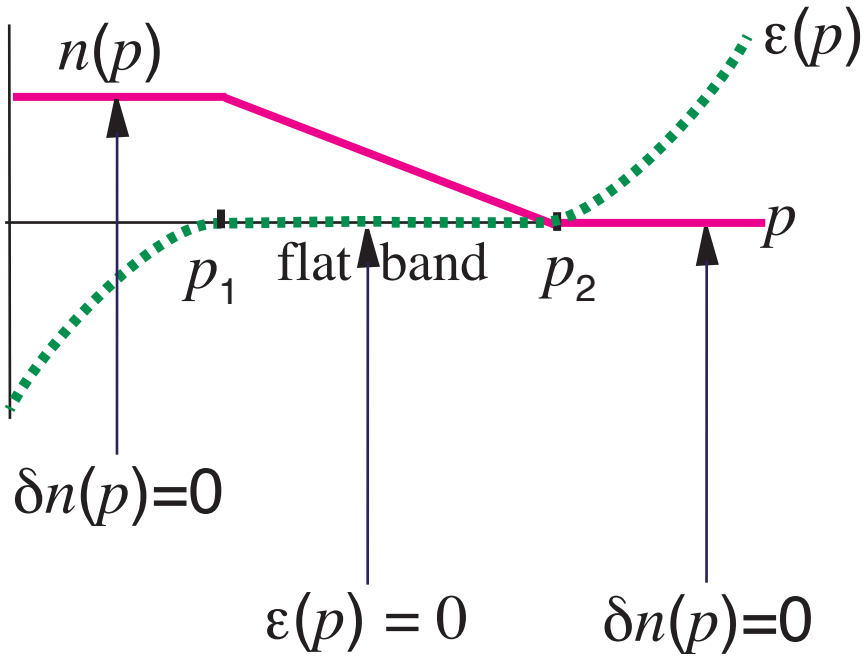
$$\delta E\{n(p)\} = \int \varepsilon(p)\, \delta n(p)\, d^d p = 0$$

two solutions: $\delta n(p)=0$ & $\varepsilon(p) = 0$

weak interaction:
Fermi surface (Landau Fermi-liquid)



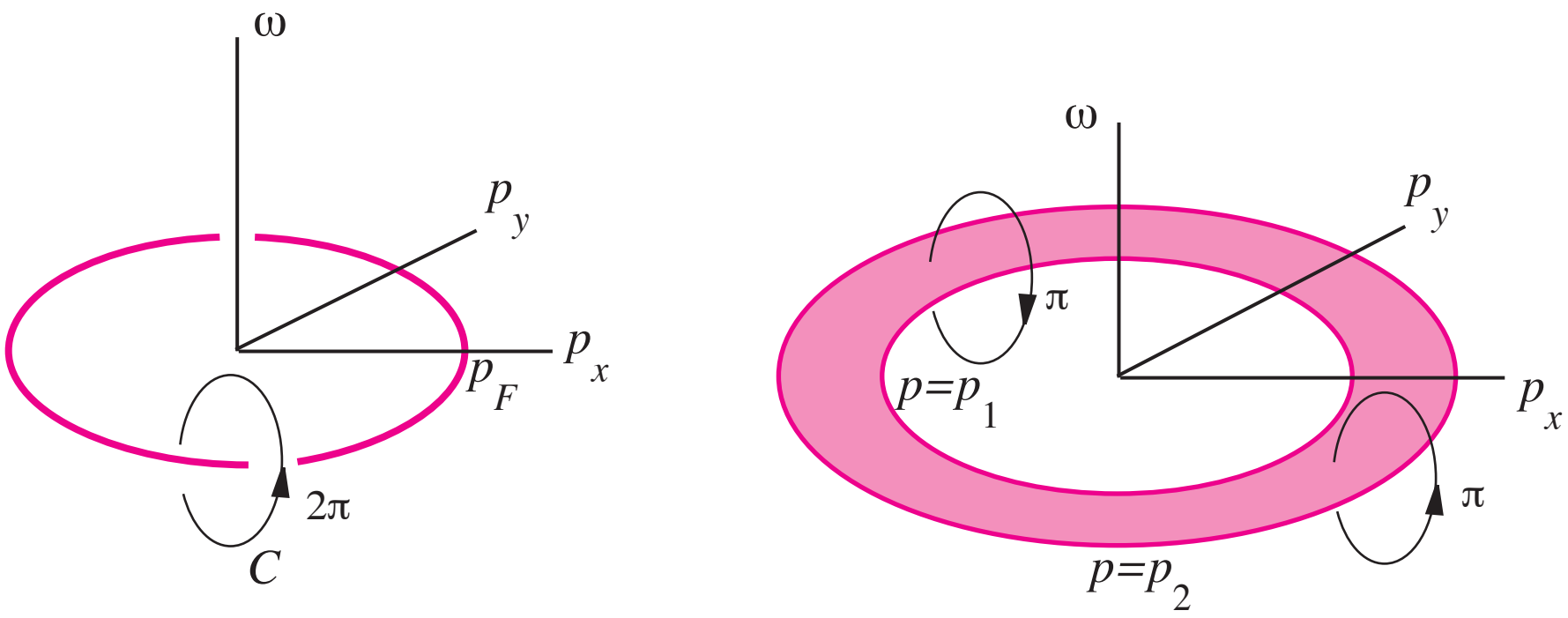
strong interaction:
Fermi ball (flat band)



S.-S. Lee
Non-Fermi liquid from a charged black hole: A critical Fermi ball
PRD 79, 086006 (2009)

anti-de Sitter/conformal field theory correspondence

From Fermi surface topology to flat band topology



p-space vortex splits into two half-quantum vortices (p-space Alice strings), separated by p-space domain wall (p-space soliton)

GV 1991

superconducting transition temperature: Fermi surface vs flat band

$$1 = g \int \frac{d^3p}{2h^3} \frac{1}{E(p)}$$

$$E^2(p) = \Delta^2 + \varepsilon^2(p)$$

finite DoS of Fermi surface gives superconducting transition

$$E^2(p) = \Delta^2 + v_F{}^2 (p - p_F)^2$$

normal superconductors:
exponentially suppressed
transition temperature

$$T_c = T_F \exp(-1/g\nu)$$

interaction *DOS*

extremely high DoS of flat band gives high transition temperature

$E(p) = \Delta$ in flat band

flat band superconductivity:
linear dependence
of T_c on coupling g

$$T_c \sim g V_{FB}$$

coupling *flat band volume*

PRL 112, 070403 (2014) PHYSICAL REVIEW LETTERS week ending 21 FEBRUARY 2014

Fermi Condensation Near van Hove Singularities Within the Hubbard Model on the Triangular Lattice

Dmitry Yudin,[1] Daniel Hirschmeier,[2] Hartmut Hafermann,[3] Olle Eriksson,[1] Alexander I. Lichtenstein,[2] and Mikhail I. Katsnelson[4,5]

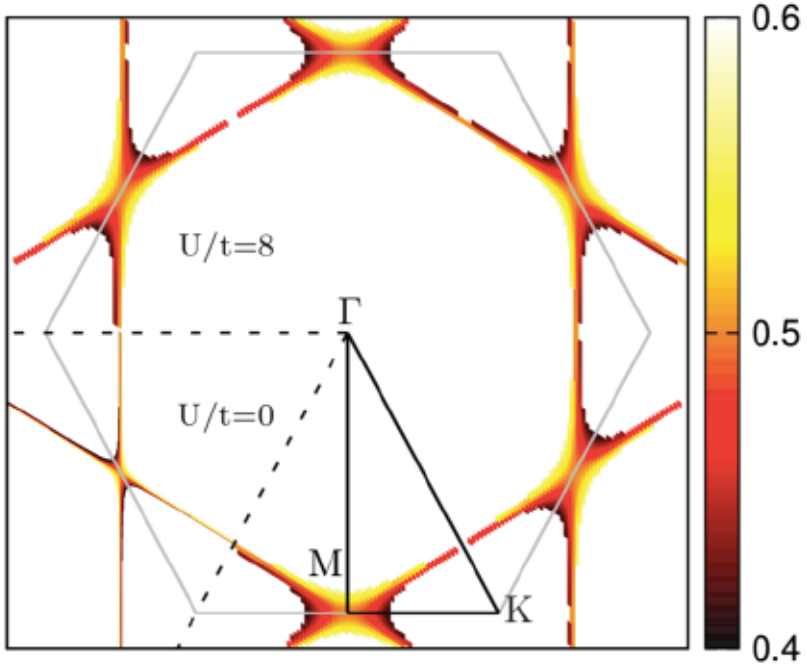
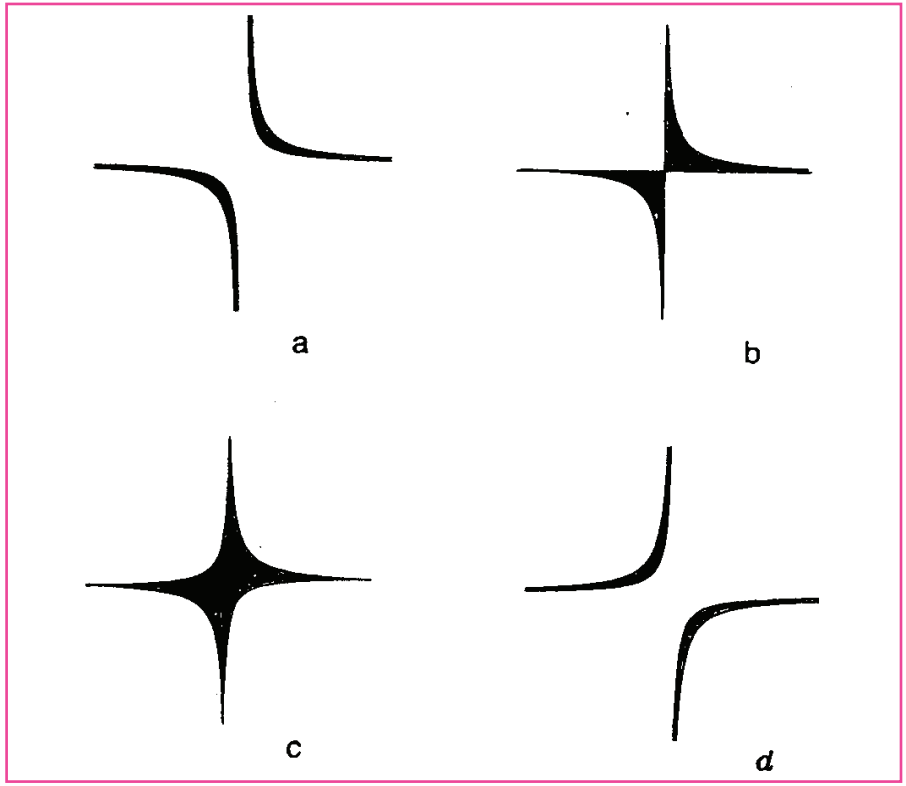


FIG. 4 (color online). Broadened Fermi surface within ±0.1 electrons for $U/t = 8$ and $T/t = 0.1$. The lower left sextant shows the noninteracting result.

flat band near Lifshitz transition



Письма в ЖЭТФ, том 59, вып.11, стр.798 - 802, ©1994 г. 10 июня

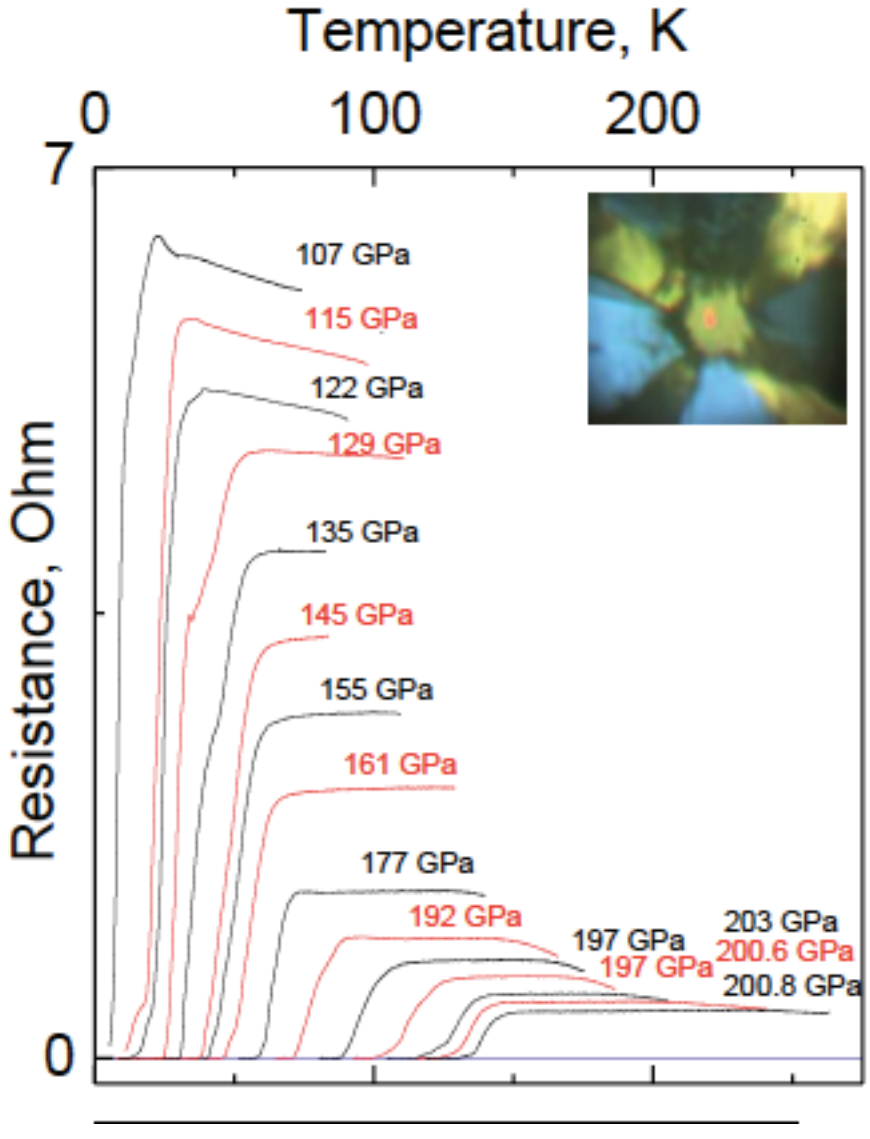
ON FERMI CONDENSATE: NEAR THE SADDLE POINT AND WITHIN THE VORTEX CORE

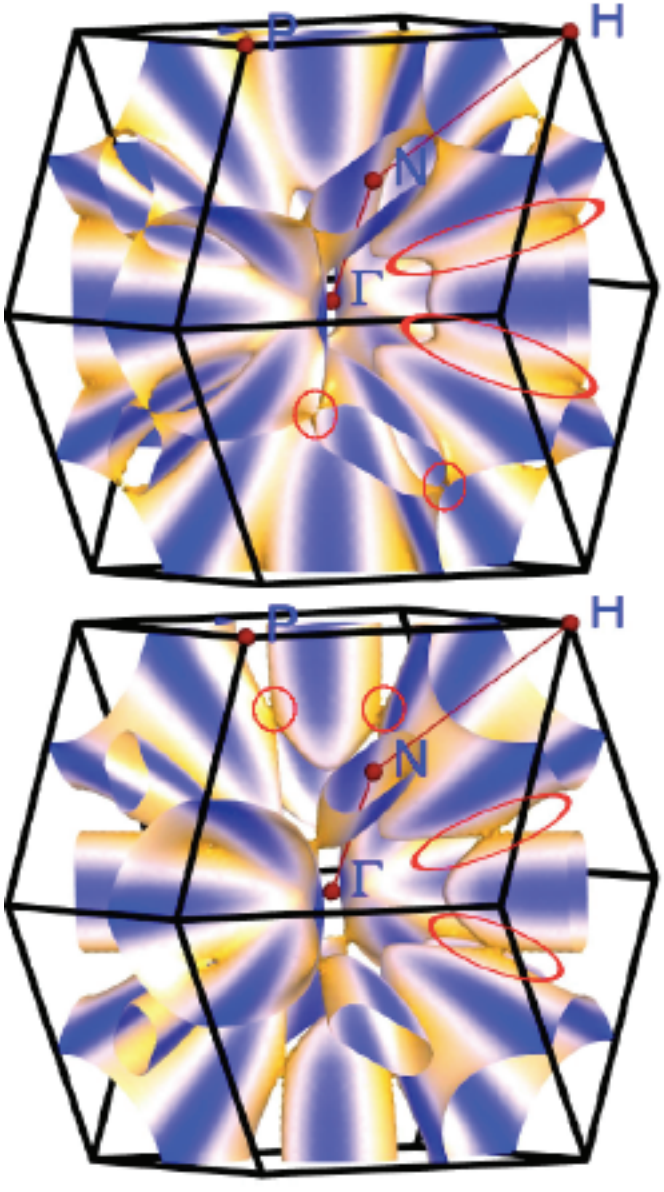
G.E. Volovik

van Hove singularity & room-T superconductivity

Conventional superconductivity at 203 K at high pressures

A.P. Drozdov[1]*, M. I. Eremets[1]*, I. A. Troyan[1], V. Ksenofontov[2], S. I. Shylin[2]



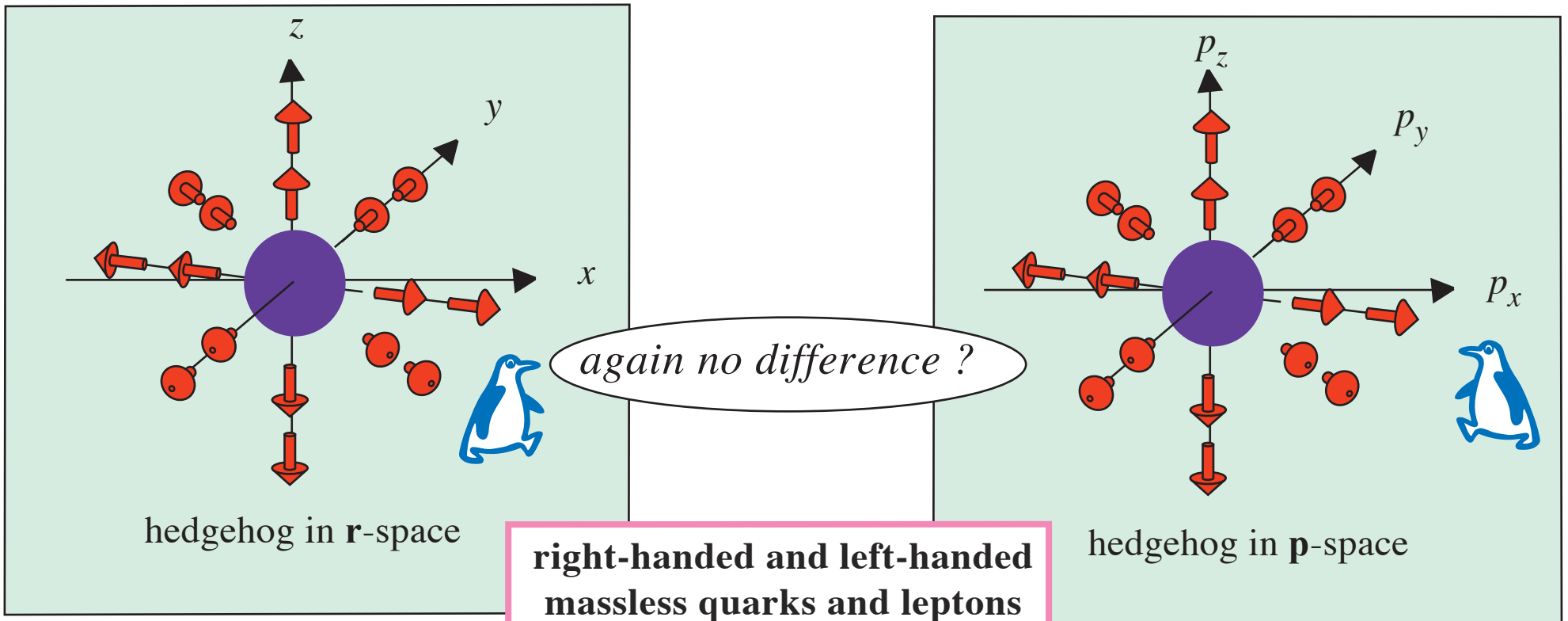


Impact of van Hove singularities in the strongly coupled high temperature superconductor H_3S

Yundi Quan and Warren E. Pickett*

superfluid 3He-A & Standard Model in massless phase: chiral Weyl fermion as hedgehog in p-space

magnetic hedgehog vs right-handed Weyl electron



$\sigma(\mathbf{r}) = \hat{\mathbf{r}}$

right-handed and left-handed massless quarks and leptons are elementary particles in Standard Model

$\sigma(\mathbf{p}) = \hat{\mathbf{p}}$

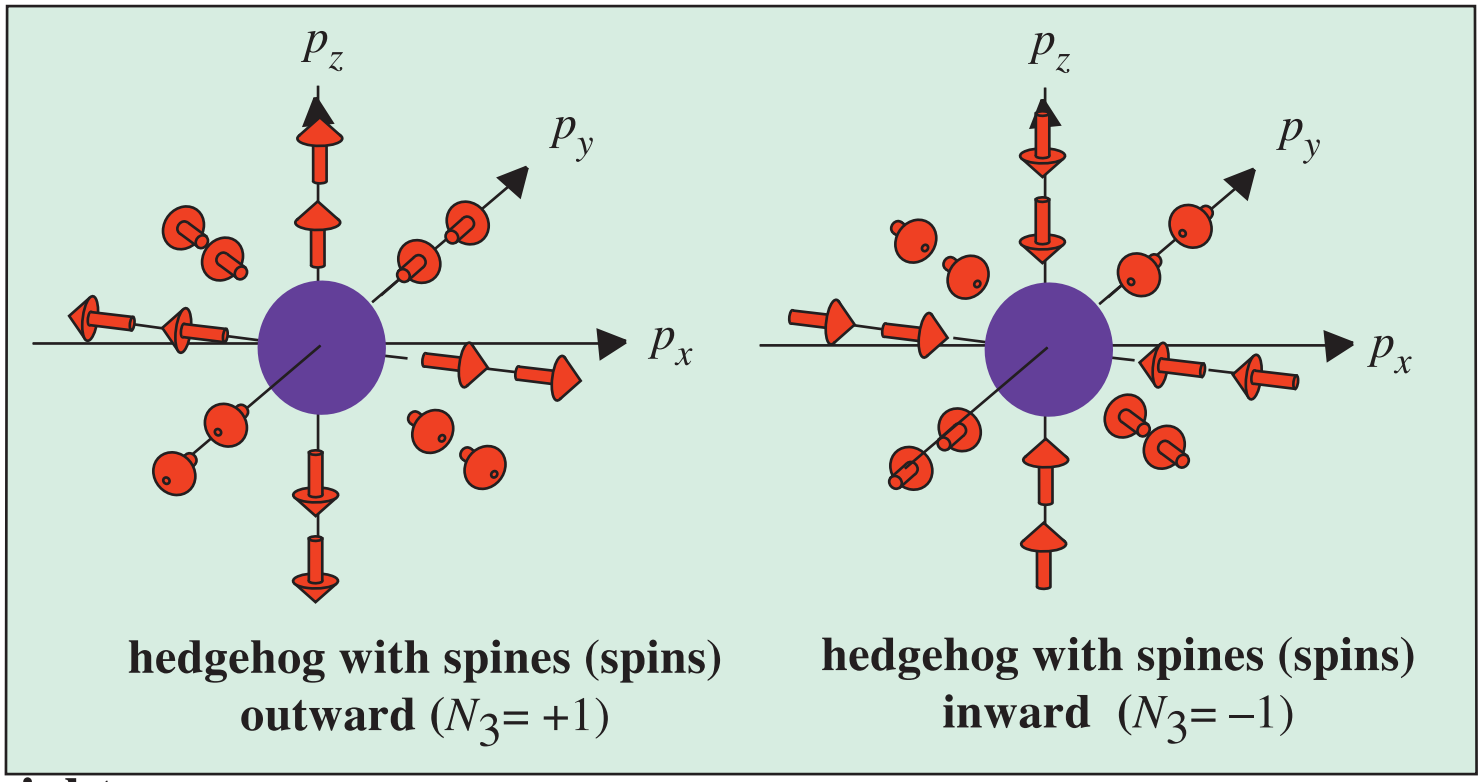
Landau CP symmetry is emergent

close to Fermi point

$$H = + c\, \sigma \cdot \mathbf{p}$$

right-handed electron =
hedgehog in **p**-space with spines = spins

Topological invariant for right and left elementary particles



hedgehog with spines (spins) outward ($N_3 = +1$)

hedgehog with spines (spins) inward ($N_3 = -1$)

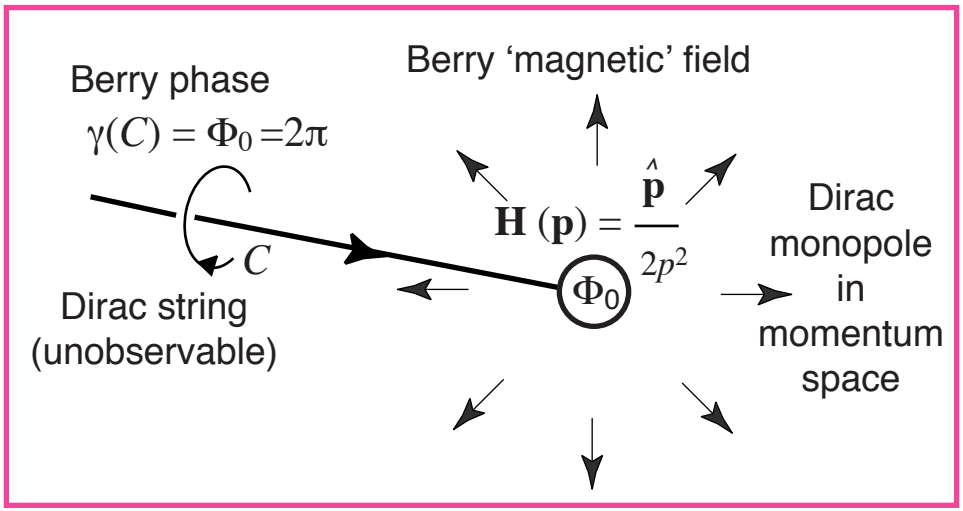
right $H = +c\,\sigma \cdot \mathbf{p}$ $\mathbf{g}(\mathbf{p}) = +\,c\mathbf{p}$

$H = \sigma \cdot \mathbf{g}(\mathbf{p})$

$H = -c\,\sigma \cdot \mathbf{p}$ $\mathbf{g}(\mathbf{p}) = -\,c\mathbf{p}$ **left**

$$N_3 = \frac{1}{8\pi}\, e_{ijk} \int dS^i\, \hat{\mathbf{g}} \cdot (\partial^j \hat{\mathbf{g}} \times \partial^k \hat{\mathbf{g}})$$

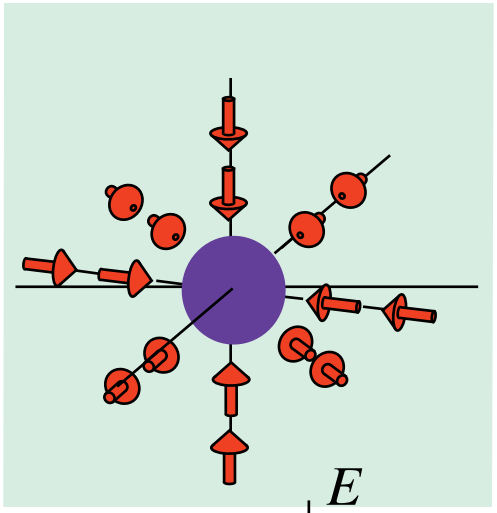
over 2D surface around Fermi point



Weyl fermions in 3+1 gapless topological cond-mat

topologically protected Weyl points in:

topological semi-metal or Weyl metals (Abrikosov-Beneslavskii 1971), ^{3}He-A (1982), triplet Fermi gases, $CoSb_3$ (arXiv:1204.5905)

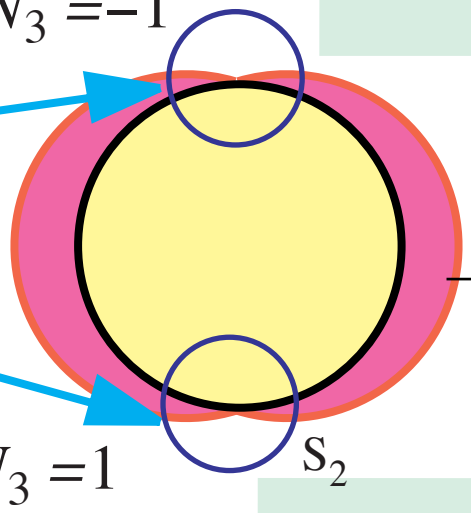


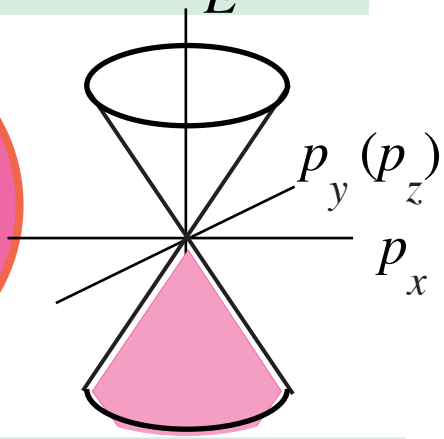
Gap node - Weyl point (anti-hedgehog)

$$N_3 = -1$$

$$N_3 = \frac{1}{8\pi} e_{ijk} \int dS^k \; \hat{\mathbf{g}} \cdot (\nabla_{p_i} \hat{\mathbf{g}} \times \nabla_{p_j} \hat{\mathbf{g}})$$

over 2D surface S in 3D p-space

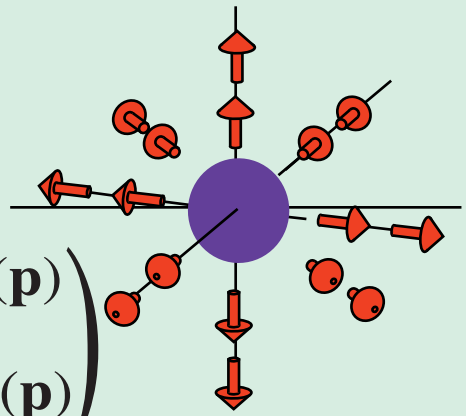




$$N_3 = 1$$

Gap node - Weyl point (hedgehog)

$$p^2 = p_x^2 + p_y^2 + p_z^2$$



$$H = \begin{pmatrix} \frac{p^2}{2m} - \mu & c(p_x + ip_y) \\ c(p_x - ip_y) & -\frac{p^2}{2m} + \mu \end{pmatrix} = \begin{pmatrix} g_3(\mathbf{p}) & g_1(\mathbf{p}) + i\, g_2(\mathbf{p}) \\ g_1(\mathbf{p}) - i\, g_2(\mathbf{p}) & -g_3(\mathbf{p}) \end{pmatrix}$$

emergence of relativistic chiral Weyl fermions near Fermi points

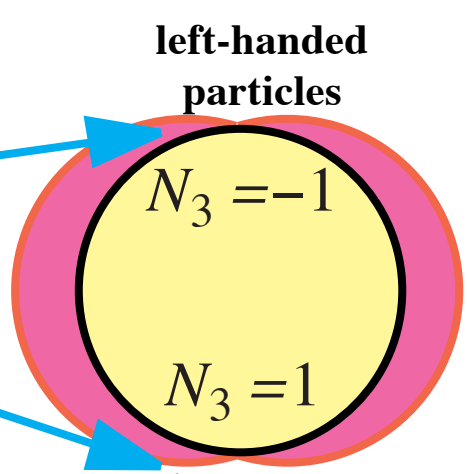
original non-relativistic Hamiltonian

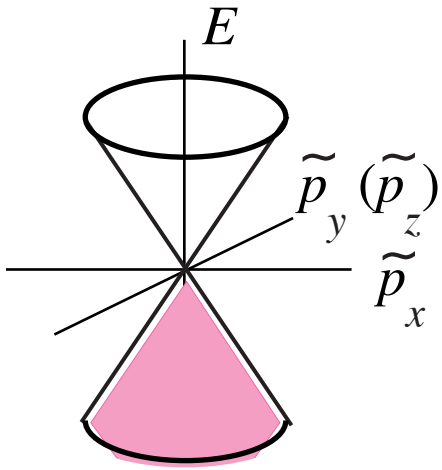
$$H = \begin{pmatrix} \frac{p^2}{2m} - \mu & c(p_x + ip_y) \\ c(p_x - ip_y) & -\frac{p^2}{2m} + \mu \end{pmatrix} = \begin{pmatrix} g_3(\mathbf{p}) & g_1(\mathbf{p}) + i\, g_2(\mathbf{p}) \\ g_1(\mathbf{p}) - i\, g_2(\mathbf{p}) & -g_3(\mathbf{p}) \end{pmatrix} = \tau \cdot \mathbf{g}(\mathbf{p})$$

close to nodes, i.e. in low-energy corner relativistic chiral fermions emerge

$$H = N_3\, c\, \tau \cdot (\mathbf{p} - \mathbf{p}_0)$$

$$E = -c\tilde{p}$$





chirality is emergent ??

top. invariant determines chirality in low-energy corner

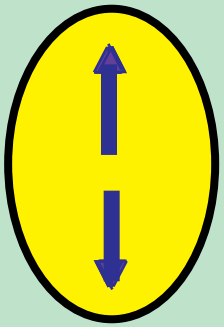
what else is emergent ?

relativistic invariance as well

bosonic collective modes in two generic fermionic vacua

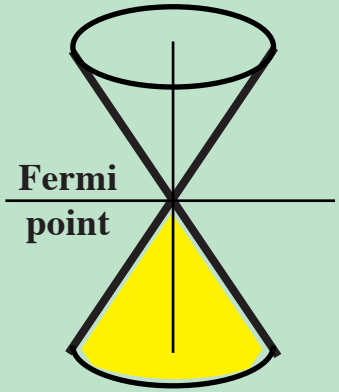
Landau theory of Fermi liquid

collective Bose modes of fermionic vacuum:
propagating
oscillation of **shape**
of Fermi surface



Landau, ZhETF **32**, 59 (1957)

Standard Model + gravity

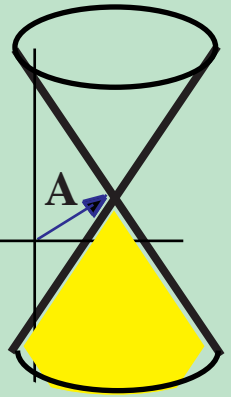


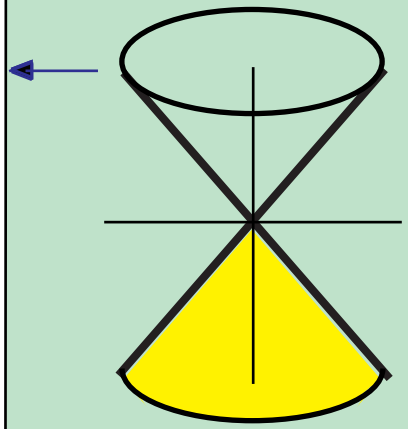
collective Bose modes:

propagating
oscillation of **position**
of Fermi point

$\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A}$

form effective dynamic
electromagnetic field

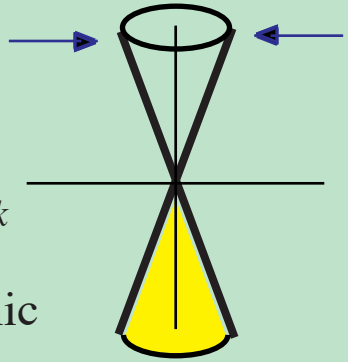




propagating
oscillation of **slopes**

$E^2 = c^2p^2 \rightarrow g^{ik}p_i p_k$

form effective dynamic
gravity field



two generic quantum field theories of interacting bosonic & fermionic fields

relativistic quantum fields & gravity emerging near Weyl point

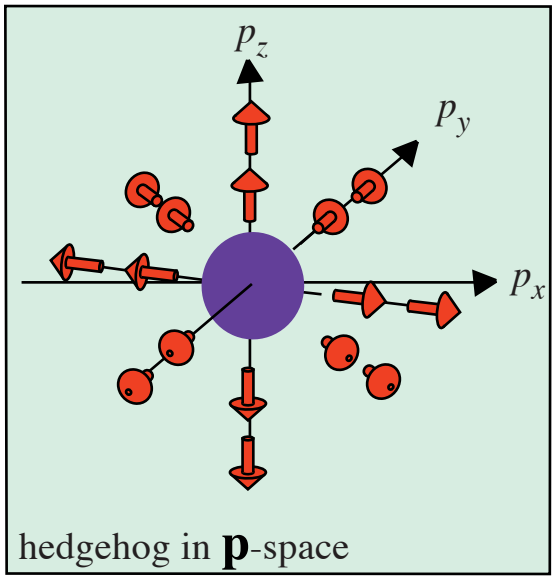
Atiyah-Bott-Shapiro construction:
linear expansion of Hamiltonian near the nodes in terms of Dirac Γ-matrices

primary object:
tetrad
$e_a{}^\mu$

$E = v_F (p - p_F)$ emergent relativity $H = e_a{}^k \Gamma^a \cdot (p_k - p_k^0)$

linear expansion near Fermi surface

linear expansion near Weyl point



secondary object:
metric
$g^{\mu\nu} = \eta^{ab}\, e_a{}^\mu\, e_b{}^\nu$

$$g^{\mu\nu}(p_\mu - eA_\mu - e\tau \cdot \mathbf{W}_\mu)(p_\nu - eA_\nu - e\tau \cdot \mathbf{W}_\nu) = 0$$

effective metric: emergent gravity

effective electromagnetic field

effective $SU(2)$ gauge field

effective electric charge

$e = +1$ or -1

effective isotopic spin

all ingredients of Standard Model :
chiral fermions & gauge fields
emerge in low-energy corner
together with spin, Dirac Γ–matrices,
gravity & physical laws:
Lorentz & gauge invariance,
equivalence principle, etc

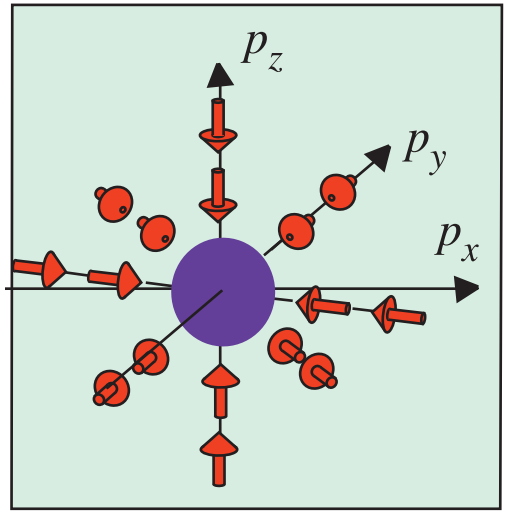
gravity & gauge fields
are collective modes
of vacua with Weyl point

Einstein-Cartan-Sciama-Kibble theory
with tetrads, spin connection & torsion

Chiral Weyl fermions in Standard Model

Family #1 of quarks and leptons

left particles



hedgehog with spines (spins) inward ($N_3 = -1$)

+2/3 u_L +1/6	-1/3 d_L +1/6	quarks	+2/3 u_R +2/3	-1/3 d_R -1/3	$SU(3)_C$
+2/3 u_L +1/6	-1/3 d_L +1/6		+2/3 u_R +2/3	-1/3 d_R -1/3	
+2/3 u_L +1/6	-1/3 d_L +1/6		+2/3 u_R +2/3	-1/3 d_R -1/3	
$SU(2)_L$					
0 ν_L -1/2	-1 e_L -1/2	leptons	0 ν_R 0	-1 e_R -1	

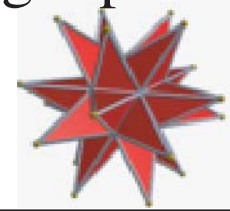
$H = -c\,\sigma\cdot\mathbf{p}$

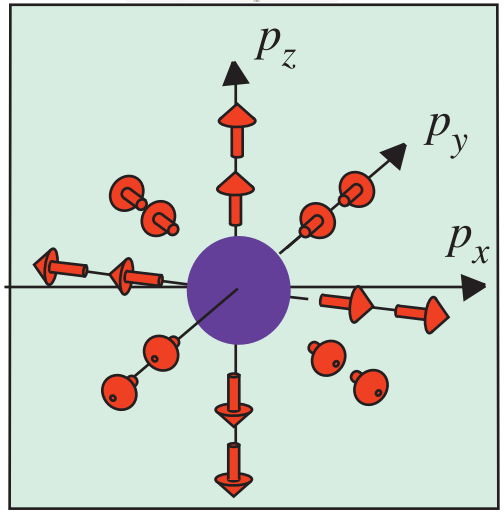
$N_3 = -1$

$H = +c\,\sigma\cdot\mathbf{p}$

$N_3 = +1$

right particles





hedgehog with spines (spins) outward ($N_3 = +1$)

$$N_3 = \frac{1}{24\pi^2}\, e_{\mu\nu\lambda\gamma}\, \mathbf{tr} \int dS^{\gamma}\, \mathbf{G}^{\mu}\, \mathbf{G}^{-1}\, \mathbf{G}^{\nu}\, \mathbf{G}^{-1} \mathbf{G}^{\lambda}\, \mathbf{G}^{-1}$$

over 3D surface S in 4D momentum space

general topological invariant in terms of Green's function for interacting systems

Standard Model topological invariant

Topological invariant protected by symmetry

$$N_K = \frac{1}{24\pi^2} e_{\mu\nu\lambda}\, \mathbf{tr} \int_{\text{over } S^3} dV\, \mathbf{K}\, \mathbf{G}\, \nabla^{\mu}\, \mathbf{G}^{-1}\, \mathbf{G}\, \nabla^{\nu}\, \mathbf{G}^{-1} \mathbf{G}\, \nabla^{\lambda}\, \mathbf{G}^{-1}$$

G is Green's function, K is symmetry operator

$$\mathbf{GK} = +/- \mathbf{KG}$$

for Standard Model vacuum K is Z_2 center group

$$K = \exp 2\pi i \tau_3$$

τ_3 weak isotopic spin of SU(2)

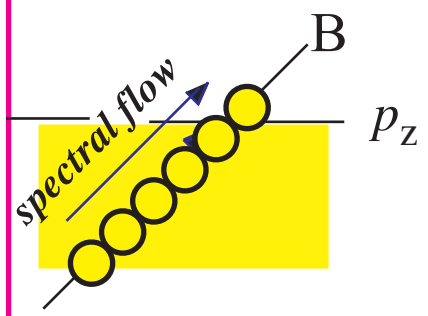
$$N_K = 16\, n_g$$

16 massless Weyl particles in one generation are protected by combined symmetry and topology

Weyl point (Dirac monopole) as origin of chiral anomaly

example:
baryon production from vacuum by hypermagnetic field

baryonic charge B due spectral flow
along anomalous branch
in parallel **B** and **E**



$$\dot{p}_z = eE_Y$$

chiral anomaly equation
(Adler, Bell, Jackiw)

$$\dot{B} = \frac{1}{4\pi^2} N_B \, \mathbf{B}_Y \cdot \mathbf{E}_Y$$

$$N_B = \frac{1}{24\pi^2} e_{\mu\nu\lambda} \mathbf{tr} \int_{\text{over } S^3} dV \; \mathbf{B}\mathbf{Y}^2 \, \mathbf{G} \nabla^{\mu} \mathbf{G}^{-1} \, \mathbf{G} \nabla^{\nu} \mathbf{G}^{-1} \mathbf{G} \nabla^{\lambda} \mathbf{G}^{-1}$$

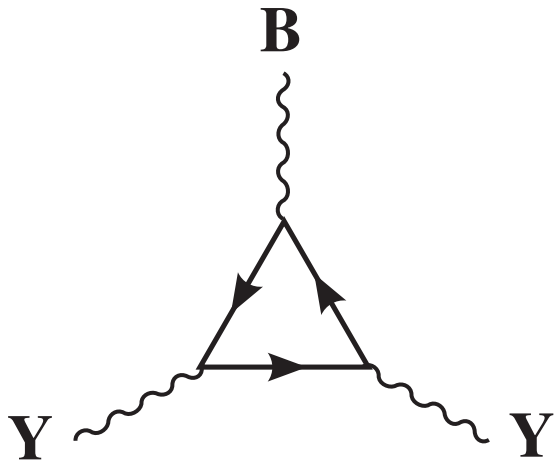
matrix of baryonic charge **matrix of hypecharge**

$$\dot{B} = \frac{1}{4\pi^2} \, \mathbf{B}_Y \cdot \mathbf{E}_Y \sum_a B_a C_a Y_a^2$$

B_a-- **baryonic charge**
Y_a-- **hypercharge**

C_a = +1 **for right**
-1 **for left**

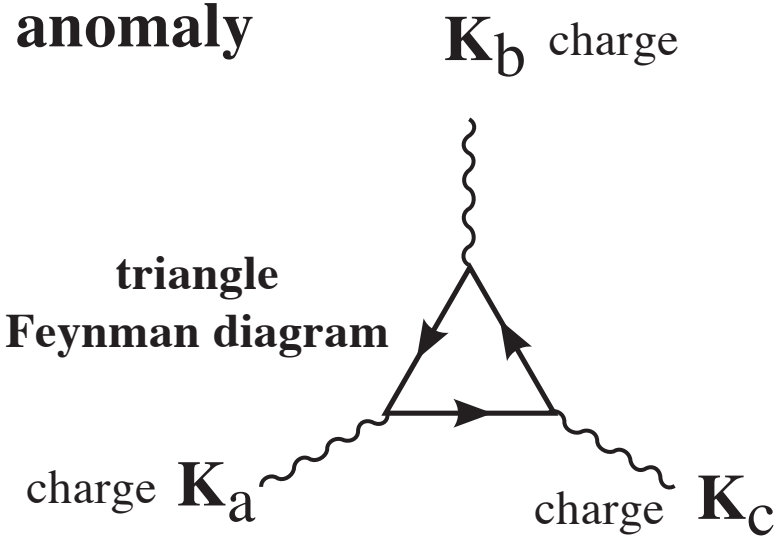
triangle
Feynman diagram



earlier indications of axial anomaly in 3He-A from Weyl point:
anomaly in orbital dynamics & paradox of angular & linear momenta
Leggett-Takagi, Ann. Phys. **110**, 353 (1978); GV & Mineev, JETP **56**, 579 (1982)

Symmetry protected invariants and chiral anomaly

prefactors of ***r***-space topological terms
are ***p***-space topological invariant



chiral anomaly from 3+1 Weyl points

$$K_{abc} = \frac{1}{24\pi^2}\, e_{\mu\nu\lambda}\, \mathbf{tr} \int_{\text{over } S^3} dV\, \mathbf{K}_a\mathbf{K}_b\mathbf{K}_c\, \mathbf{G}\, \nabla^{\mu}\, \mathbf{G}^{-1}\, \mathbf{G}\nabla^{\nu}\, \mathbf{G}^{-1}\mathbf{G}\, \nabla^{\lambda}\, \mathbf{G}^{-1}$$

prefactor of Chern-Simons term in 2+1 gapped topological matter

$$K_{ab} = \frac{1}{24\pi^2}\, e_{\mu\nu\lambda}\, \mathbf{tr} \int_{\text{over 2+1 p-space}} dV\, \mathbf{K}_a\mathbf{K}_b\, \mathbf{G}\, \nabla^{\mu}\, \mathbf{G}^{-1}\, \mathbf{G}\nabla^{\nu}\, \mathbf{G}^{-1}\mathbf{G}\, \nabla^{\lambda}\, \mathbf{G}^{-1}$$

spin quantum Hall effect:

planar phase film of 3He & 2D topological insulator

$$\mathrm{H} = \begin{pmatrix} \frac{p^2}{2m} - \mu & c(p_x + i\, p_y \sigma_z) \\ c(p_x - i\, p_y \sigma_z) & -\frac{p^2}{2m} + \mu \end{pmatrix}$$

$$N_3 = \frac{1}{24\pi^2} e_{\mu\nu\lambda} \mathbf{tr} \left[\int d^2p\, d\omega\, \mathbf{G} \nabla^{\mu} \mathbf{G}^{-1}\, \mathbf{G} \nabla^{\nu} \mathbf{G}^{-1} \mathbf{G} \nabla^{\lambda} \mathbf{G}^{-1} \right] = 0$$

$$N_{se} = \frac{1}{24\pi^2} e_{\mu\nu\lambda} \mathbf{tr} \left[\int d^2p\, d\omega\, \mathbf{K}\, \mathbf{G} \nabla^{\mu} \mathbf{G}^{-1}\, \mathbf{G} \nabla^{\nu} \mathbf{G}^{-1} \mathbf{G} \nabla^{\lambda} \mathbf{G}^{-1} \right]$$

$$\mathbf{K} = \sigma_z \qquad N_3^{+} = +1 \qquad N_3^{-} = -1$$

$$N_3 = N_3^{+} + N_3^{-} = 0 \qquad N_{se} = N_3^{+} - N_3^{-} = 2$$

spin current $J_x^z = \frac{1}{4\pi} N_{se} E_y$ spin-charge QHE

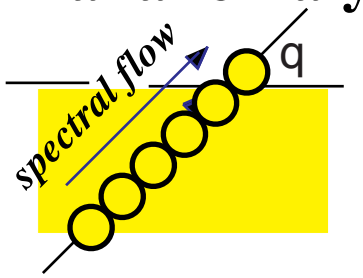
$$\sigma_{xy}^{spin/charge} = \frac{N_{se}}{4\pi} \qquad N_{se} = 2$$

GV & Yakovenko

J. Phys. CM **1**, 5263 (1989)

experimental verification of chiral anomaly equation

measurement of *Kopnin force*



momentum from vacuum of fermion zero modes

spectral flow produces

baryons from vacuum

$$\dot{\mathbf{P}} = \sum \mathbf{P}_a \dot{n}_a$$

$$\dot{B} = \sum B_a \dot{n}_a$$

$\mathbf{P}_a$ -- **momentum of Weyl point (fermionic charge)**

e_a -- **effective electric charge**

B_a -- **baryonic charge**

Y_a -- **hypercharge**

$$\dot{\mathbf{P}} = (1/4\pi^2)\, \mathbf{B}\bullet\mathbf{E} \sum_a \mathbf{P}_a C_a e_a^2$$

applied to ^{3}He-A

C_a = +1 **for right**

-1 **for left**

chiral anomaly equation (Adler, Bell, Jackiw)

$$\dot{B} = (1/4\pi^2)\, \mathbf{B}_Y\bullet\mathbf{E}_Y \sum_a B_a C_a Y_a^2$$

applied to Standard Model

C_a = +1 **for right**

-1 **for left**

quasiparticles move from vacuum to the positive energy world, where they are scattered by quasiparticles in bulk and transfer momentum from vortex to normal component

this is the source of Kopnin spectral flow force

Bevan, Manninen, Cook, Hook, Hall, Vachaspati & GV

Momentum creation by vortices in superfluid 3He as a model of primordial baryogenesis, Nature **386**, 689 (1997)

Experimental check of Adler-Bell-Jackiw equation for chiral anomaly in 3He-A

(Bevan et al. 1997)

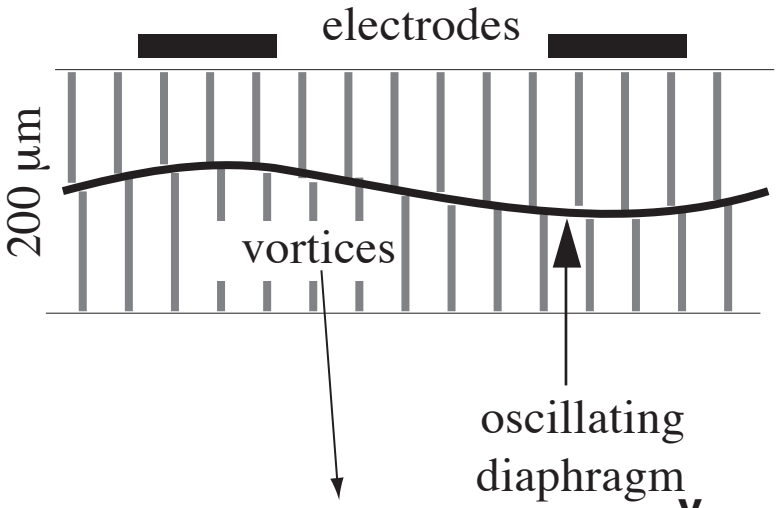
$$\partial_t j_i - \nabla_k \pi_{ik} = -\frac{3}{2} C_0 \hat{l}_i (\partial_t \hat{\mathbf{l}} \cdot \nabla \times \hat{\mathbf{l}})$$

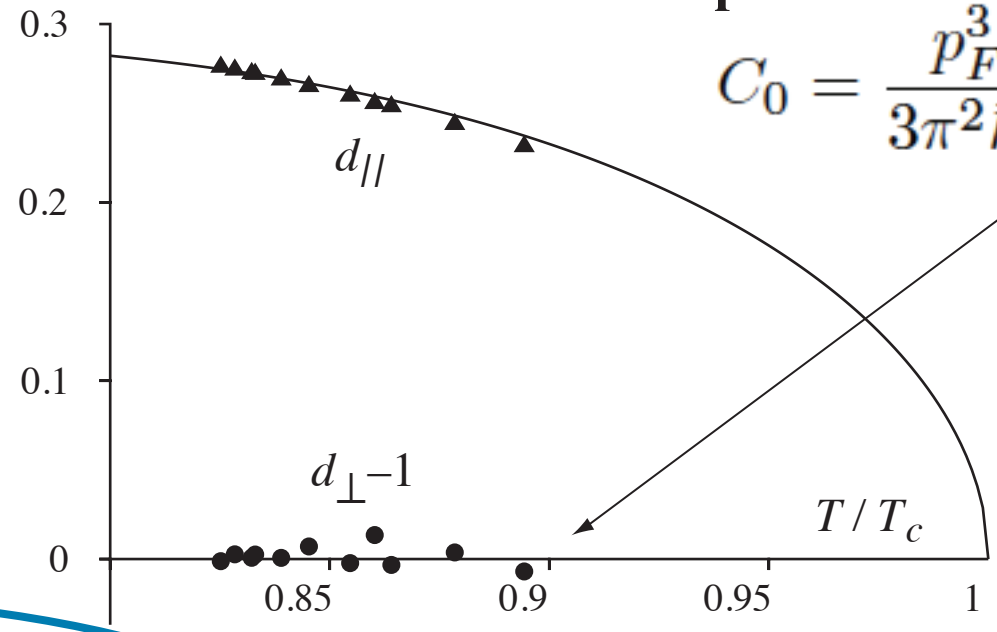
anomalous momentum transfer produces extra force acting on skyrmions

$$\hat{\mathbf{z}} \times (\mathbf{v_L} - \mathbf{v_s}) + d_\perp \hat{\mathbf{z}} \times (\mathbf{v_n} - \mathbf{v_L}) + d_{||} (\mathbf{v_n} - \mathbf{v_L}) = 0$$

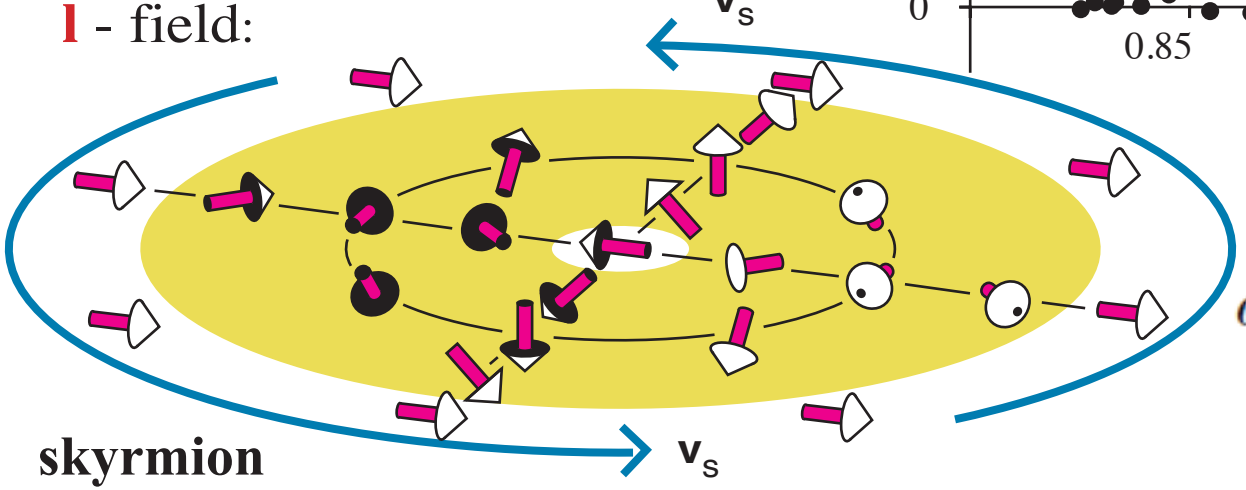
anomaly parameter

$$C_0 = \frac{p_F^3}{3\pi^2\hbar^2} \approx n$$





l - field:

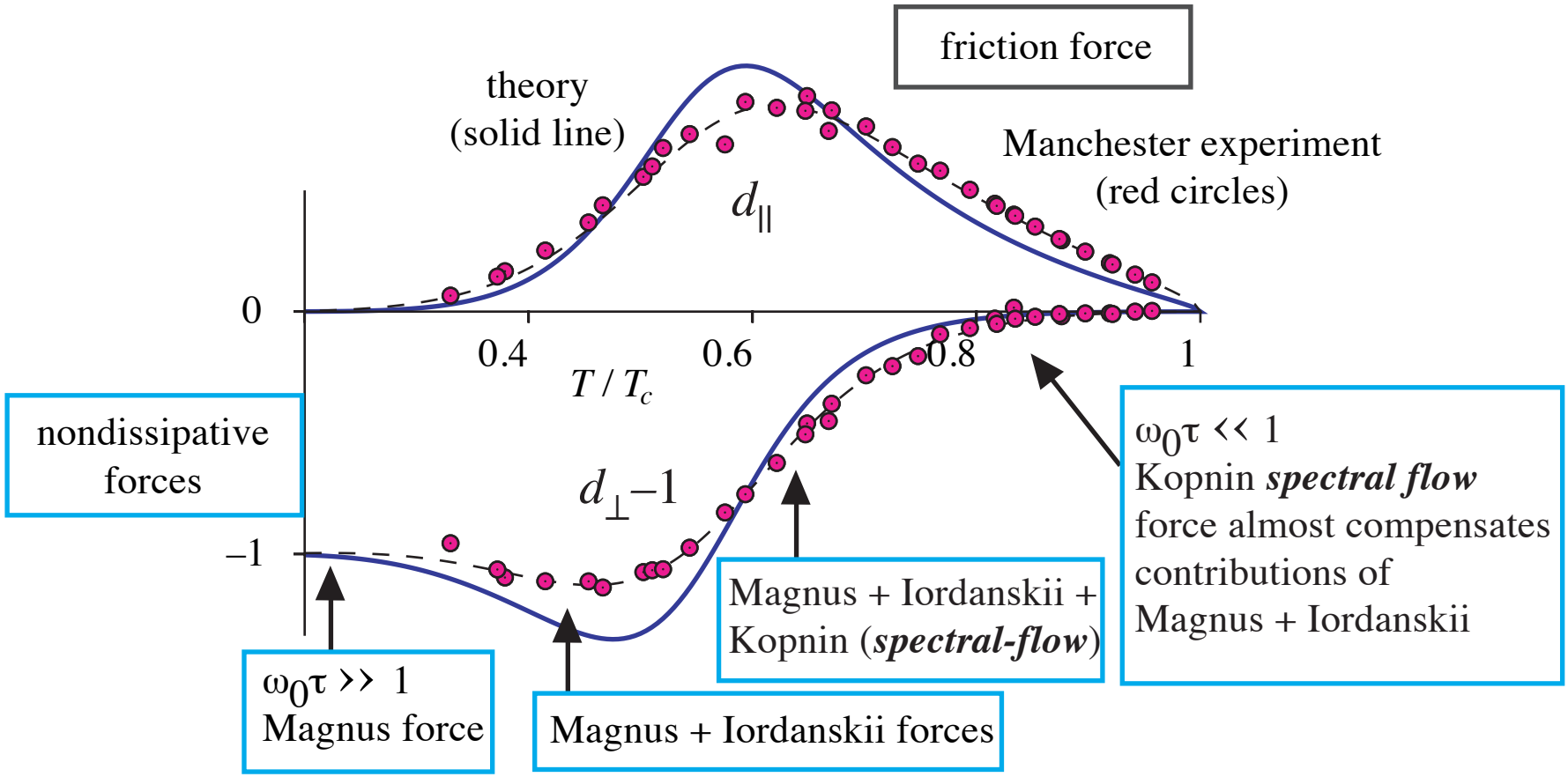


skyrmion

anomaly parameter

$$d_\perp = 1 - \frac{n - C_0}{n_s(T)}, \quad d_{||} = \frac{\gamma}{\pi n_1 n_s(T)}$$

Experimental chiral anomaly: spectral flow force on a vortex



superfluid Reynolds number

$$\mathrm{Re}_\alpha(T) = \frac{1 - d_\perp}{d_{||}} \approx \omega_0\tau$$

$\mathrm{Re}_\alpha(T_c) = 0$ $\mathrm{Re}_\alpha(T \sim 0.6\, T_c) \sim 1$ $\mathrm{Re}_\alpha(0) = \infty$

laminar vortex flow turbulent vortex flow

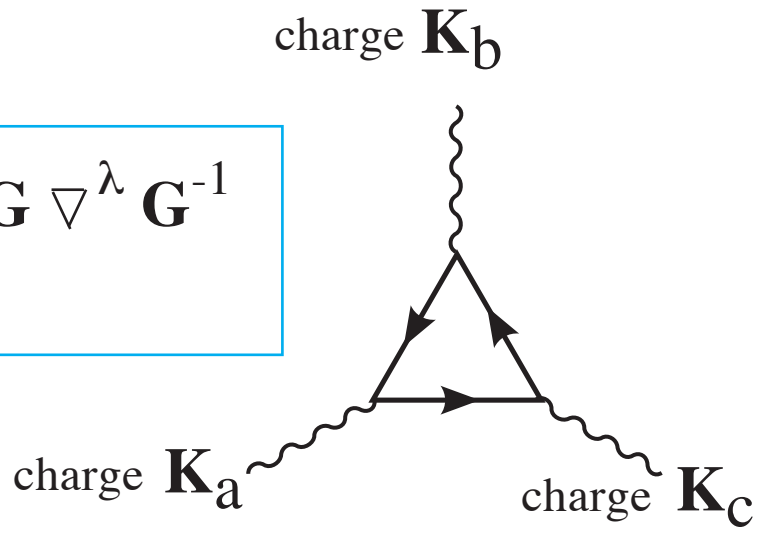
Symmetry protected p-space topological invariants & chiral anomaly

$$K_{abc} = \frac{1}{24\pi^2}\, e_{\mu\nu\lambda}\, \mathbf{tr} \int_{\text{over } S^3} dS\, \mathbf{K_a K_b K_c\, G}\, \nabla^{\mu}\, \mathbf{G}^{-1}\, \mathbf{G}\, \nabla^{\nu}\, \mathbf{G}^{-1} \mathbf{G}\, \nabla^{\lambda}\, \mathbf{G}^{-1}$$

integral is around Weyl point

$\mathbf{G}(p_\mu)$ is Green's function matrix

$\mathbf{K_c}$ are charges of fields acting on fermions
(electric, weak, baryonic, hypercharge, etc.)



triangle Feynman diagram

these invariants determine chiral anomaly effects due 3+1 Weyl points:
electroweak baryogenesis, chiral magnetic effect, Kopnin force, chiral vortical effect, spin quantum Hall effect, ...

p-space invariants are prefactors of topological terms in ***r***-space action

topological origin
of quantization of physical parameters

Chiral effects in QCD quark matter & Weyl superfluid 3He-A

Important part of heavy-ion experimental programs
of RHIC at BNL & of LHC at CERN
is devoted to studies of effects of strong magnetic field.
The most interesting effect is CME

Chiral magnetic effect (CME) due to chiral imbalance

$$F_{CME} = (\mu_L - \mu_R) / 8\pi^2 \int dV \, \mathbf{A}\cdot\mathbf{B}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

current **J** is
along magnetic field **B**

$$\mathbf{J} = \delta F/\delta \mathbf{A} = \mathbf{B}\,(\mu_L - \mu_R) / 4\pi^2$$

current is quantized in terms of p-space topological invariant

$$F_{CME} = N_{CME} / 8\pi^2 \int dV \, \mathbf{A}\cdot\mathbf{B}$$

$$N_{CME} = \frac{1}{24\pi^2}\, e_{\alpha\beta\gamma}\, \mathbf{tr} \int dS\, \mathbf{Q}^2 \mu\, \mathbf{G}^{,\alpha}\, \mathbf{G}^{-1}\, \mathbf{G}^{,\beta}\, \mathbf{G}^{-1} \mathbf{G}^{,\gamma}\, \mathbf{G}^{-1}$$

μ matrix of chemical potentials $\mathbf{Q}$ matrix of electric charges

Analog of chiral magnetic effect in 3He-A

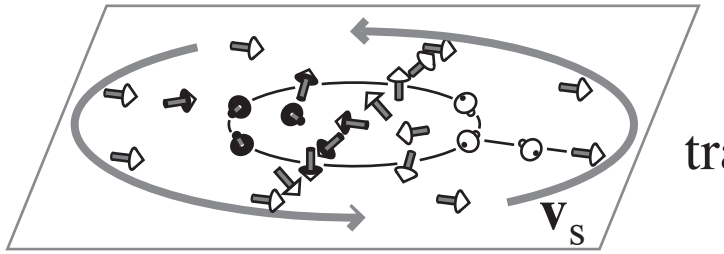
$$F_{CME} = (\mu_L - \mu_R) / 8\pi^2 \int dV \mathbf{A}\cdot\mathbf{B}$$

$$\mathbf{A} = p_F \mathbf{l} \qquad \mathbf{B} = p_F \nabla \times \mathbf{l}$$

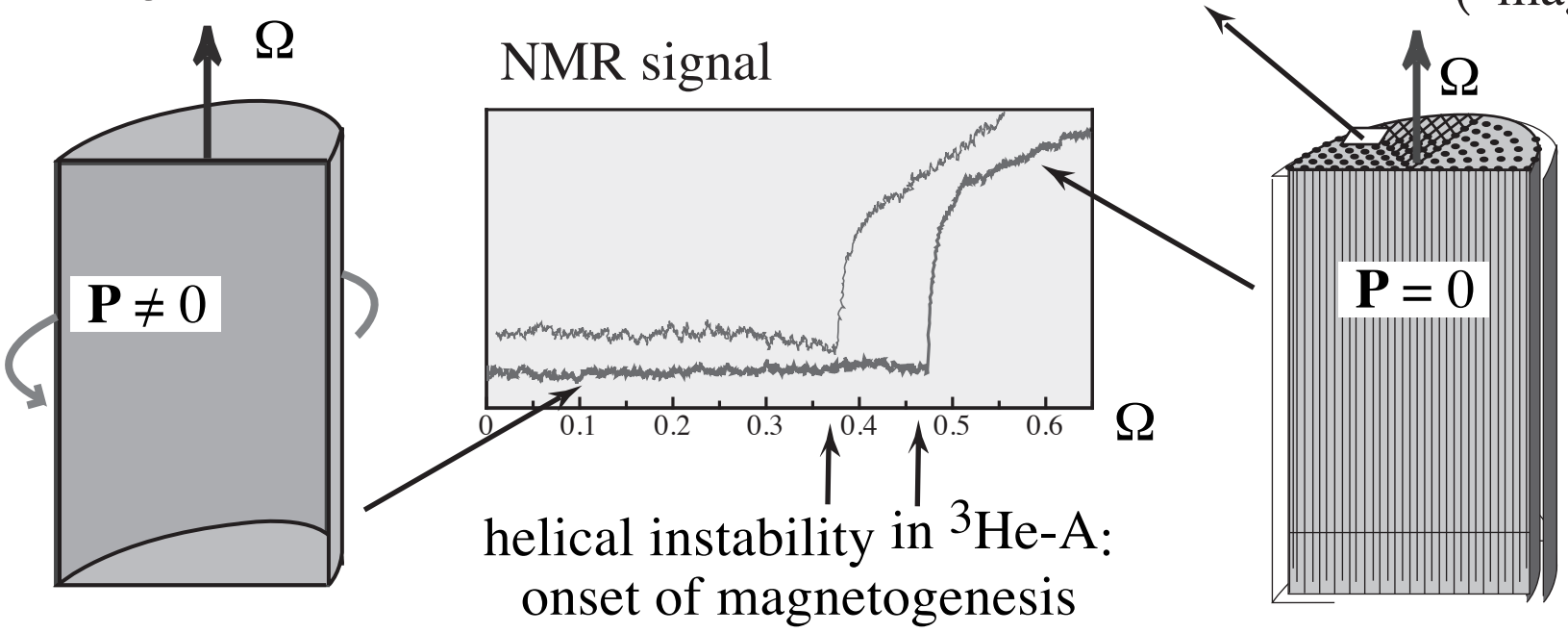
$$\mu_L - \mu_R = 2\, p_F\, \mathbf{l}\cdot(\mathbf{v}_n - \mathbf{v}_s)$$

analog of CME leads to helical instability in 3He-A (magnetogenesis)

chiral imbalance is produced by counterflow:
$\mu_L - \mu_R = 2\, p_F\, \mathbf{l}\cdot(\mathbf{v}_n - \mathbf{v}_s)$
no skyrmions
(no "magnetic" field)



$\mu_L - \mu_R = 0$;
chiral instability
transforms counterflow to
lattice of skyrmions
("magnetic" field **B**)



Helsinki 1996

quantum turbulence in chiral 3He-A in future ROTA experiments
effect of Weyl fermions, chiral anomaly, CME, CVE, Majorana core fermions

Chiral vortical effect (CVE) in quark liquid due to chiral imbalance

$$F_{CVE} = (\mu_L^2 - \mu_R^2) / 8\pi^2 \int dV \mathbf{A}\cdot\mathbf{B}_g$$

$\mathbf{B}_g$ is gravimagnetic field

current is quantized
in terms of p-space topological invariant

current **J** is
along rotation axis $\Omega = \mathbf{B}_g / c^2$

$$\mathbf{J} = \delta F/\delta \mathbf{A} = \Omega\, (\mu_L^2 - \mu_R^2) / 4\pi^2 c^2$$

$$F_{CVE} = N_{CVE} / 8\pi^2 \int dV \mathbf{A}\cdot\mathbf{B}_g$$

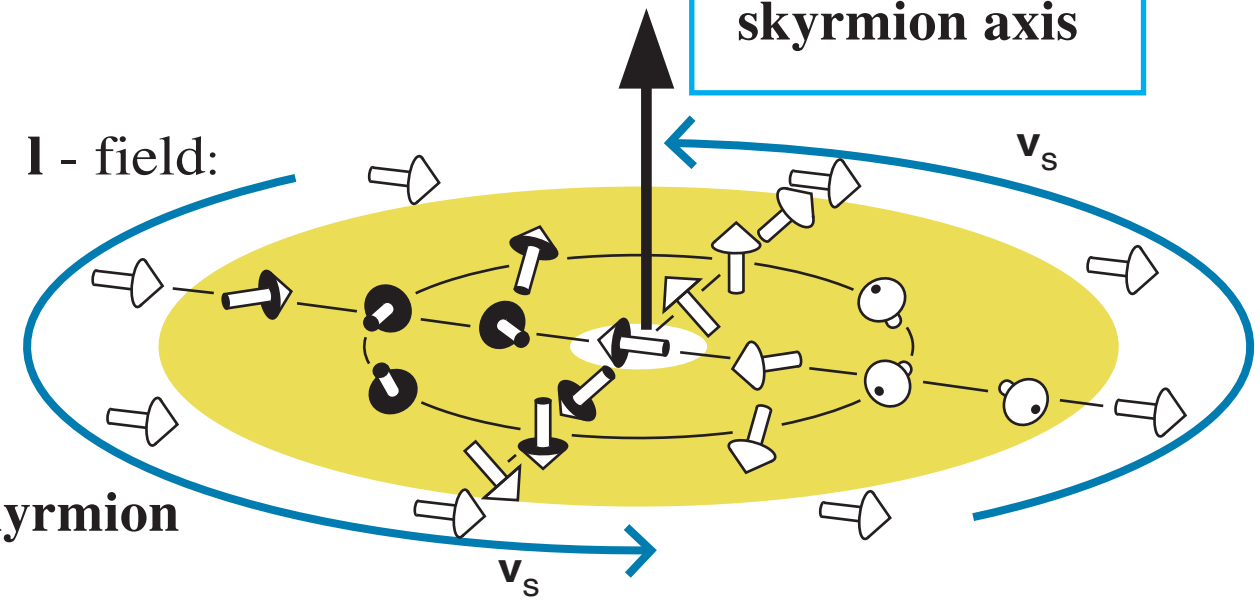
$$N_{CVE} = \frac{1}{24\pi^2}\, e_{\alpha\beta\gamma}\, \mathbf{tr} \int dS\, \mathbf{Q}\mu^2\, \mathbf{G}^{,\alpha}\, \mathbf{G}^{-1}\, \mathbf{G}^{,\beta}\, \mathbf{G}^{-1} \mathbf{G}^{,\gamma}\, \mathbf{G}^{-1}$$

μ matrix of chemical potentials
Q matrix of electric charges

suggestion for ROTA group:
CVE in rotating 3He-A

current J along skyrmion axis

l - field:

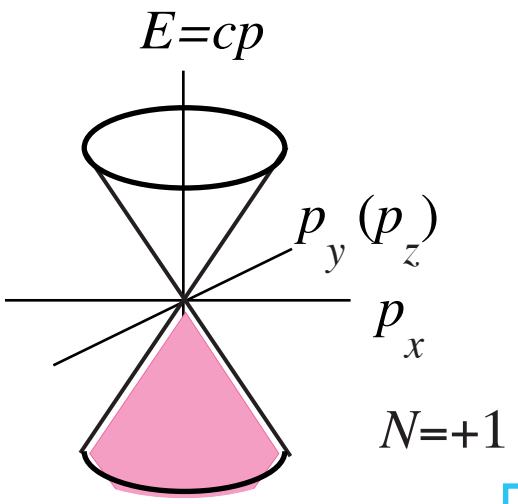


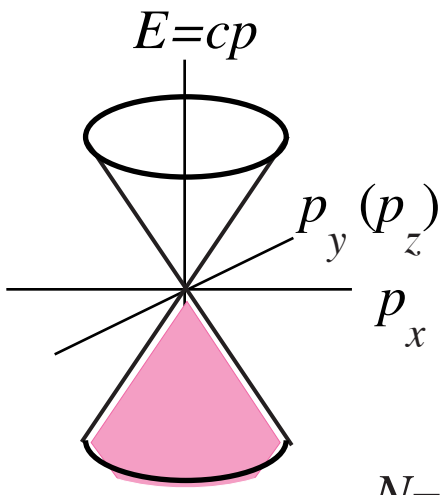
continuous vortex - skyrmion

From massless Weyl particles to massive Dirac particles

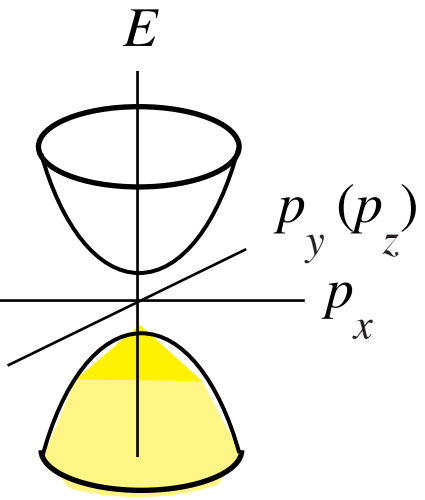
where are massive Dirac particles?

Dirac particle - composite object:
mixture of left and right Weyl particles





$T_{ew} \sim 1$ TeV$\sim 10^{16}$K



$N=+1-1=0$

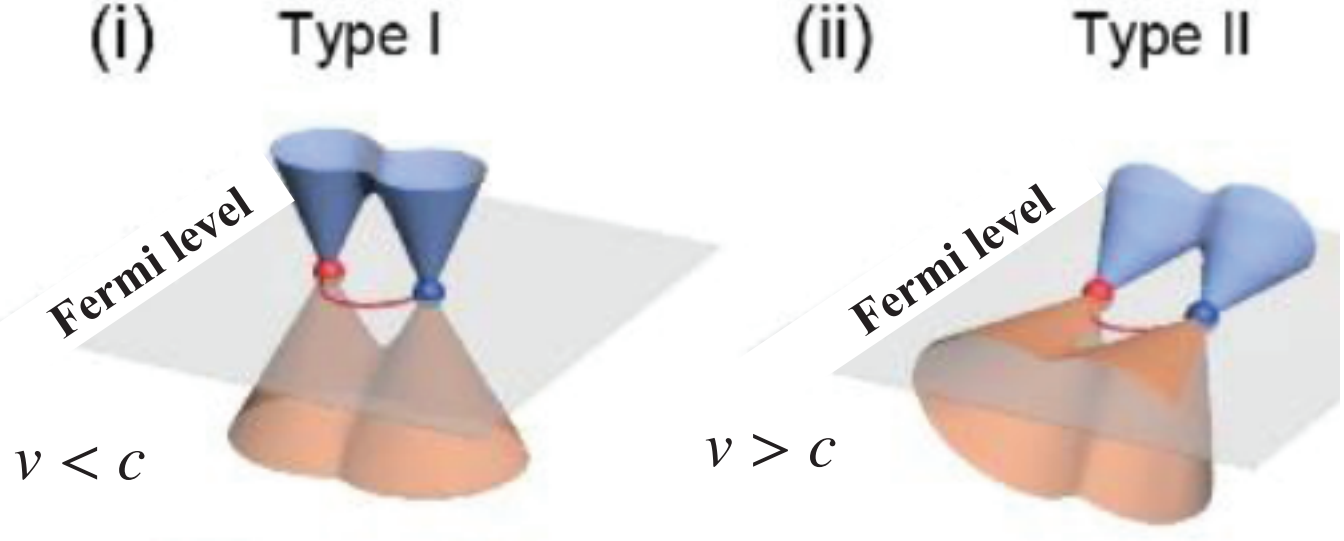
$E^2 = c^2p^2 + m^2$

is Dirac vacuum topologically trivial ?

fully gapped vacua
can be also topologically nontrivial
(3He-B, topological insulators, ...)

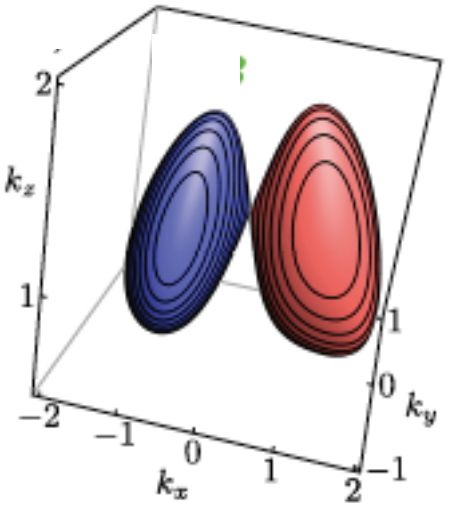
type II Weyl fermions in semimetals

$$H = c\,\sigma\cdot\mathbf{p} - vp_z$$



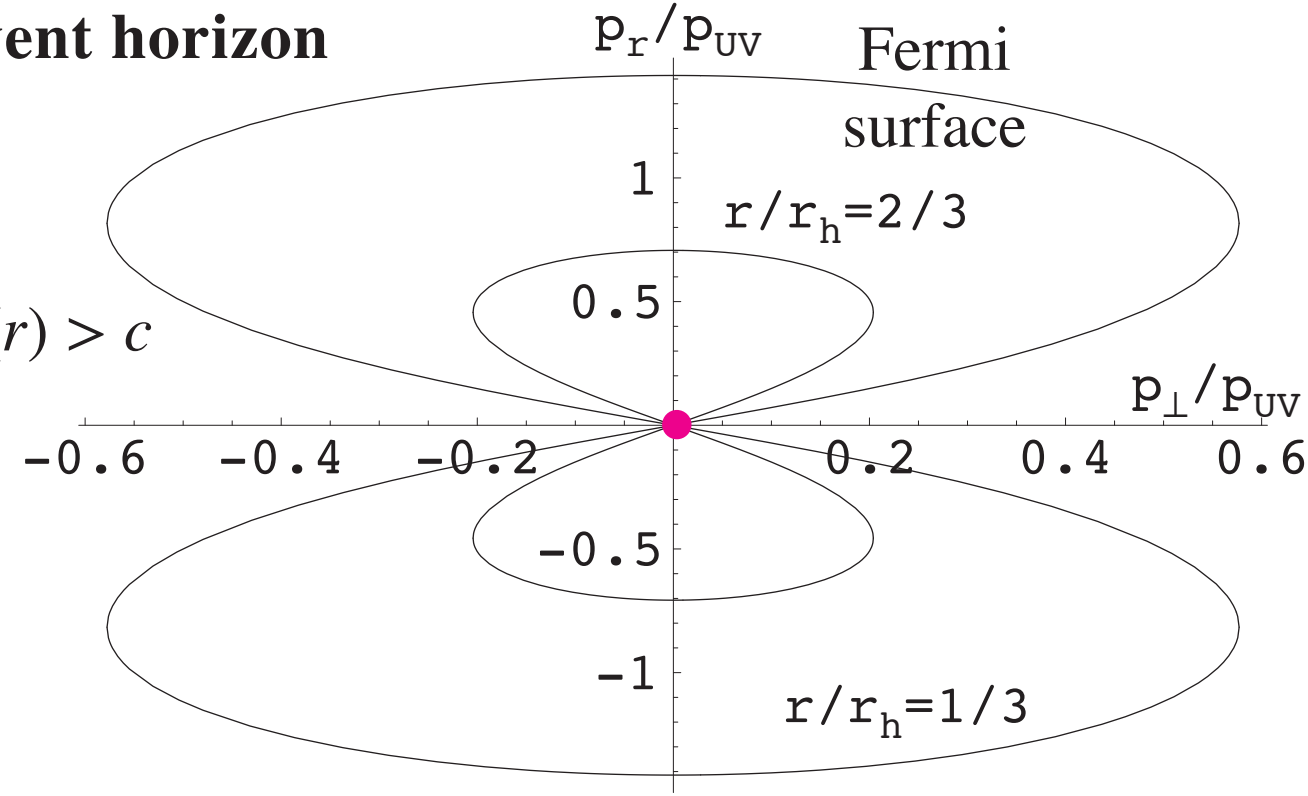
GV & Zubkov
Emergent Weyl spinors in multi-fermion systems
NPB **881**, 514 (2014)

Fermi surface



type II Weyl fermions behind event horizon

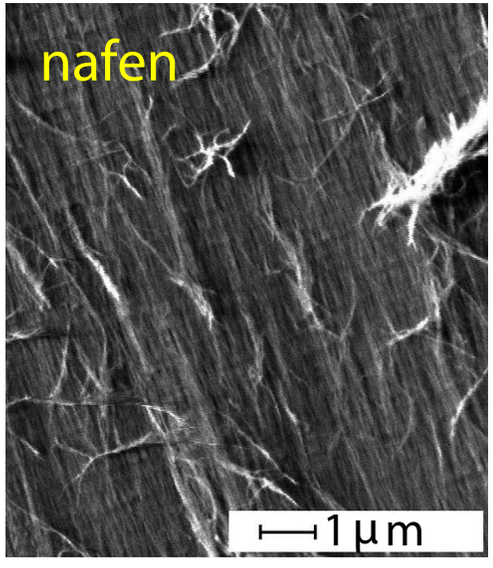
$$H = c\,\sigma\cdot\mathbf{p} - v(r)p_r \qquad v(r) > c$$



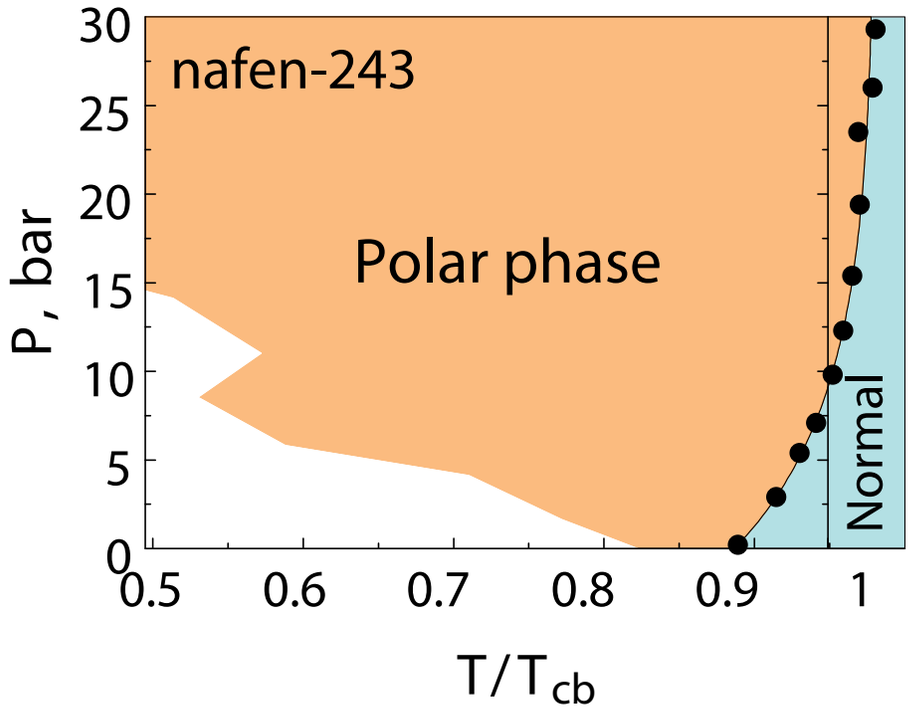
Huhtala & GV
Fermionic microstates
within Painleve-Gullstrand black hole
JETP **94**, 853 (2002)

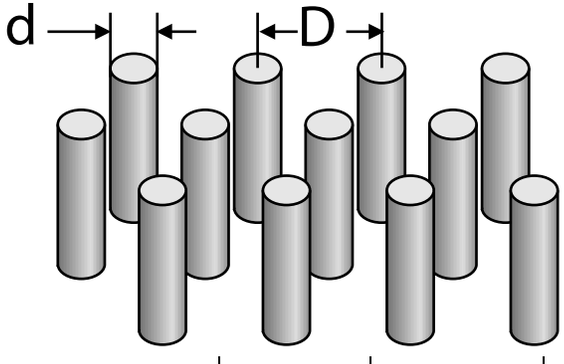
POLAR PHASE OF SUPERFLUID 3He in NAFEN

Engineering new topological superfluids by nanostructural confinement.



$$A_{\mu j} = \Delta d_{\mu} m_j e^{i\phi}$$





	open	d, nm	D, nm
nafen-90	98%	8	47
nafen-243	94%	9	32

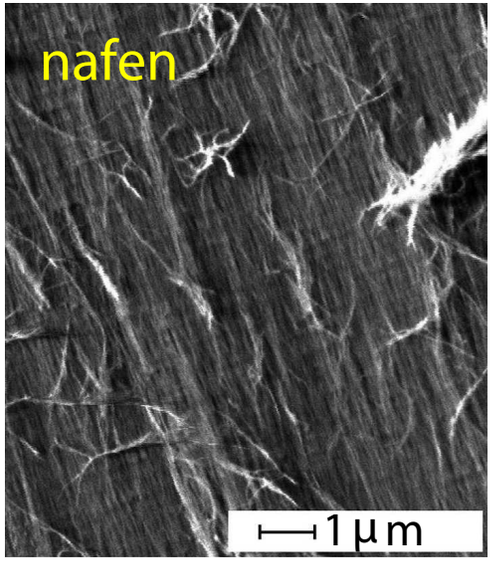
$$H = \left(\frac{p^2}{2m} - \mu\right)\tau_3 - c\tau_1\sigma_z p_z$$

Dmitriev et al, PRL (2015) [arXiv:1507.04275]

Dirac nodal line & surface flat band in polar phase of 3He in NAFEN

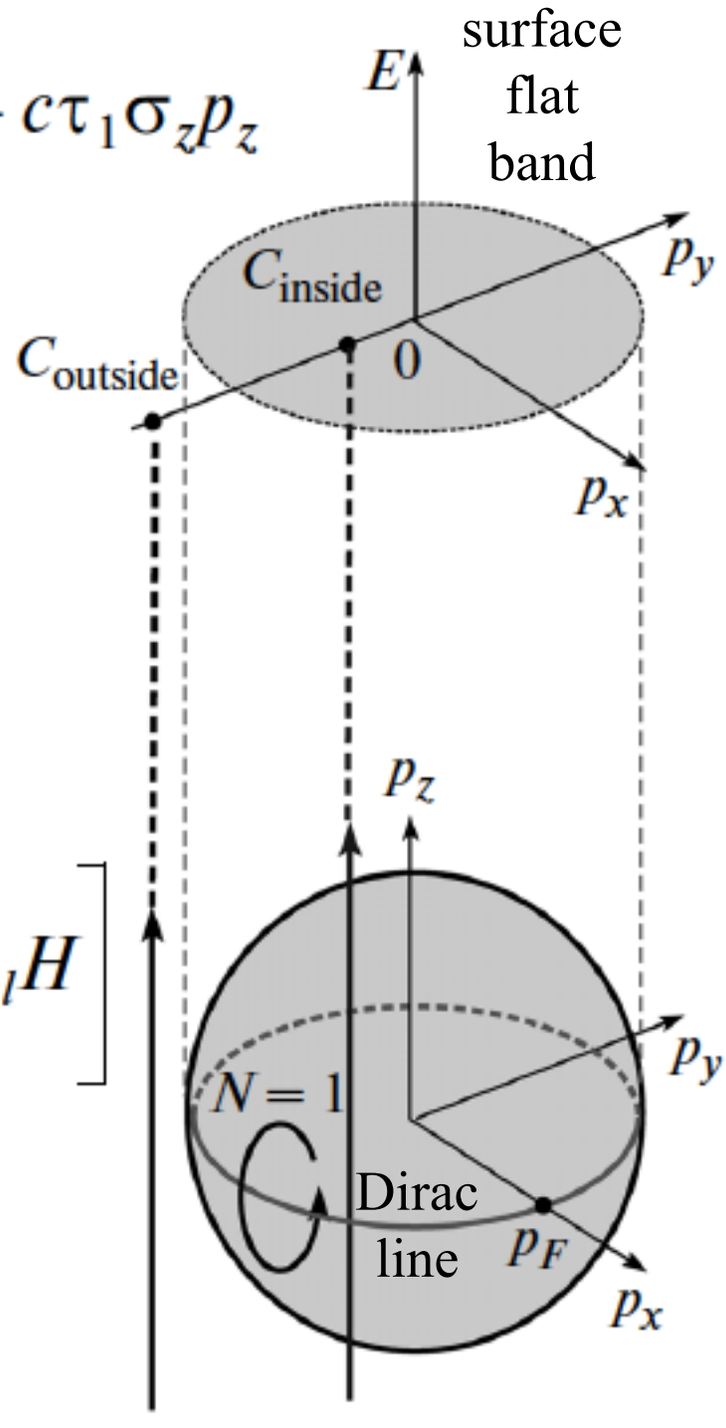
$$A_{\mu j} = \Delta d_{\mu} m_j e^{i\phi}$$

$$H = \left(\frac{p^2}{2m} - \mu\right)\tau_3 - c\tau_1\sigma_z p_z$$



topological invariant
for line node

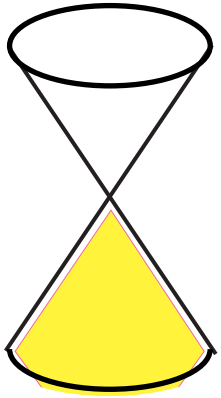
$$N_K = \frac{1}{4\pi i}\mathrm{Tr}\left[K\oint_C dl\, H^{-1}\nabla_l H\right]$$

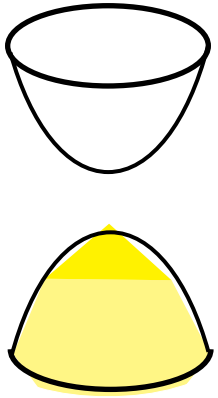


3D topological superfluids / insulators / semiconductors / vacua

gapless topologically nontrivial vacua

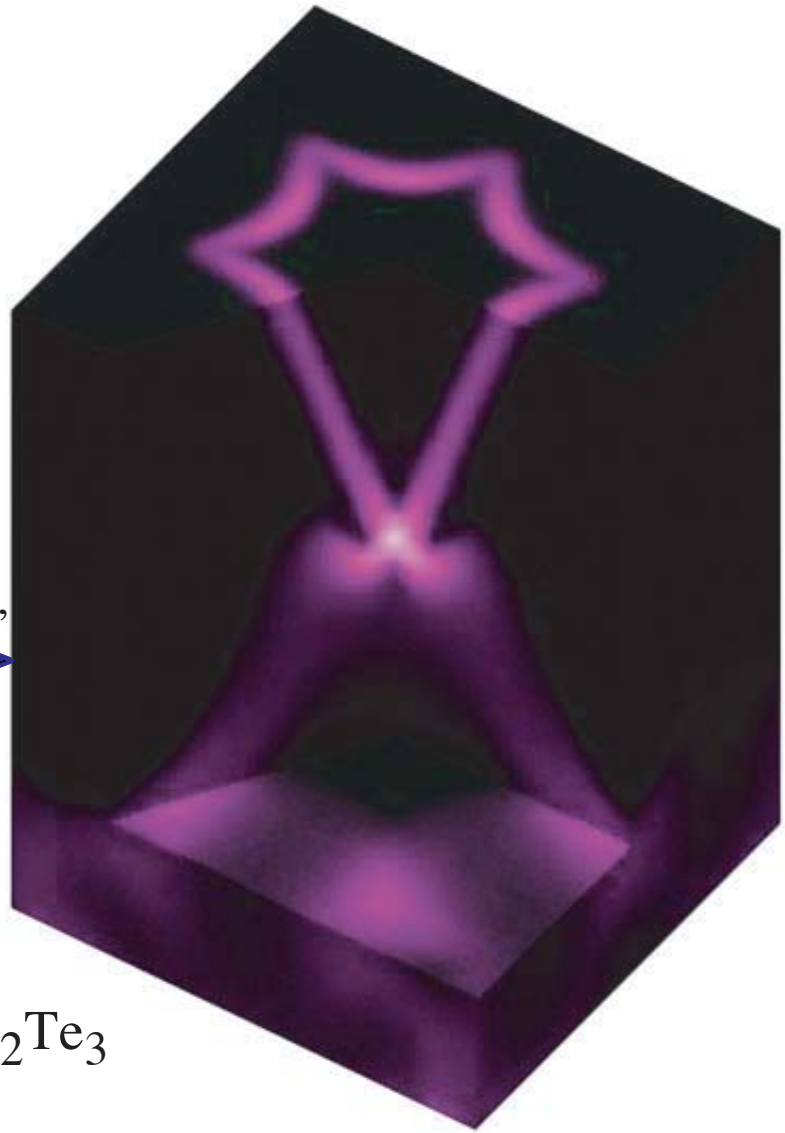
fully gapped topologically nontrivial vacua





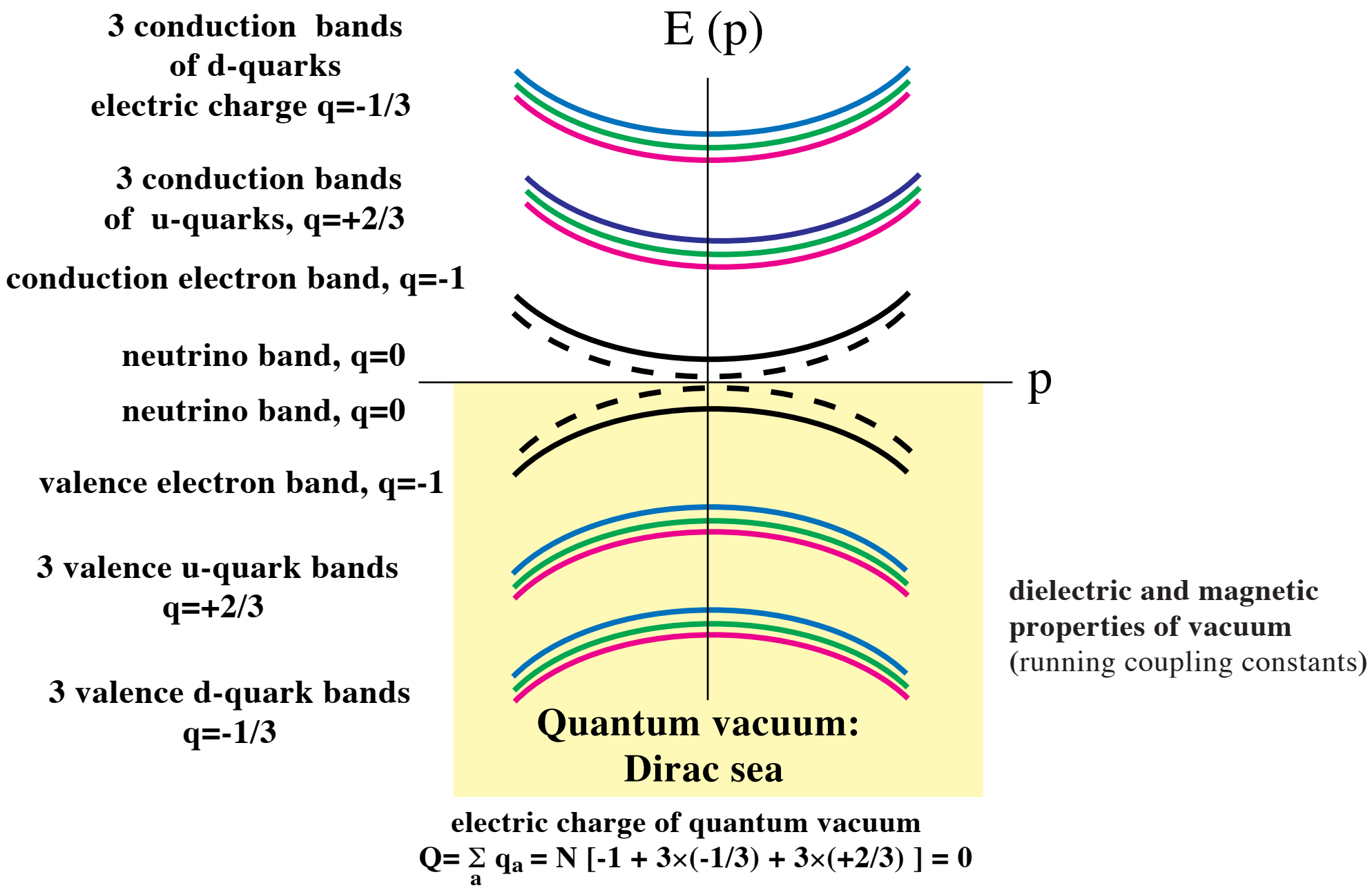
3He-A,
Standard Model
above electroweak transition,
semimetals,
4D graphene
(cryocrystalline vacuum)

3He-B,
Standard Model
below electroweak transition,
topological insulators,
triplet & singlet
chiral superconductor, ...



Bi_2Te_3

Present vacuum as semiconductor or insulator



electric charge of quantum vacuum

$$Q = \sum_a q_a = N\,[-1 + 3\times(-1/3) + 3\times(+2/3)\,] = 0$$

fully gapped 3+1 topological matter

superfluid ^{3}He-B, topological insulator Bi_2Te_3 , present vacuum of Standard Model

* Standard Model vacuum as topological insulator

Topological invariant protected by symmetry

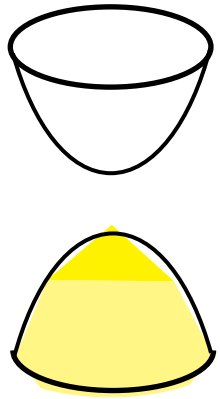
$$N_K = \frac{1}{24\pi^2} e_{\mu\nu\lambda} \mathbf{tr} \int_{\text{over 3D momentum space}} dV\, K\, \mathbf{G}\nabla^{\mu}\mathbf{G}^{-1}\,\mathbf{G}\nabla^{\nu}\mathbf{G}^{-1}\mathbf{G}\nabla^{\lambda}\mathbf{G}^{-1}$$

G is Green's function at $\omega=0$, K is symmetry operator $\mathbf{GK} = +/- \mathbf{KG}$

Standard Model vacuum: $K=\gamma_5$ $\mathbf{G}\gamma_5 = -\gamma_5\mathbf{G}$

$$N_K = 8n_g$$

8 massive Dirac particles in one generation

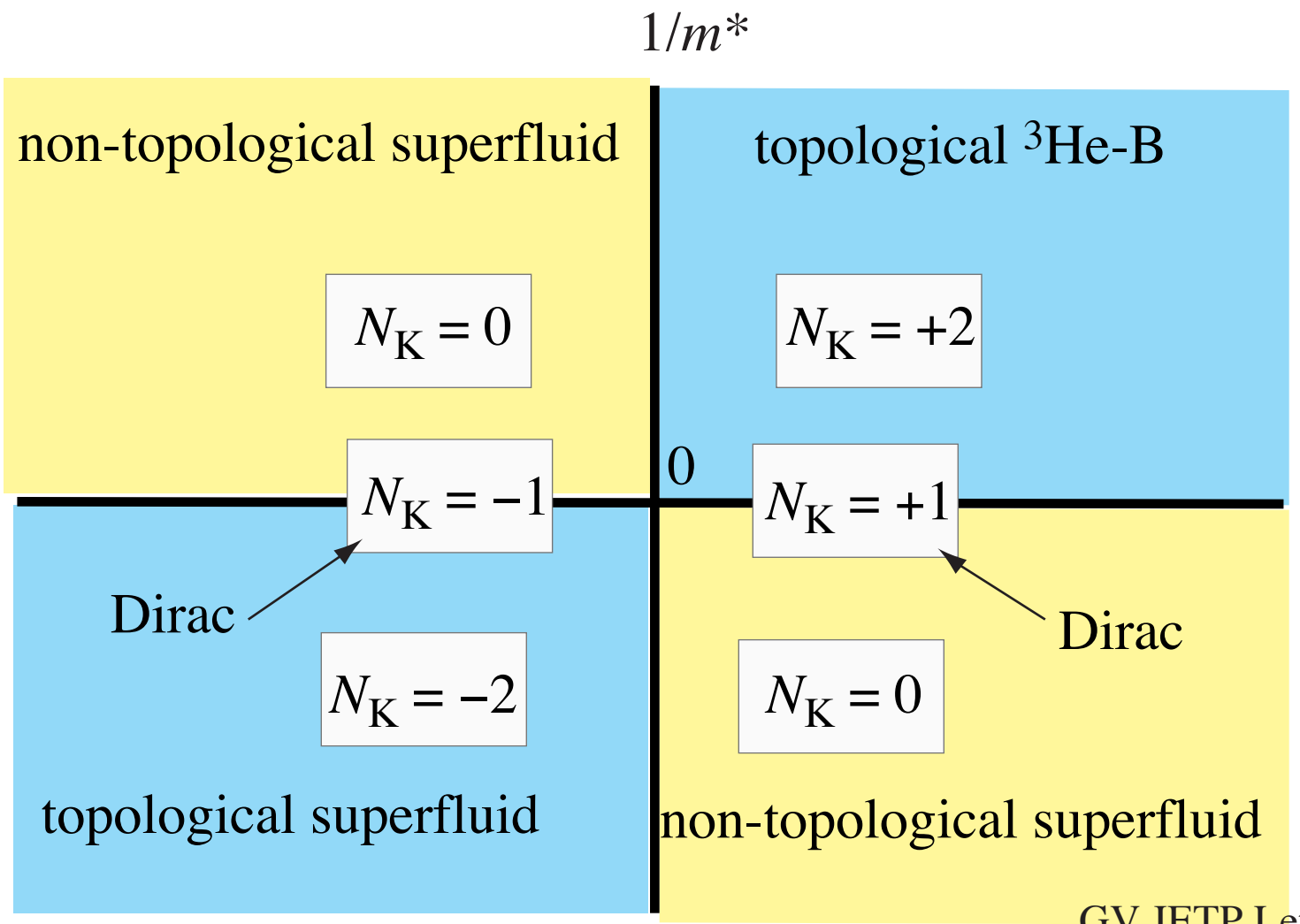


topological superfluid ^{3}He-B

$$H\tau_2 = -\tau_2 H$$

$$K = \tau_2$$

$$H = \begin{pmatrix} \frac{p^2}{2m^*} - \mu & c_B \sigma\cdot\mathbf{p} \\ c_B \sigma\cdot\mathbf{p} & -\frac{p^2}{2m^*} + \mu \end{pmatrix} = \left(\frac{p^2}{2m^*} - \mu \right)\tau_3 + c_B \sigma\cdot\mathbf{p}\, \tau_1$$



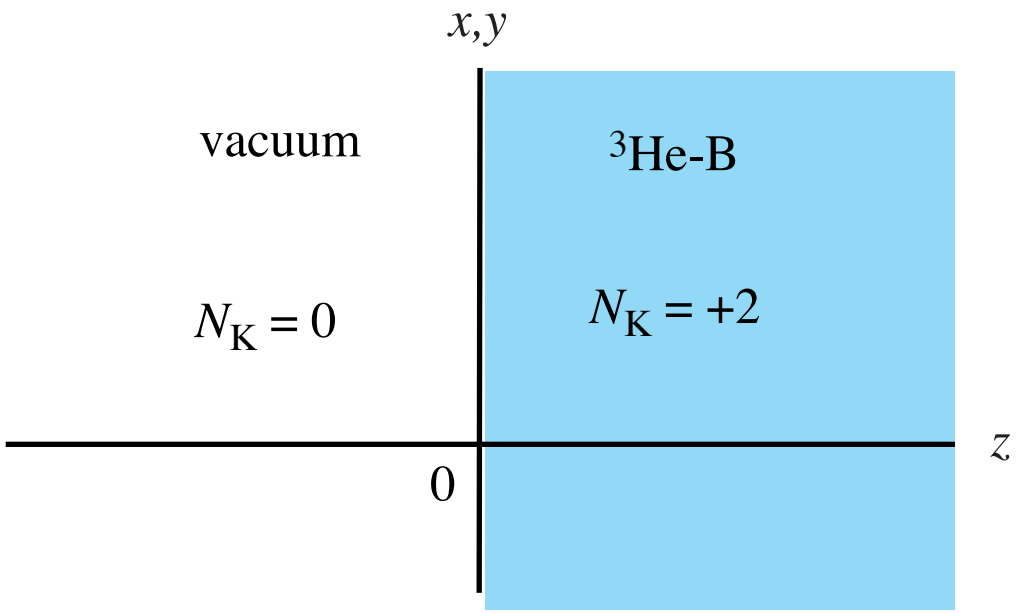
Dirac vacuum

$1/m^* = 0$

$$H = \begin{pmatrix} -M & c_B \sigma\cdot\mathbf{p} \\ c_B \sigma\cdot\mathbf{p} & +M \end{pmatrix}$$

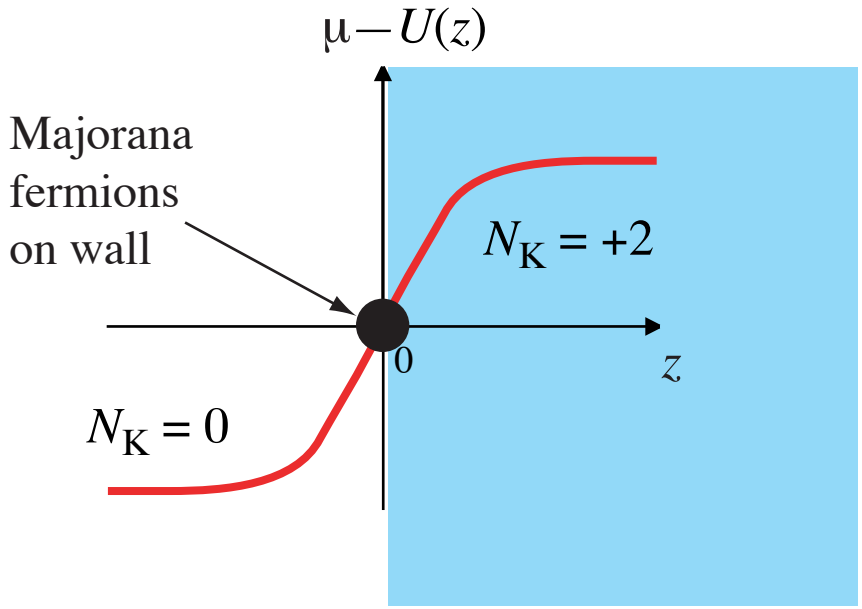
GV JETP Lett. **90**, 587 (2009)

Boundary of 3D gapped topological superfluid



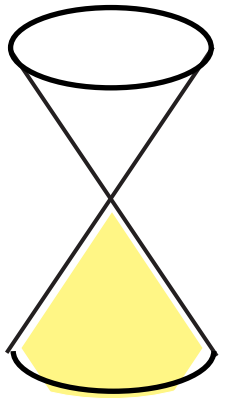
$$H = \begin{pmatrix} \frac{p^2}{2m^*} - \mu + U(z) & c_B \sigma \cdot \mathbf{p} \\ c_B \sigma \cdot \mathbf{p} & -\frac{p^2}{2m^*} + \mu - U(z) \end{pmatrix}$$

spectrum of Majorana fermion zero modes

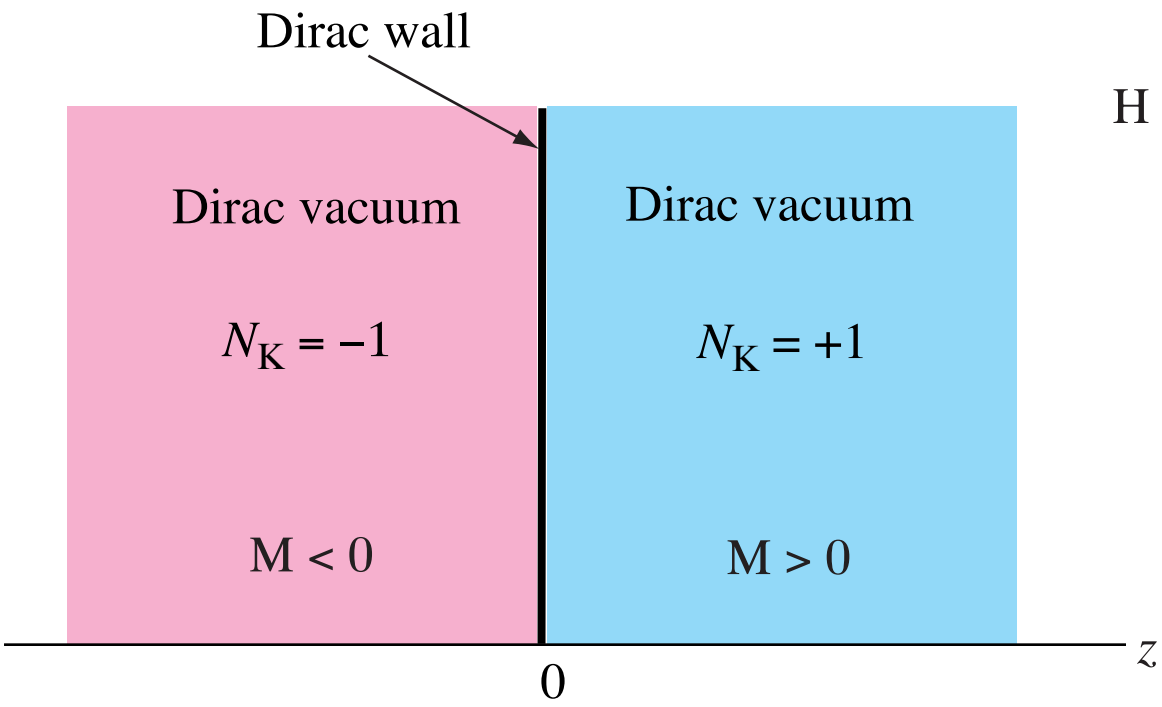


$$H_{zm} = c_B \hat{\mathbf{z}} \cdot \sigma \times \mathbf{p} = c_B (\sigma_x p_y - \sigma_y p_x)$$

helical fermions

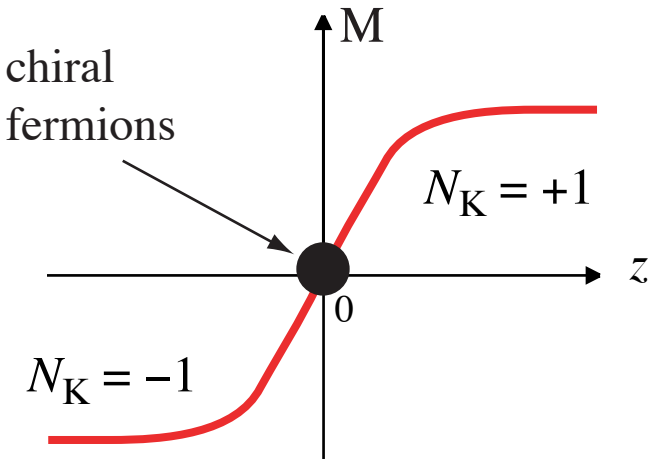


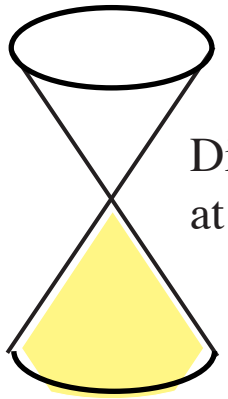
fermion zero modes on Dirac wall

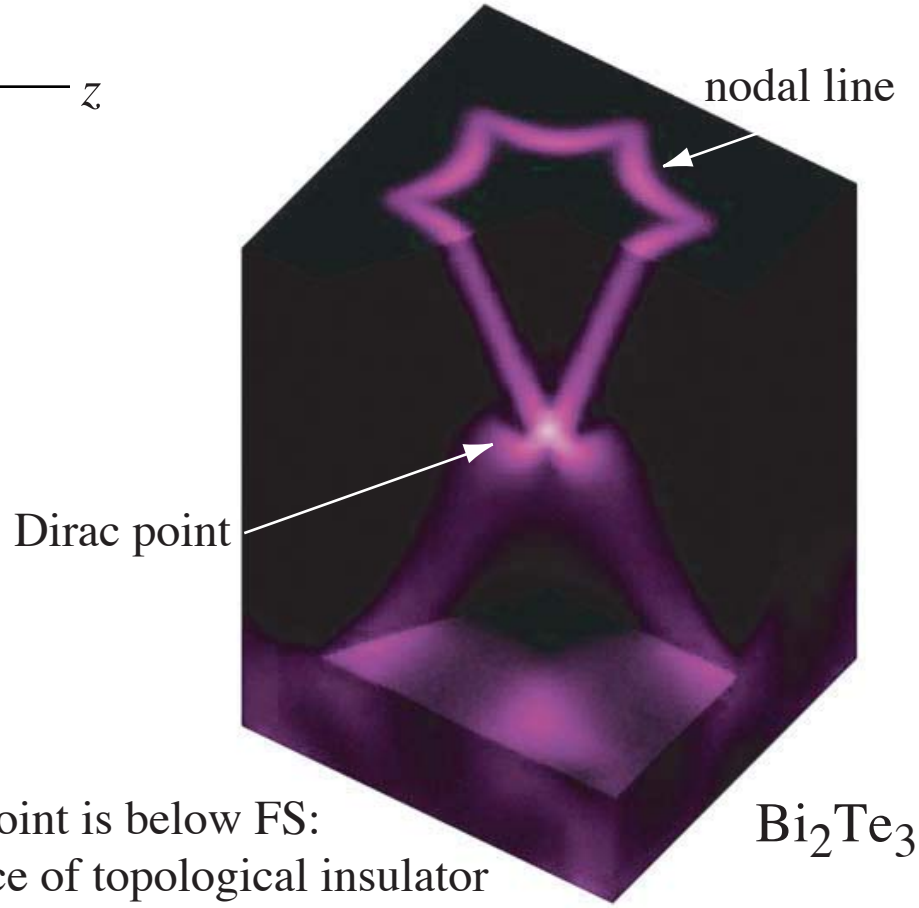


$$H = \begin{pmatrix} -M(z) & c\sigma\cdot\mathbf{p} \\ c\sigma\cdot\mathbf{p} & +M(z) \end{pmatrix}$$

Volkov-Pankratov,
2D massless fermions
in inverted contacts
JETP Lett. **42**, 178 (1985)





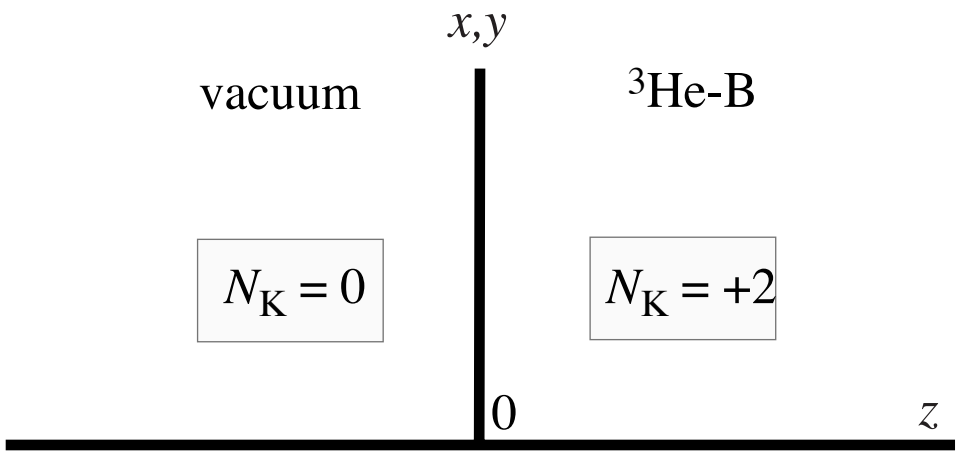


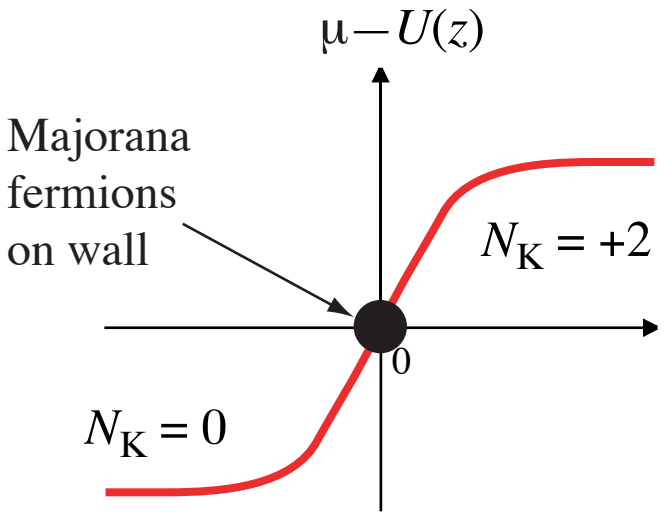
in Bi_2Te_3 Dirac point is below FS:
nodal line on surface of topological insulator

Majorana fermions: edge states on the boundary of 3D gapped topological matter

$$H = \begin{pmatrix} \frac{p^2}{2m^*} - \mu + U(z) & c\sigma\cdot\mathbf{p} \\ c\sigma\cdot\mathbf{p} & \frac{p^2}{2m^*} + \mu - U(z) \end{pmatrix}$$

* boundary of topological superfluid ^{3}He-B



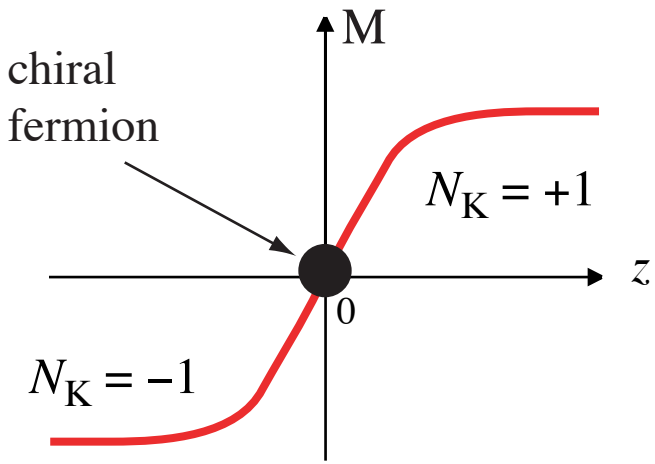


spectrum of fermion zero modes

$$H_{zm} = c\,(\sigma_x p_y - \sigma_y p_x)$$

helical fermions

* Dirac domain wall

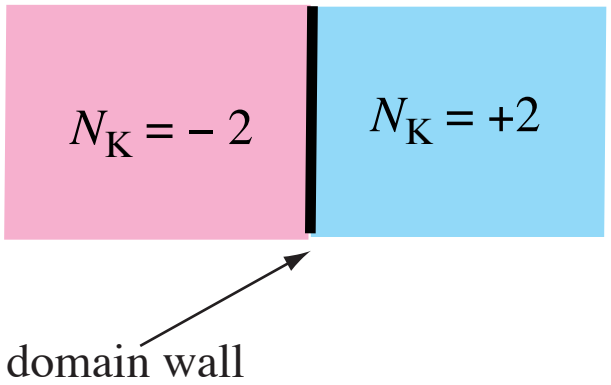


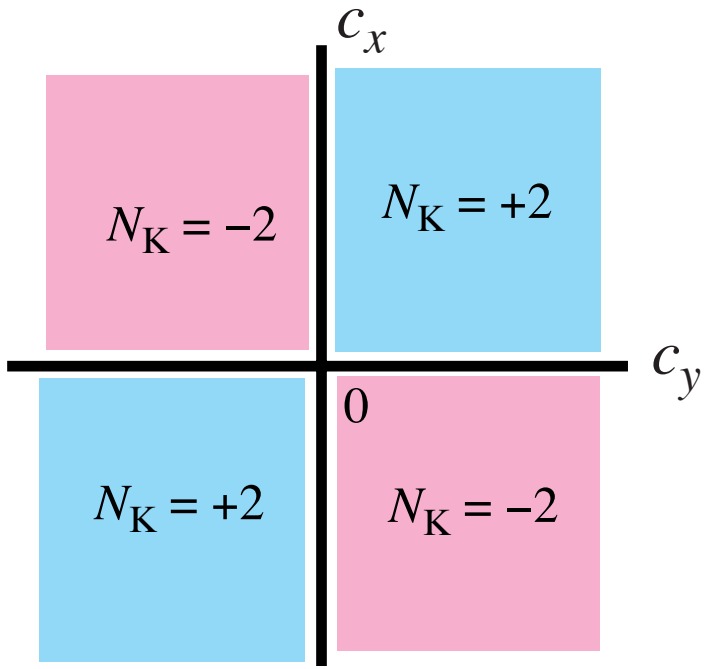
$$H = \begin{pmatrix} -M(z) & c\sigma\cdot\mathbf{p} \\ c\sigma\cdot\mathbf{p} & +M(z) \end{pmatrix}$$

Volkov-Pankratov,
2D massless fermions
in inverted contacts
JETP Lett. **42**, 178 (1985)

Majorana fermions on interface in topological superfluid ^{3}He-B

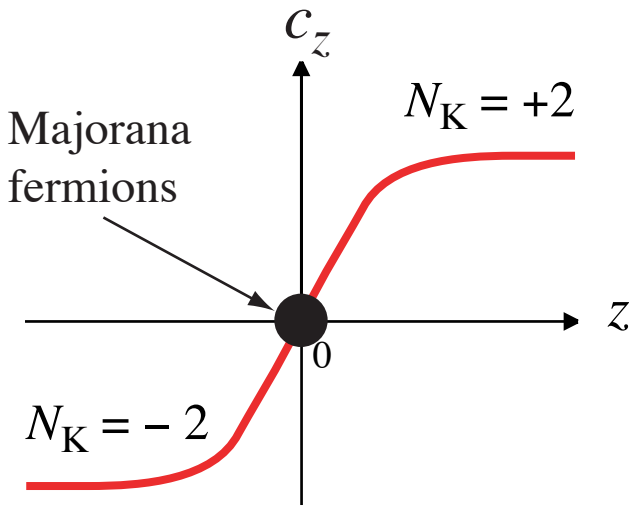
$$H = \begin{pmatrix} \frac{p^2}{2m^*} - \mu & \sigma_x c_x p_x + \sigma_y c_y p_y + \sigma_z c_z p_z \\ \sigma_x c_x p_x + \sigma_y c_y p_y + \sigma_z c_z p_z & -\frac{p^2}{2m^*} + \mu \end{pmatrix}$$

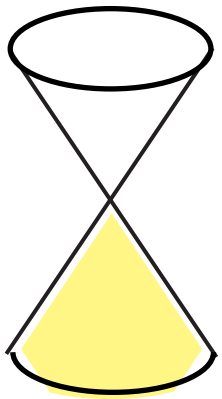




phase diagram

one of 3 "speeds of light" changes sign across wall





spectrum of fermion zero modes

$$H_{zm} = c\,(\sigma_x p_y - \sigma_y p_x)$$

Zero energy states in the core of vortices in topological superfluids

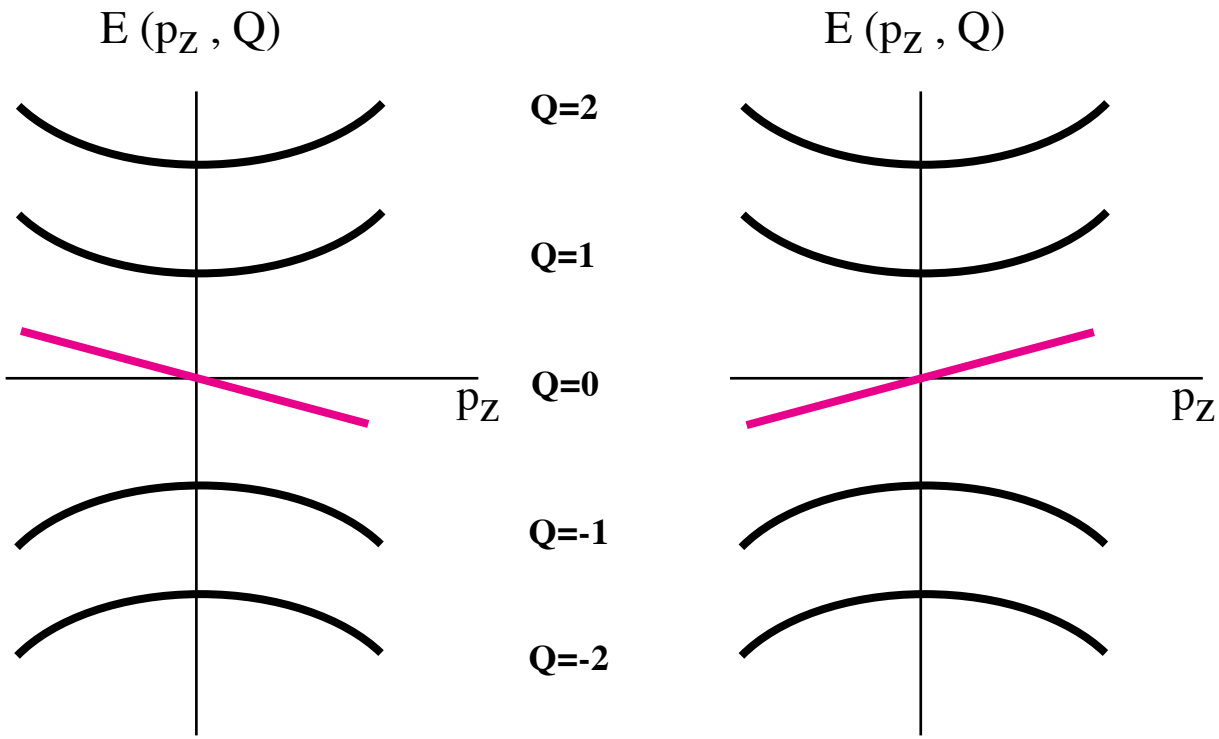
vortices in fully gapped 3+1 system

fermion zero modes in vortex core

Bound states of fermions on cosmic strings and vortices

Spectrum of quarks in core of electroweak cosmic string

quantum numbers: Q - angular momentum & p_z - linear momentum



$E(p_z) = - cp_z$ for d quarks $\qquad$ $E(p_z) = cp_z$ for u quark

asymmetric branches cross zero energy

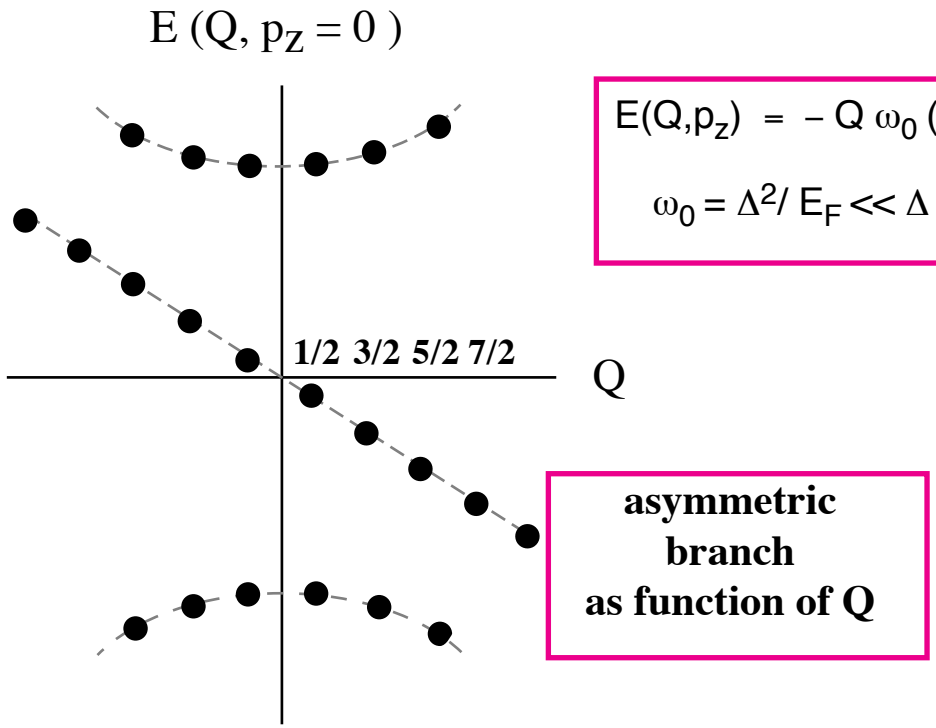
Index theorem: Number of asymmetric branches = N
N is vortex winding number

Jackiw & Rossi
Nucl. Phys. B**190**, 681 (1981)

Bound states of fermions on vortex in s-wave superconductor

Caroli, de Gennes & J. Matricon, Phys. Lett. **9** (1964) 307

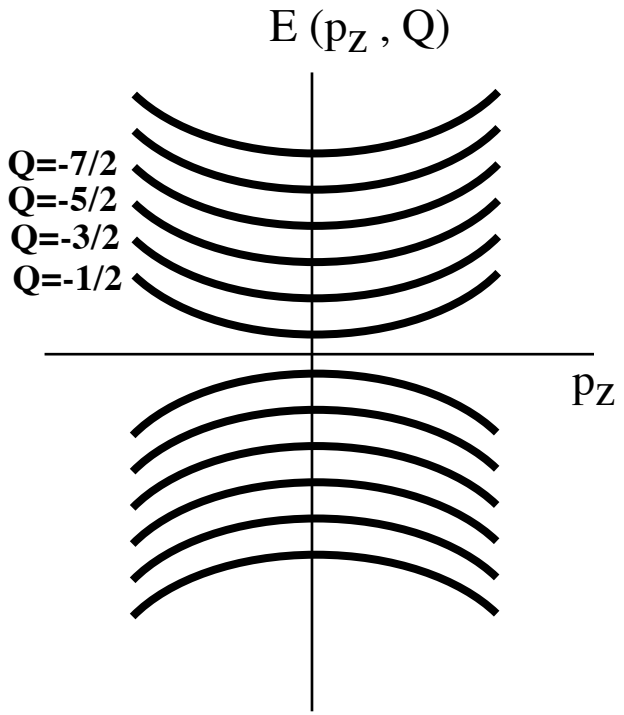
$N_K = 0$



$E(Q,p_z) = -Q\,\omega_0(p_z)$

$\omega_0 = \Delta^2 / E_F << \Delta$

asymmetric branch as function of Q



Angular momentum Q is half-odd integer in s-wave superconductor

no true fermion zero modes: no asymmetric branch as function of p_z

Index theorem for approximate fermion zero modes:

Number of asymmetric Q-branches = 2N
N is vortex winding number

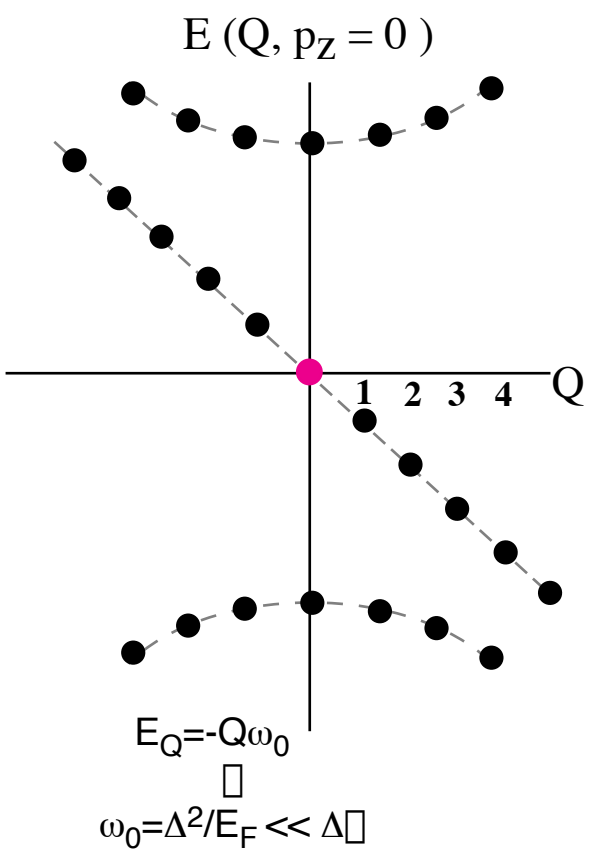
GV JETP Lett. **57**, 244 (1993)

Index theorem for true fermion zero modes?

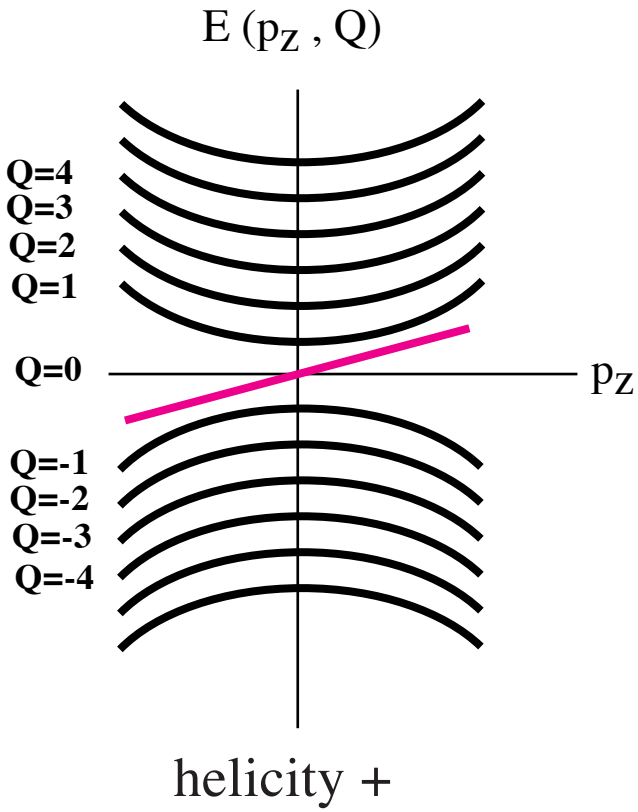
is the existence of fermion zero modes related to topology in bulk?

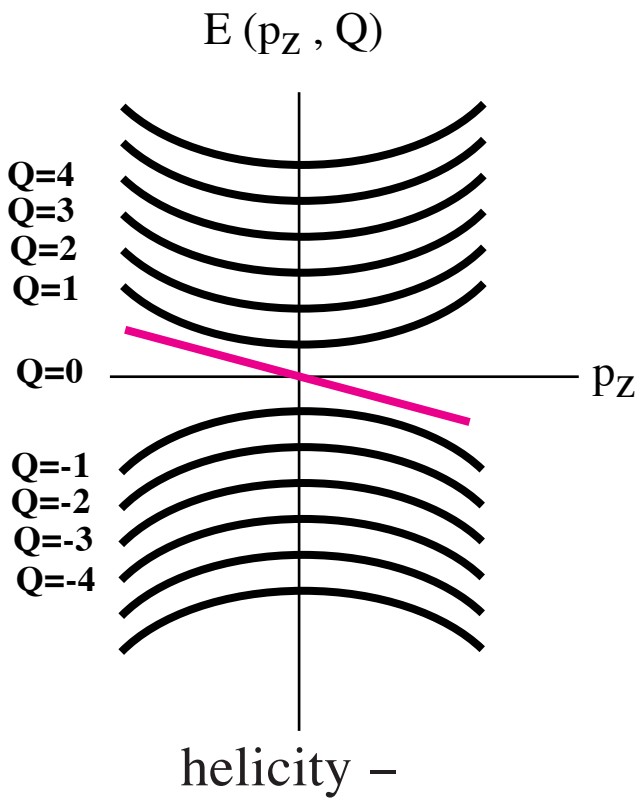
fermions zero modes on symmetric vortex in 3He-B

topological ^{3}He-B at $\mu > 0$: $N_K = 2$



Q is integer
for p-wave superfluid ^{3}He-B





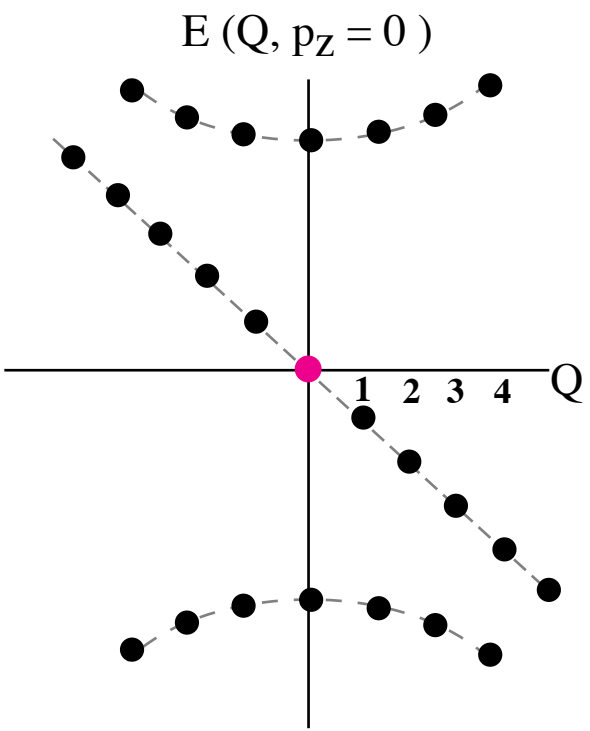
gapless fermions on Q=0 branch form

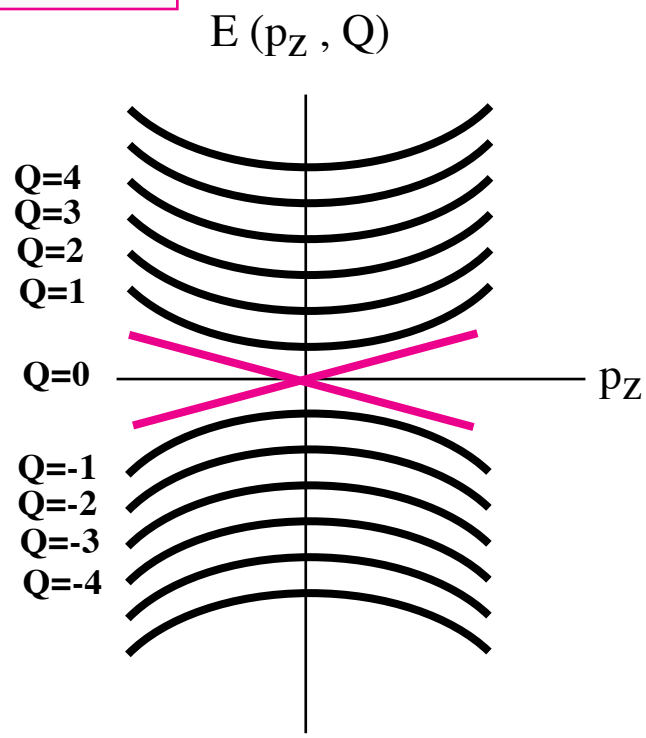
1D Fermi-liquid

Misirpashaev & GV
Fermion zero modes in symmetric vortices in superfluid 3He,
Physica B **210**, 338 (1995)

fermions zero modes on symmetric vortex in 3He-B

topological ^{3}He-B at $\mu > 0$: $N_K = 2$





$E_Q = -Q\omega_0$

$\omega_0 = \Delta^2/E_F << \Delta$

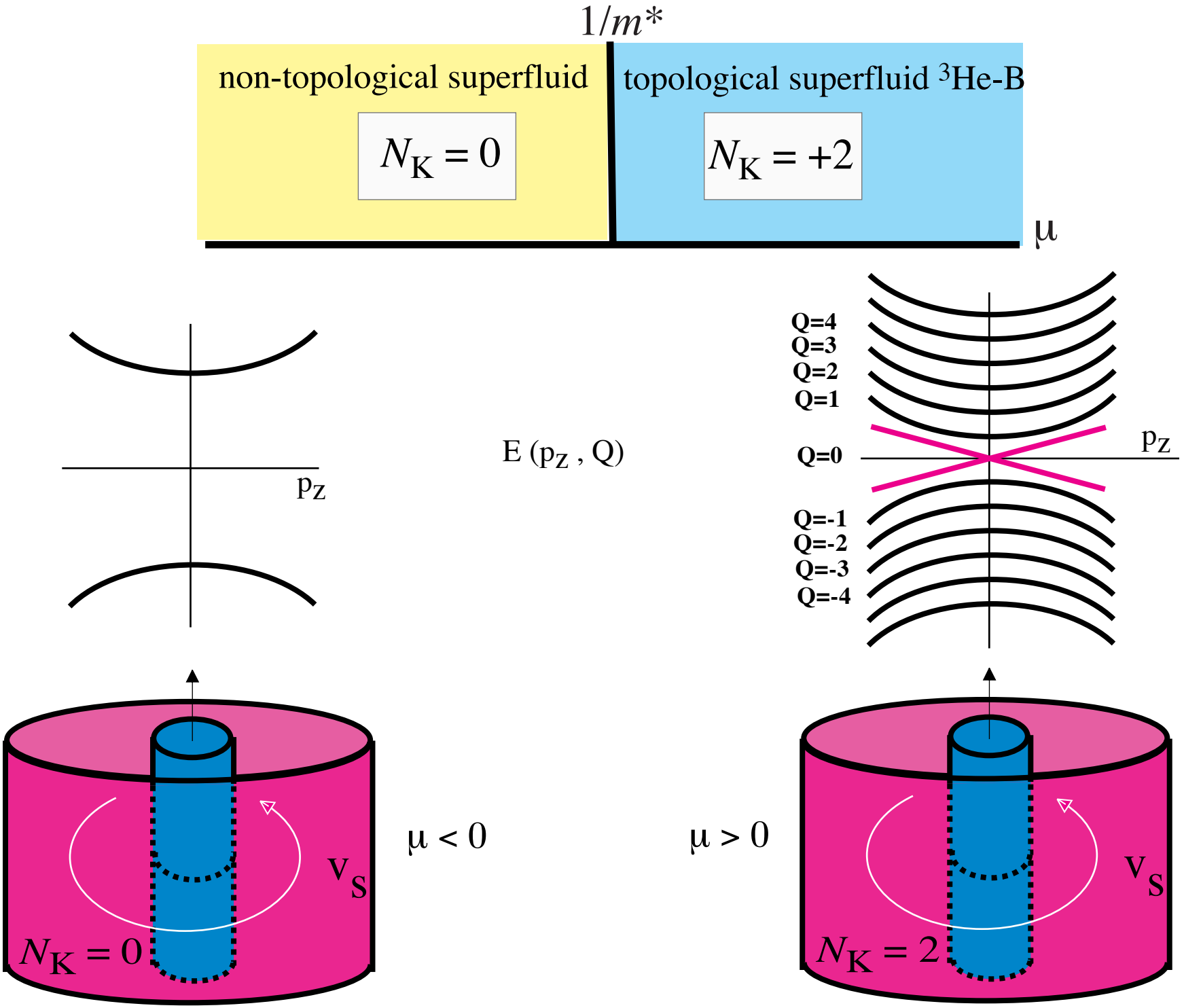
Q is integer
for p-wave superfluid ^{3}He-B

gapless fermions on Q=0 branch form

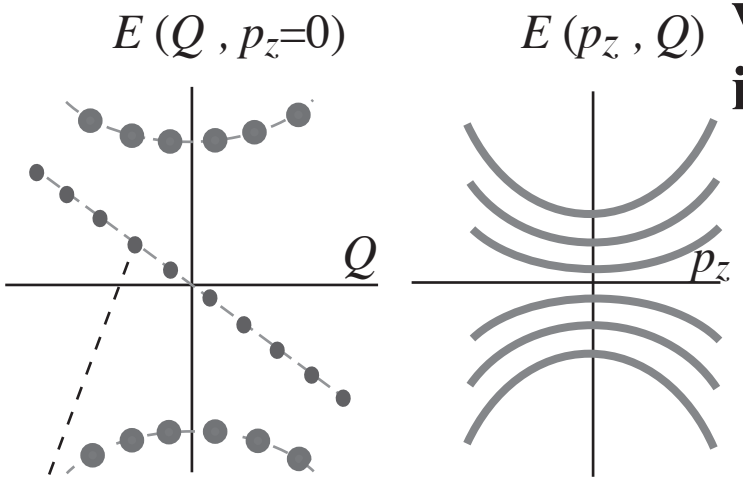
1D Fermi-liquid

Misirpashaev & GV
Fermion zero modes in symmetric vortices in superfluid 3He,
Physica B **210**, 338 (1995)

topological quantum phase transition in bulk & in vortex core



Electrons in vortex
in non-topological *s*-wave superconductors



Asymmetric branch

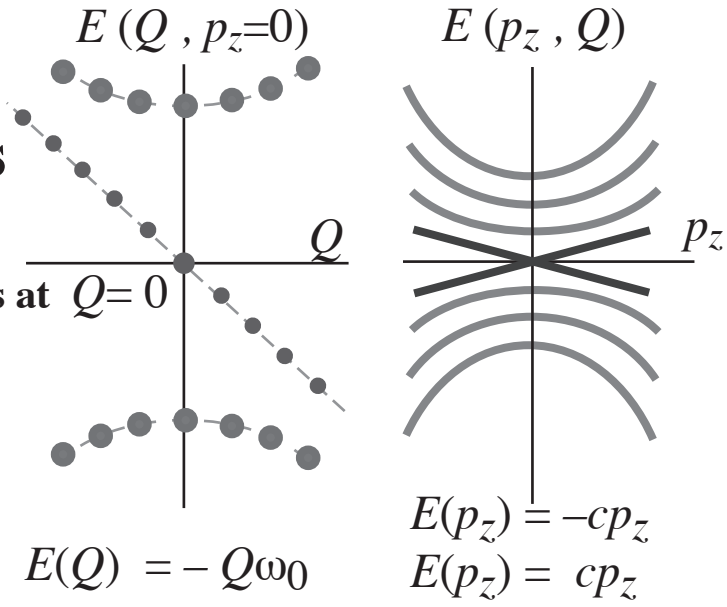
$E(Q) = -Q\omega_0\,(p_z)$

$\omega_0 = \Delta^2/\,E_F \;\ll \Delta$

vortex core states in non-topological materials

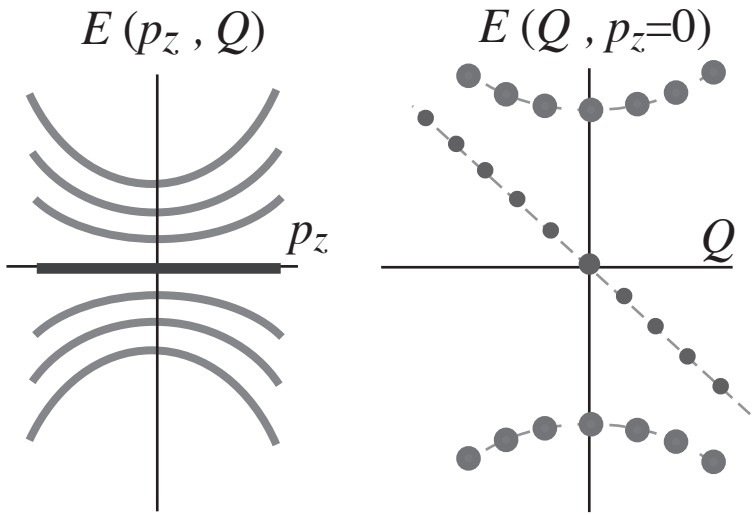
vortex core states in topological materials

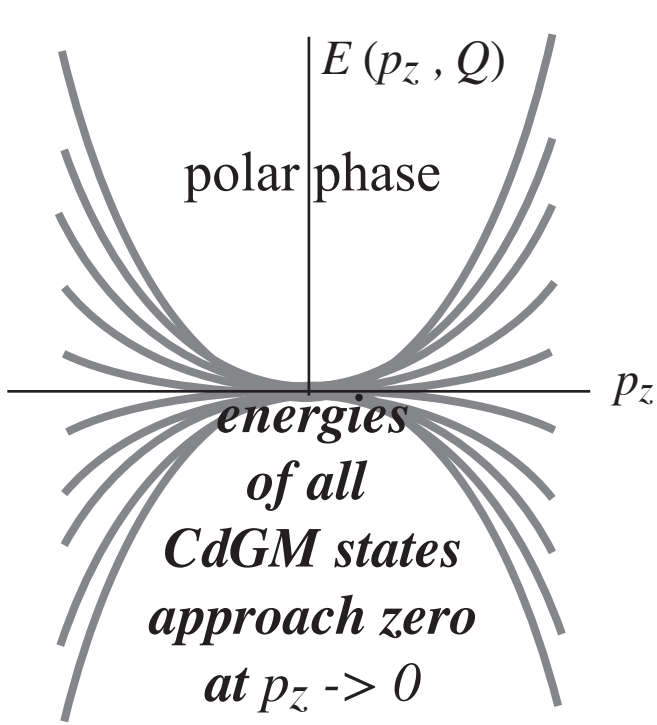
Quasiparticles in ^{3}He-B vortex

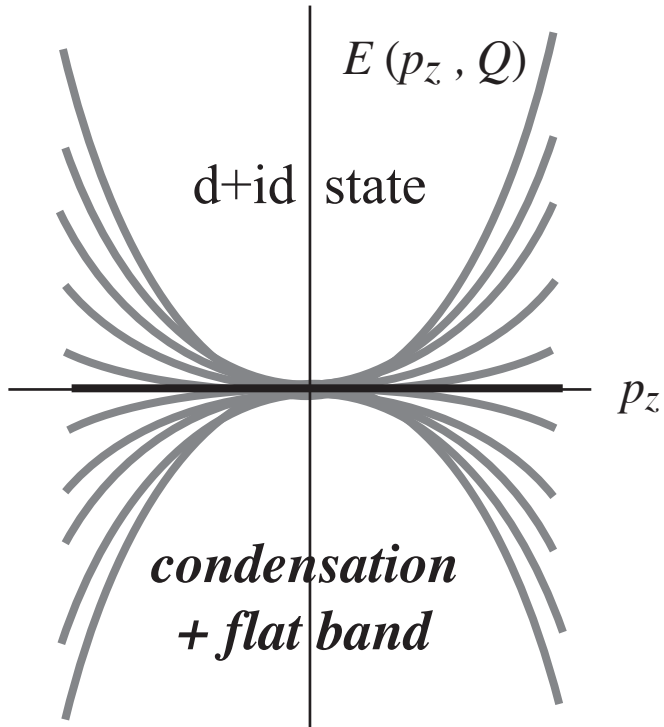


condensation of Majorana fermions in vortex core in superfluid/superconductors with Dirac line

flat band of Majorana fermions
in vortex in chiral Weyl ^{3}He-A

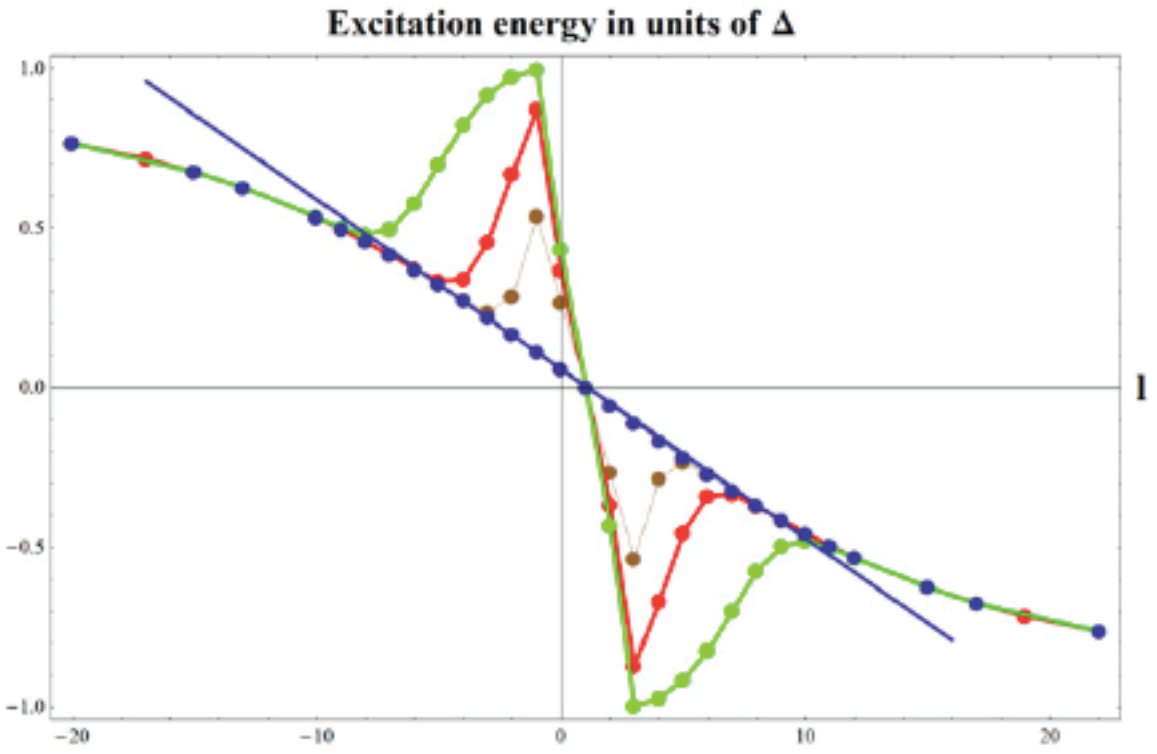




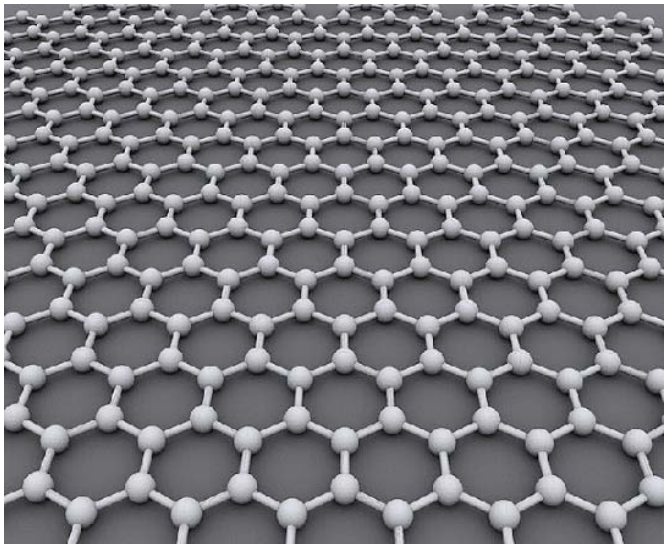


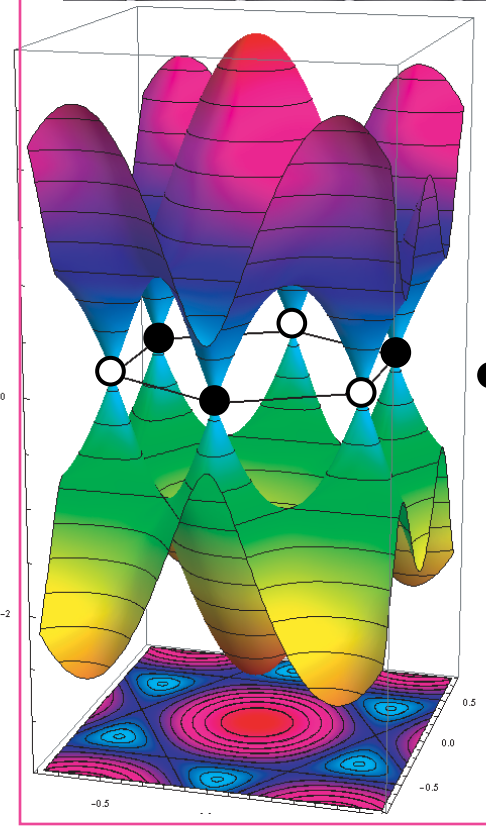
Effect of nanoholes on the vortex core fermion spectrum and heat transport in p-wave superconductors

B Rosenstein[1,2], I Shapiro[3] and B Ya Shapiro[3]



graphene:
2+1 gapless Dirac fermions

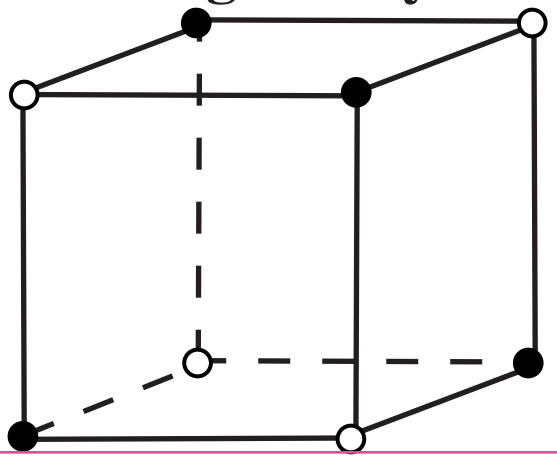




○ Dirac point
$N = +1$

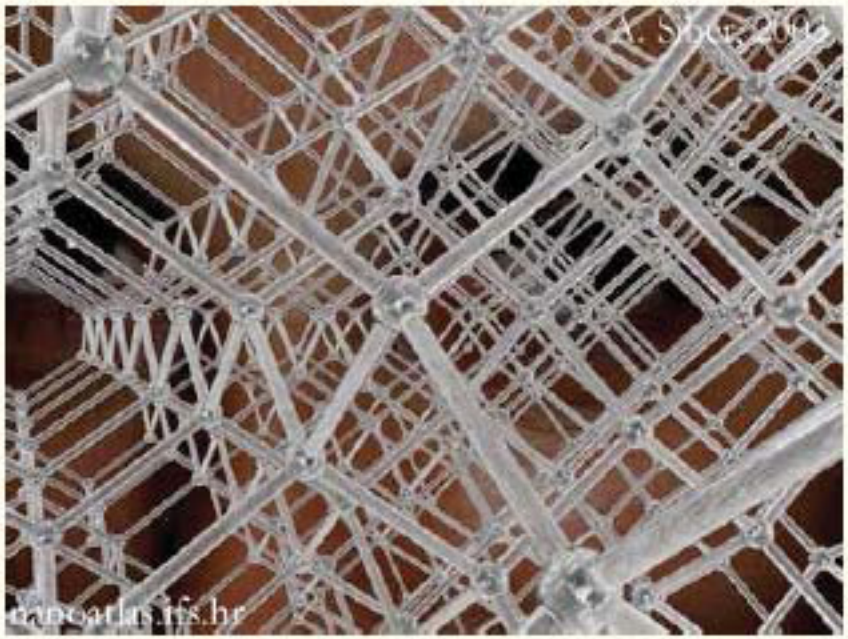
● Dirac point
$N = -1$

α-phase: 3D analog of graphene
4 left and 4 right Weyl fermions



4D graphene in lattice QCD

Kaplan 1993, 2011, Creutz JHEP 04 (2008) 017



8 left and 8 right Weyl fermions
as in Standard Model

connection to Z_8 & Z_{16} topological classification ?