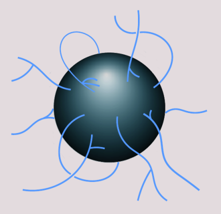


Black Holes in Loop Quantum Gravity
Microstates and Entropy

Karim Noui

LMPT, Tours

An History of connections

Ashtekar connection (1986)

- The connection is a complex sl$(2,\mathbb{C})$ valued $(\gamma = \pm i)$
- Gravity similar to (complex) Yang-Mills with polynomial constraints
- Remove second class constraints of first order formulation
- But : reality conditions are highly non-linear constraints
- Starting the quantization ? No Hilbert space with $SL(2,\mathbb{C})$

Ashtekar-Barbero connections (1995)

- Transforming Ashtekar connection to real $SU(2)$ connection
- Via a one parameter (γ) family of canonical transformations
- No more reality conditions but constraints are no more polynomial
- Now : we can start the quantization... leaving aside Hamiltonian

Barbero-Immirzi parameter

A free parameter γ which enters in Poisson brackets

$$\{A_a^i(x), E_j^b(y)\} = \gamma\, \delta_a^b\, \delta_j^i\, \delta^3(x-y)$$

It is irrelevant at classical level

- It is the parameter in a family of canonical transformations
- From Holst action, it disappears on shell ($T(e,\omega)=0$)

$$S_H[e,\omega] = \int \langle e \wedge e \wedge \star F(\omega) + \frac{1}{\gamma} e \wedge e \wedge F(\omega) \rangle$$

It is of prime importance at quantum (kinematical) level

- It enters in physical predictions at kinematical level
- Area (volume) spectrum : area gap $= 4\pi \ell_p^2 \gamma \sqrt{3} \Longrightarrow$ Black Holes
- Cosmology : density at Planck scale $\rho_c^{-1} = 32\pi^2 G^2 \hbar \gamma^3 / \sqrt{3}$

The puzzle : classical vs quantum

One of the main recurrent questions concerning LQG

A closely related point : discreteness of quantum geometry

- γ real transforms $SL(2,\mathbb{C})$ into $SU(2)$ in the time gauge
- $SU(2)$ is compact and responsible for discrete spectra
- What happens if we consider (non compact) $SL(2,\mathbb{C})$ instead ?

Some proposals

- A quantum ambiguity : Rovelli-Thiemann (1997)
- Choice of phase space in discrete gravity : Dittrich-Ryan (2012)
- etc $\cdots$

Our proposal : γ should be fixed to the complex value

$$\gamma = \pm i$$

From quantum geometry to Quantum Black Holes

Loop Quantum Gravity in a nutshell

- Classical phase space
- Holonomy-Flux algebra : polymer states
- Quantization : discreteness of space

Looking at Black Holes

- Classical (real) calculation of the entropy
- Analytic continuation of black holes entropy to complex γ
- Case of rotating black holes

Phase space of Ashtekar gravity

Action : first order gravity with Immirzi parameter

$$S_H[e,\omega] = \int \langle e \wedge e \wedge \star F(\omega) + \frac{1}{\gamma} e \wedge e \wedge F(\omega) \rangle$$

Canonical variables : Connection and Electric field

$$\{A_a^i(x), E_j^b(y)\} = (8\pi\gamma G)\, \delta_a^b\, \delta_j^i\, \delta^3(x,y)$$

Constraints and Symmetries

- Gauss constraint $G = D_a E^a$: complex $SL(2,\mathbb{C})$ gauge symmetry
- Vectorial constraint $H_a = E^b \cdot F_{ab}$: space diffeomorphisms
- Scalar constraint $H = E^a \times E^b \cdot F_{ab} + \cdots$: time reparametrizations

Polymer states hypothesis

Holonomy-Flux algebra

- Phase space : $\mathcal{P} = T^*(\mathcal{A})$ with $\mathcal{A} = \{SU(2)\, connections\}$
- Holonomy-flux algebra associated to edges e and surfaces S

$$A(e) = P\exp(\int_e A) \quad \text{and} \quad E_f(S) = \int_S \text{Tr}(f \star E)\,.$$

- Cylindrical functions : $f \in \text{Cyl}$ is a function of $A(e)$ with $e \subset \gamma$
- $E_f(S)$ acts as a vector field on f if $S \cap \gamma \neq 0$

Action of symmetries

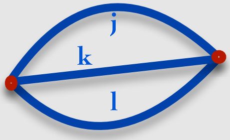
- Gauss constraint : $f(A(e)) \longmapsto f(g(s(e))^{-1}A(e)g(t(e)))$
- Diffeomorphisms : $f(A(e)) \longmapsto f(A(\varphi(e)))$
- Similar action for the variables $E_f(S)$

Kinematical states

Unicity of the kinematical quantization

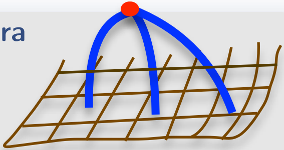
The quantum algebra $\mathcal{A}$ admits an unique Diff-invariant unitary irreducible representation : GNS construction.

The Spin-Networks basis



- Generalization of $SU(2)$ Wilson loops
- Constructed from $SU(2)$ harmonic analysis
- Edges colored with spins and Vertices with intertwiners

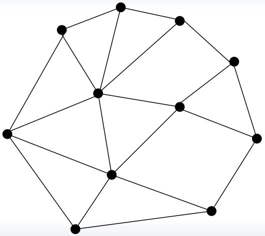
Geometric operators have discrete spectra

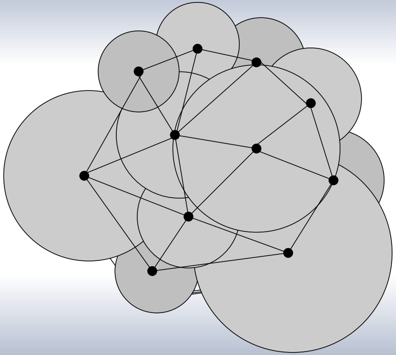


- Area on edges : $A = 8\pi\gamma\ell_p^2 \sum_\ell \sqrt{\ell(\ell+1)}$
- Volume on nodes : discrete spectrum

Picture of space at Planck scale

From the kinematics, Space is discrete...

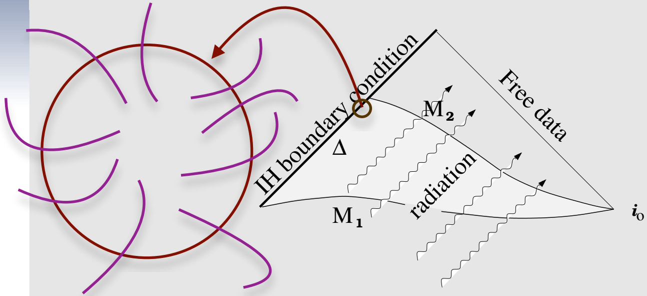




... Edges carry quanta of area, nodes carry quanta of volume

Description of Black Holes

The horizon as a boundary



- Geometric conditions : null-surface, no-expansion, $F \propto E$ etc...
- Restricted here to spherical black holes
- Result : Symplectic structure of an $SU(2)$ Chern-Simons theory

Engle-KN-Perez (2009), Engle-KN-Perez-Pranzetti (2010-2011)

Black Holes and Chern-Simons theory

$SU(2)$ Chern-Simons theory

- Black Holes are closely related to $SU(2)$ Chern-Simons theory

$$S(A) = \frac{k}{2\pi} \int \langle A \wedge dA + \frac{2}{3} A \wedge A \wedge A \rangle$$

- On a punctured 2-sphere $(j_1, \cdots, j_n)$ such that

$$a_H = 8\pi \ell_p^2 \gamma \sum_{\ell} \sqrt{j_\ell (j_\ell + 1)}$$

- With level k (coupling constant) related to horizon area a_H

$$k \propto \frac{a_H}{2\pi \gamma \ell_p^2}$$

Quantization of $SU(2)$ Chern-Simons

Path integral quantization : Witten (1989)

$\mathcal{Z}_k(M;K)$ related to manifold and knots invariants. Note that k must be an integer for the path integral to be well-defined

Hamiltonian (combinatorial) quantization

- Quantum states are constructed from representation theory of $U_q(su(2))$ where $q = \exp(i\pi/(k+2))$
- Irreps of $U_q(su(2))$ are labelled by $j \leq k/2$ with $d_j = 2j+1$
- Physical Hilbert space $\mathcal{H}(j_1, \cdots, j_n) = \mathrm{Inv}_q(V_{j_1} \otimes \cdots \otimes V_{j_n})$ with

$$\dim(\mathcal{H}) = \frac{2}{k+2} \sum_{\ell=0}^{k/2} (\sin(\frac{\pi d_\ell}{k+2}))^{2-N} \prod_{i=1}^{N} \sin(\frac{\pi d_\ell d_{j_i}}{k+2}) \, .$$

Generalization to $SU(N)$ with any surface topology (conformal blocks) ...

Counting microstates

Large area $a_H \gg \ell_p^2$ limit : level $k \to \infty$

The quantum group becomes classical

- Black Hole microstates are (classical) $SU(2)$ intertwiners
- Verlinde formula is a Riemann sum of the integral

$$\dim(\mathcal{H}) = \frac{2}{\pi} \int d\theta \sin^2\theta \left(\prod_{\ell=1}^{n} \frac{\sin(d_\ell \theta)}{\sin\theta} \right)$$

Asymptotic analysis and Bekenstein-Hawking entropy

- Rovelli (1996) for spin 1/2. Provided $\gamma = \gamma_0$

$$S(a_H) = \log(\dim(\mathcal{H})) \sim \frac{a_H}{4\ell_p^2} - \frac{3}{2}\log(\frac{a_H}{\ell_p^2})$$

The intriguing role of γ ?

Why fixing γ to recover Black Hole entropy ? It might exist different fixings coming from cosmology, Kerr black hole entropy, coupling to matter or others etc...

Different scenario to address this issue

- Bianchi (2012) : entropy from Spin-Foam models
- Frodden-Ghosh-Perez (2011-$\cdots$) : quasi local energy which allows a (grand) canonical approach from observers near horizon
- BenAchour-Frodden-Geiller-KN-Perez (2012-$\cdots$) : why not $\gamma = \pm i$? Can we make sense of this idea ?

Complex Black Holes

Black Holes are described in terms of Chern-Simons theory

Analytic continuation of Chern-Simons theory

- $\gamma = \pm i \implies k \in i\mathbb{R}$: complex level
- $\implies$ Chern-Simons theory with $SL(2,\mathbb{C})$: much more difficult to quantize, and only known way relies on analytic continuation techniques (Morse theory) initiated by Witten (2010)

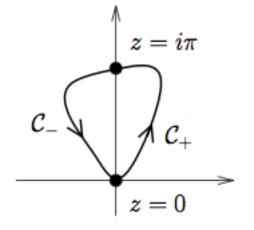
Continuing Black Hole entropy

- It is a Chern-Simons observable and can be continued importing techniques from Chern-Simons theory
- Extra condition $a_j = \gamma\sqrt{j(j+1)} \in \mathbb{R}$: $j = -1/2 + is$, $s \in \mathbb{R}$
- Here one turns $SU(2)$ into $SU(1,1)$ irreps

Complex Black Hole entropy

Verlinde formula as an integral on the complex plane

$$\frac{i}{\pi}\oint_{\mathcal{C}} dz \,\sinh^2(z)\prod_{\ell=1}^{n}\frac{\sinh(d_\ell z)}{\sinh(z)}\coth((k+2)z)$$



Analytic continuation

- $\gamma = \pm i$ and $j_\ell = -1/2 + is_\ell$
- Relies on choice of contour
- Unique non-trivial choice of contour up to discrete ambiguity
- Semi-classical limit fixes the discrete ambiguity

Semi-classical analysis

One-color model : $s_\ell = s$

- Semi-classical limit : from quasi-local approach, $s \gg 1$ and $n \gg 1$, compatible with Spin-Foam semi-classical limit
- "Large" spins dominate according to $\langle s \rangle \sim \sqrt{a_H}$
- In the center of Milky-Way, $\langle area \rangle \sim 10^{-24} m^2$ but from the point of view of a local observer

Saddle points and analysis of critical points

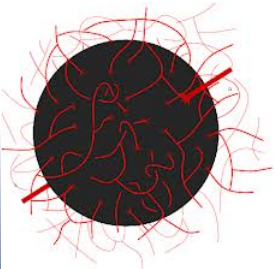
$$\frac{i}{\pi} \oint_{\mathcal{C}} dz \, \sinh^2(z) \, e^{nF(z)} \quad \text{with} \quad F(z) = \log\left(\frac{\sinh(sz)}{\sinh(z)}\right)$$

$$S(a_H) \sim \frac{a_H}{4\ell_p^2} (1 + o(1))$$

More results on Black Holes

Generalization to several situations

- Multi-color black hole : same result for leading order term in entropy
- BTZ black hole from LQG and SF models : they coincide
- Understanding Hawking radiation from this point of view



Rotating Black Holes

- Frodden-Perez-Pranzetti-Roken : Extra puncture for angular momentum
- Analytic continuation : $j = -1/2 + is$ but J remains discrete
- We recover the area law (to appear)
- This prescription does not work for real Black Holes : Bianchi (2010), Livine-Terno (2012)

Summary

Strong results which support the idea $\gamma = \pm i$

- In 3D, it is clear that imposing Hamiltonian constraint leads to eliminating γ dependency in the physics : equivalent to set $\gamma = \pm i$ from kinematics.
- In 4D, no one knows how to start Loop quantization with $\gamma = \pm i$. Therefore, we start with γ real, and at some points we perform an analytic continuation. This is formal...
- The strategy works very well for black holes (relation to CS)

Going further with quantum black holes

- Hawking radiation : scenarios have been proposed
- Information Loss paradox : Planck stars (Rovelli) and consequences