"Condensed matter physics meets relativistic quantum field theory",

Le Studium Workshop, Tours, France, 13-16 June, 2016

Anomalies, knots, and currents

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Outline

- Chiral anomaly in a background magnetic field: the Chiral Magnetic Effect (CME)

- Observation of CME in $ZrTe_5$ with Qiang Li et al

- CME and axion electrodynamics: self-similar inverse cascade of magnetic helicity towards the Chandrasekhar-Kendall state

- Quantized CME from knot reconnections

  with Yuji Hirono and Yi Yin

Classical symmetries and Quantum anomalies

Anomalies: The classical symmetry of the Lagrangian is broken by quantum effects -

examples: chiral symmetry - chiral anomaly $\partial_\mu j^\mu_A = C_{\rm A} \boldsymbol{E} \cdot \boldsymbol{B}$

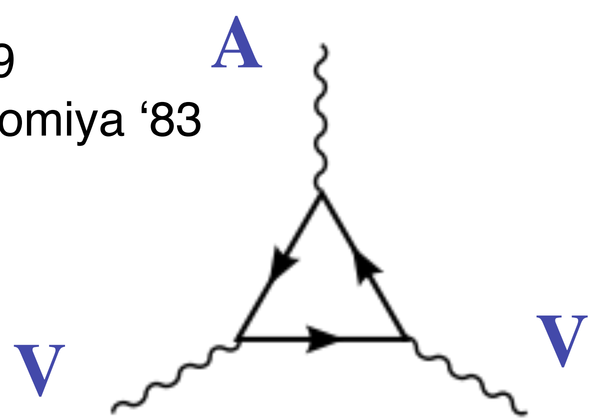
scale symmetry - scale anomaly

Anomalies imply correlations between currents:

Adler; Bell, Jackiw '69

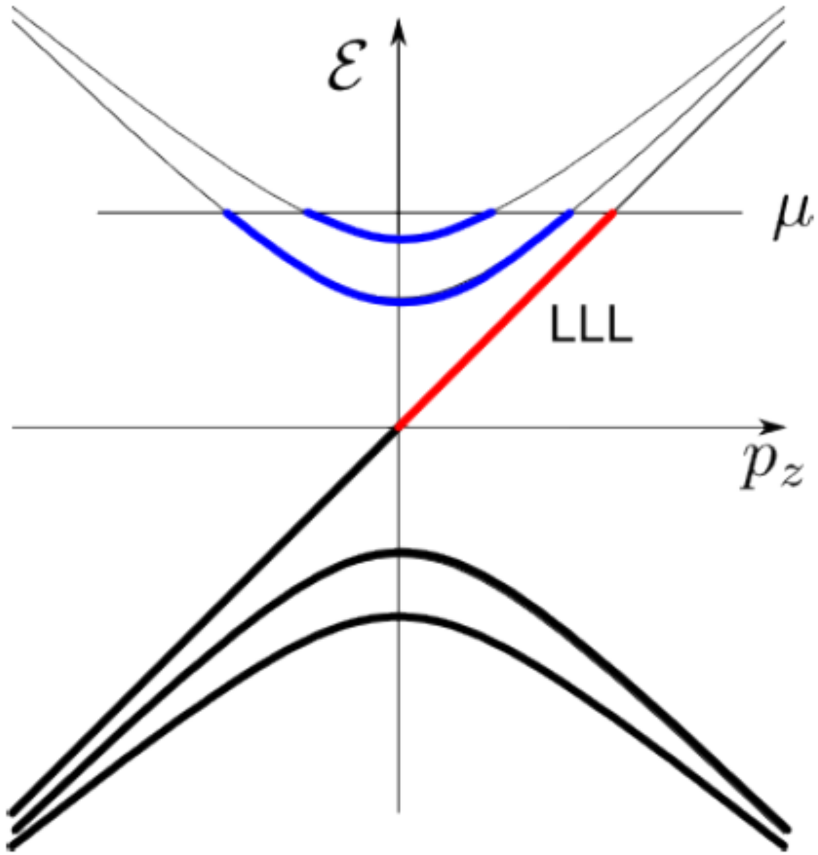
crystals: Nielsen, Ninomiya '83

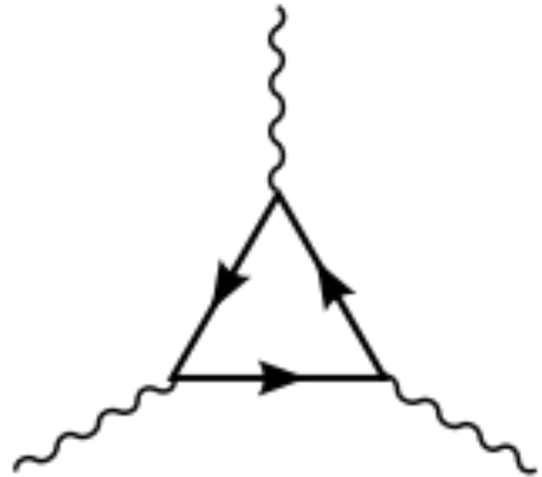
e.g. $\pi^0 \to \gamma\gamma$ decay



if A, V are background fields, V is generated!

Chiral anomaly





In classical background fields (E and B), chiral anomaly induces a collective motion in the Dirac sea

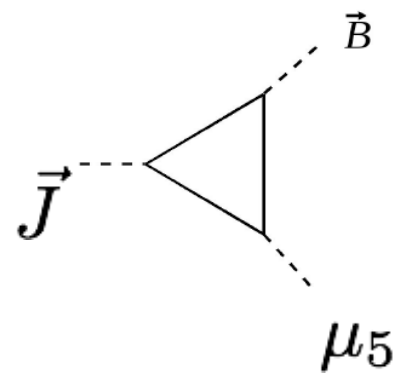
Adler; Bell, Jackiw; Nielsen, Ninomiya; …

Chiral Magnetic Effect

DK'04; K.Fukushima, DK, H.Warringa, PRD'08;
Review and list of refs: DK, arXiv:1312.3348

Chiral chemical potential is formally equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, and in the presence of B, vector e.m. current is generated:



$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2} \left(F_L^{\mu\nu} \tilde{F}_{L,\mu\nu} - F_R^{\mu\nu} \tilde{F}_{R,\mu\nu} \right)$$

Compute the current through

$$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

The result:

$$\vec{J} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

Coefficient is fixed by the axial anomaly, no corrections

Chiral magnetic conductivity: discrete symmetries

P – parity
T – time reversal

$$\vec{J} = \frac{e^2}{2\pi^2}\,\mu_5\,\vec{B}$$

P-odd
T-odd

P-even
T-odd

P-odd

P-odd effect!

Effect persists in hydrodynamics!

T-even

Non-dissipative current!

(topologically protected)

cf Ohmic conductivity:

$$\vec{J} = \sigma \vec{E}$$

T-odd, dissipative

Systematics of anomalous conductivities

	Magnetic field	Vorticity
Vector current	$\dfrac{\mu_A}{2\pi^2}$	$\dfrac{\mu\mu_A}{2\pi^2}$
Axial current	$\dfrac{\mu}{2\pi^2}$	$\dfrac{\mu^2+\mu_A^2}{4\pi^2}+\dfrac{T^2}{12}$


CME

Hydrodynamics and symmetries

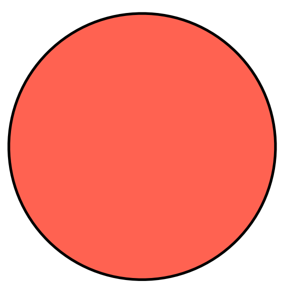
- Hydrodynamics: an effective low-energy TOE. States that the response of the fluid to slowly varying perturbations is completely determined by conservation laws (energy, momentum, charge, ...)
- Conservation laws are a consequence of symmetries of the underlying theory
- What happens to hydrodynamics when these symmetries are broken by quantum effects (anomalies of QCD and QED)?

Son, Surowka; Landsteiner, Megias, Pena-Benitez; Sadofyev, Isachenkov; Kalaydzhyan, Kirsch; DK, Yee; Zakharov; Jensen, Loganayagam, Yarom; Neiman, Oz; ….

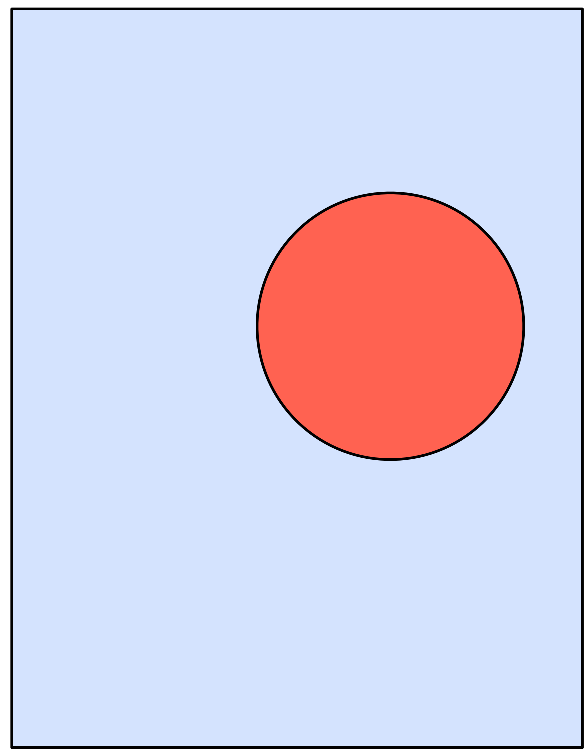
No entropy production from P-odd anomalous terms

DK and H.-U. Yee, 1105.6360

Entropy grows



$$\partial_\mu s^\mu \geq 0$$



Mirror reflection:
entropy decreases ?

$$\partial_\mu s^\mu \leq 0$$

Decrease is ruled out by 2nd law of thermodynamics

$$\partial_\mu s^\mu = 0$$

Allows to compute analytically 13 out of 18 anomalous transport coefficients in 2nd order relativistic hydrodynamics

The CME in relativistic hydrodynamics:
The Chiral Magnetic Wave

DK, H.-U. Yee, arXiv:1012.6026 [hep-th]; PRD

$$\vec{j}_V = \frac{N_c\, e}{2\pi^2}\mu_A \vec{B}; \quad \vec{j}_A = \frac{N_c\, e}{2\pi^2}\mu_V \vec{B},$$

CME Chiral separation

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c\, e\vec{B}}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix}$$

Propagating chiral wave: (if chiral symmetry is restored)

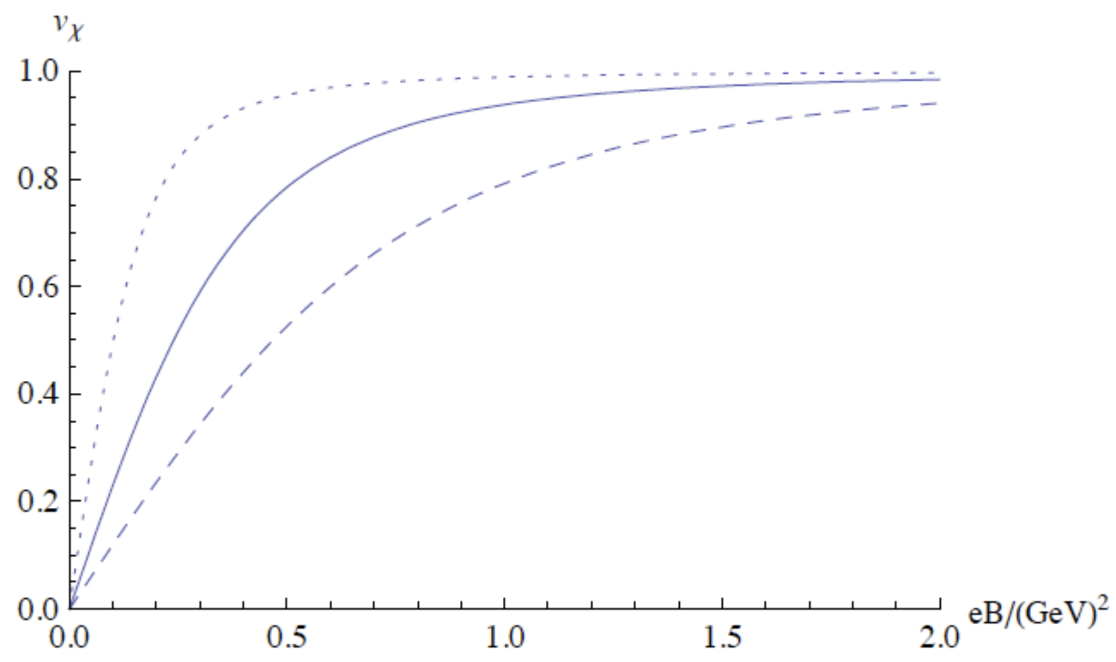
$$\left(\partial_0 \mp \frac{N_c e B \alpha}{2\pi^2}\partial_1 - D_L \partial_1^2\right) j^0_{L,R} = 0$$

Gapless collective mode is the carrier of CME current in MHD:

$$\omega = \mp v_\chi k - i D_L k^2 + \cdots$$

The Chiral Magnetic Wave:
oscillations of electric and chiral charges coupled by the chiral anomaly

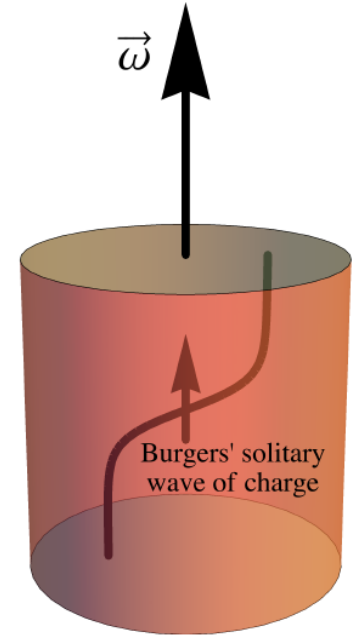
In strong magnetic field, CMW propagates with the speed of light!



DK, H.-U. Yee, Phys Rev D'11

G.Basar, DK, H.-U.Yee, PhysRevB89(2014)035142

Anomalous transport and the Burgers' equation



Consider a "hot" system (QGP, WSM) with $\frac{\mu}{T} \ll 1$

The chemical potential is then proportional to charge density:

$$\mu \approx \chi^{-1}\rho + \mathcal{O}\left(\rho^3\right)$$

the CME current is

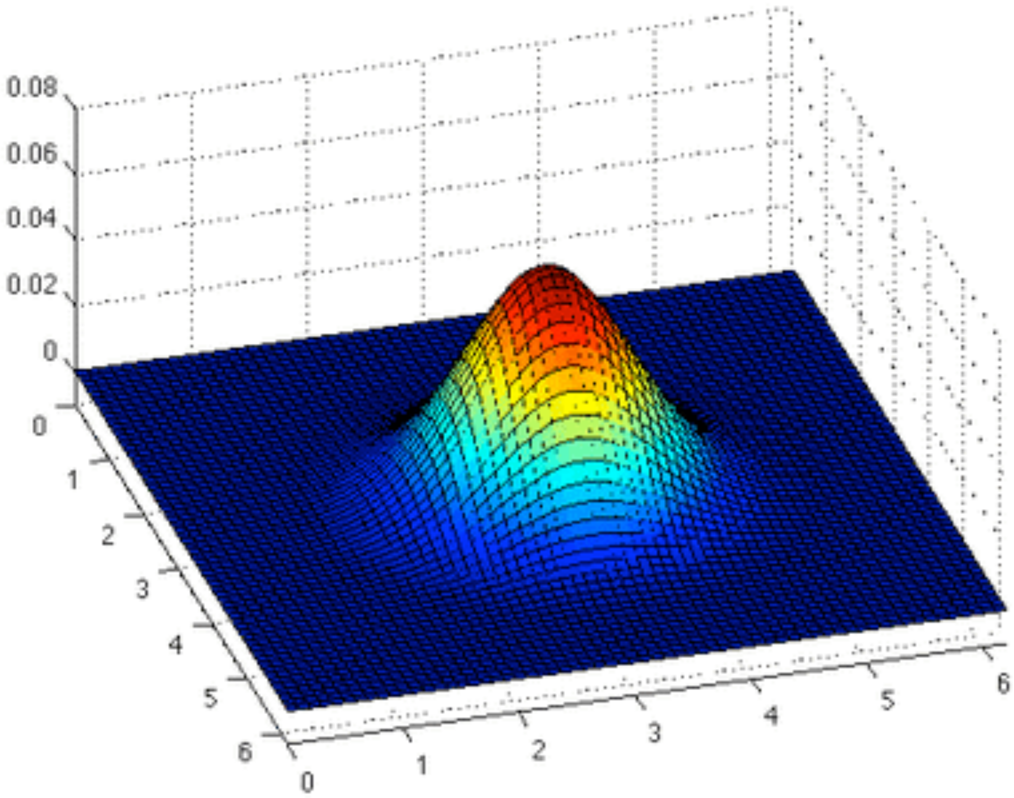
$$J^3 = \frac{ke}{4\pi^2}\left(\chi^{-2}\rho^2 + \frac{\pi^2}{3}T^2\right)\omega - D\partial_3\rho + \mathcal{O}\left(\partial^2, \rho^3\right)$$

and the charge conservation $\partial_t\rho + \partial_3 J^3 = 0$ leads to

$$\partial_t\rho + C\rho\partial_x\rho - D\partial_x^2\rho = 0 \qquad C = \frac{ke\omega}{2\pi^2\chi^2} \qquad x \equiv x^3$$

The Burgers' equation

$$\partial_t \rho + C\rho\partial_x \rho - D\partial_x^2 \rho = 0$$

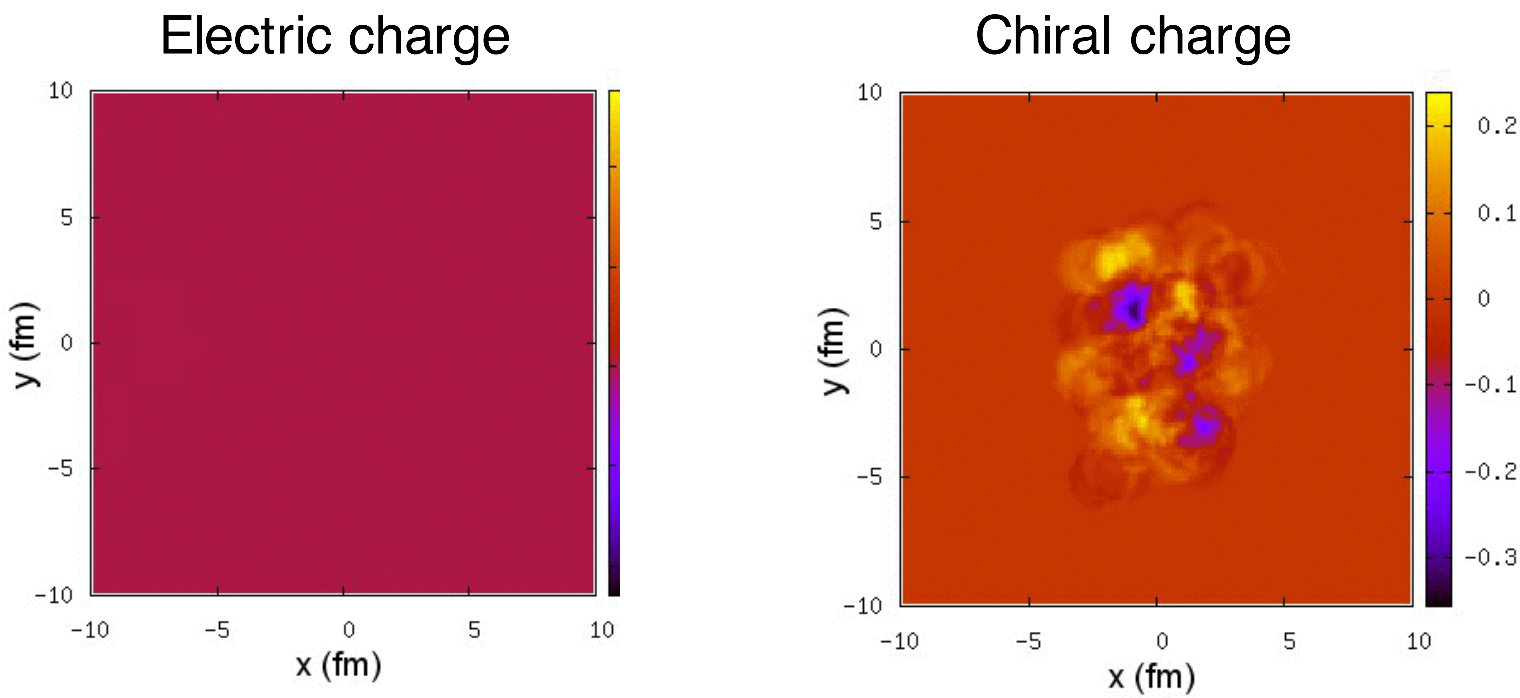


Exactly soluble by Cole-Hopf transformation -

initial value problem, integrable dynamics

describes shock waves, solitons, ...

CMHD



Y.Hirono, T.Hirano, DK, (Stony Brook – Tokyo), arxiv:1412.0311
(3+1) ideal CMHD (Chiral MagnetoHydroDynamics)

Axion electrodynamics, or Maxwell-Chern-Simons theory

$$\mathcal{L}_{\rm MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4}\, P_\mu J^\mu_{CS}$$

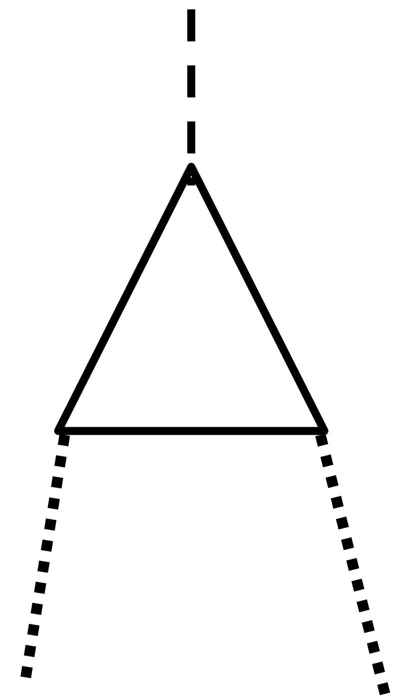
$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma} \qquad P_\mu = \partial_\mu \theta = (\dot\theta, \vec P)$$

$$\vec\nabla \times \vec B - \frac{\partial \vec E}{\partial t} = \vec J + \underline{c\left(\dot\theta \vec B - \vec P \times \vec E\right)},$$

$$\vec\nabla \cdot \vec E = \rho + \underline{c\vec P \cdot \vec B},$$

$$\vec\nabla \times \vec E + \frac{\partial \vec B}{\partial t} = 0,$$

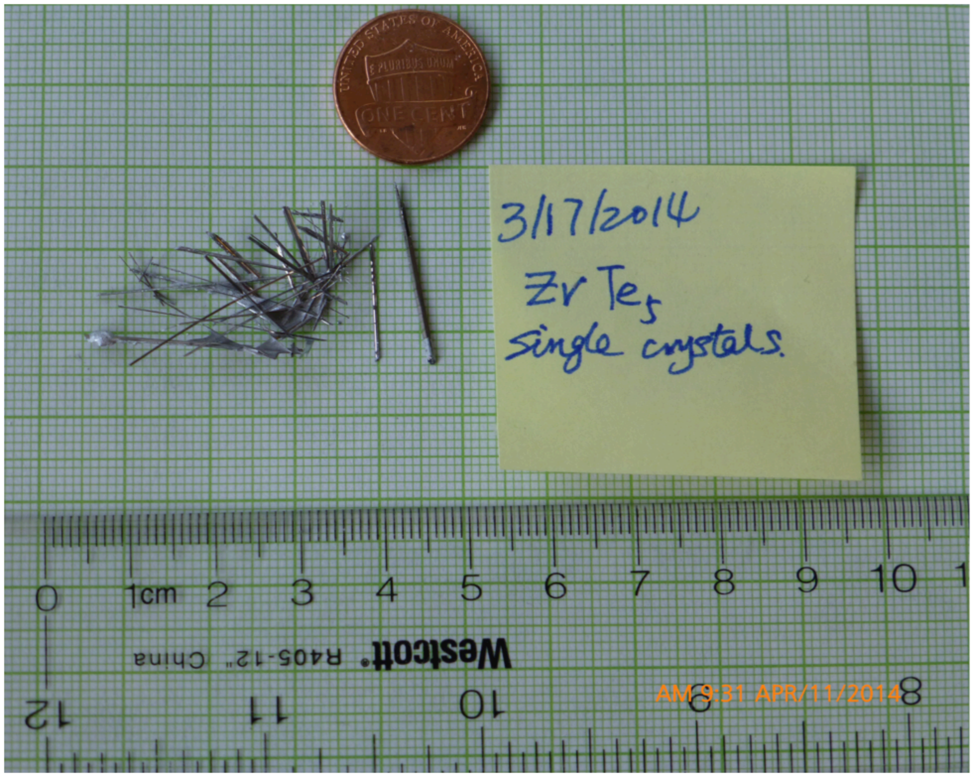
$$\vec\nabla \cdot \vec B = 0,$$

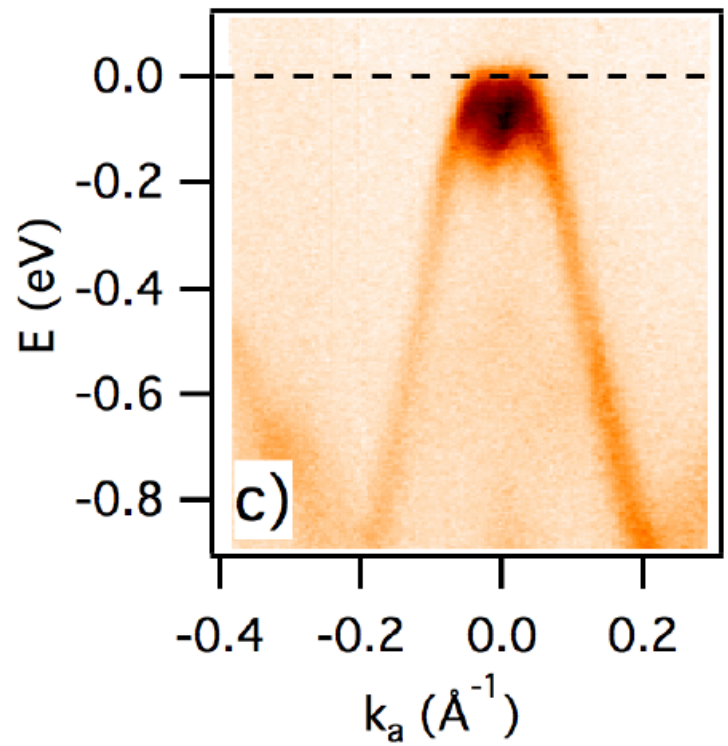


F. Wilczek, Phys.Rev.Lett.58 (1987) 1799;
DK, Ann.Phys. 325(2010) 205

Observation of the chiral magnetic effect in $ZrTe_5$

Qiang Li,[1] Dmitri E. Kharzeev,[2,3] Cheng Zhang,[1] Yuan Huang,[4] I. Pletikosić,[1,5] A. V. Fedorov,[6] R. D. Zhong,[1] J. A. Schneeloch,[1] G. D. Gu,[1] and T. Valla[1]





arXiv:1412.6543 (December 2014); Nature Physics **12**, 550 (2016)

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Put the crystal in parallel E, B fields – the anomaly generates chiral charge:

$$\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2\hbar^2 c}\vec{E}\cdot\vec{B} - \frac{\rho_5}{\tau_V}.$$

and thus the chiral chemical potential:

$$\mu_5 = \frac{3}{4}\frac{v^3}{\pi^2}\frac{e^2}{\hbar^2 c}\ \frac{\vec{E}\cdot\vec{B}}{T^2+\frac{\mu^2}{\pi^2}}\tau_V.$$

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so that there is a chiral magnetic current:

$$\vec{J}_{\rm CME} = \frac{e^2}{2\pi^2}\,\mu_5\,\vec{B}.$$

resulting in the quadratic dependence of CME conductivity on B:

$$J^i_{\rm CME} = \frac{e^2}{\pi\hbar}\,\frac{3}{8}\frac{e^2}{\hbar c}\,\frac{v^3}{\pi^3}\,\frac{\tau_V}{T^2+\frac{\mu^2}{\pi^2}}\,B^i B^k E^k \equiv \sigma^{ik}_{\rm CME}\,E^k.$$

adding the Ohmic one – negative magnetoresistance

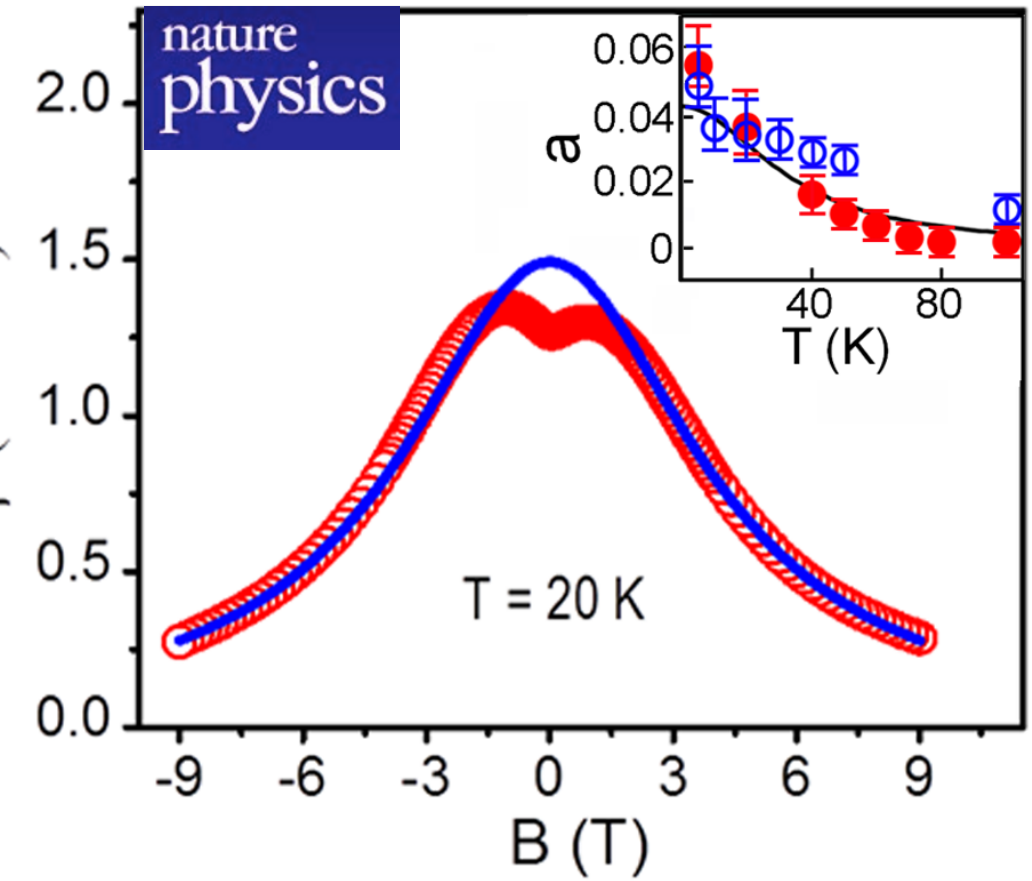
Son, Spivak, 2013

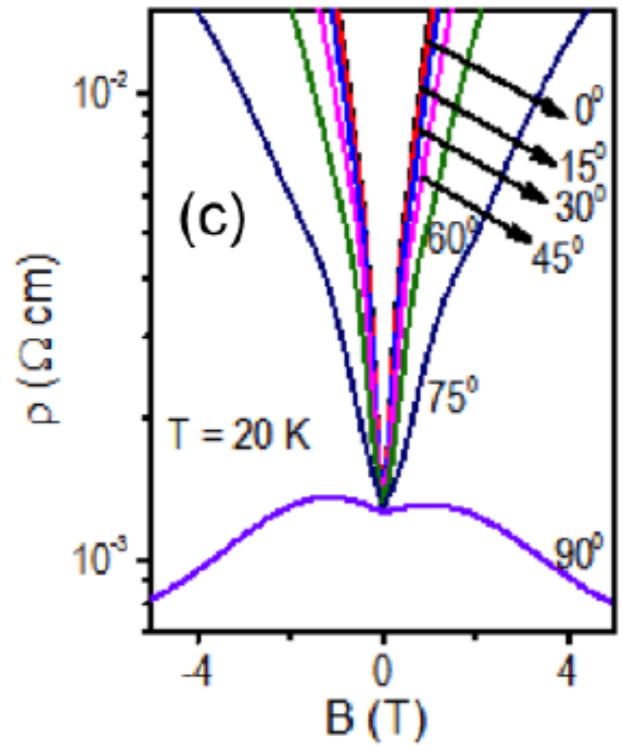
Chiral Magnetic Effect Generates Quantum Current

Separating left- and right-handed particles in a semi-metallic material produces anomalously high conductivity

February 8, 2016

Nature Physics **12**, 550 (2016)



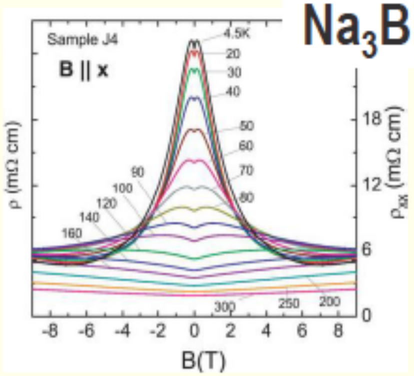


Qiang Li's Distinguished CQM lecture at Simons Center, Feb 19, 2016 on video:
http://scgp.stonybrook.edu/video_portal/video.php?id=2458

Chiral magnetic effect in Dirac/Weyl semimetals

Dirac semimetals:

Na$_3$Bi



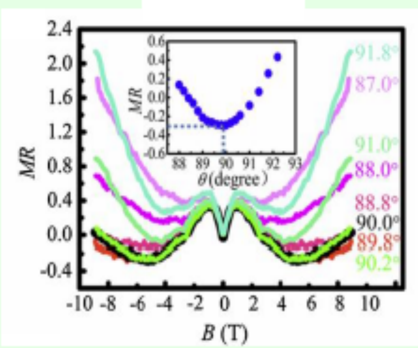
ZrTe$_5$ - Q. Li, D. Kharzeev, et al (BNL and Stony Brook Univ.) arXiv:**1412.6543**; doi:10.1038/NPHYS3648

Na$_3$Bi - J. Xiong, N. P. Ong et al (Princeton Univ.) arxiv:**1503.08179**; Science 350:413,2015

Cd$_3$As$_2$- C. Li et al (Peking Univ. China) arxiv:**1504.07398**; Nature Commun. 6, 10137 (2015).

Weyl semimetals

TaAs



TaAs - X. Huang et al (IOP, China) arxiv:**1503.01304**; Phys. Rev. X 5, 031023

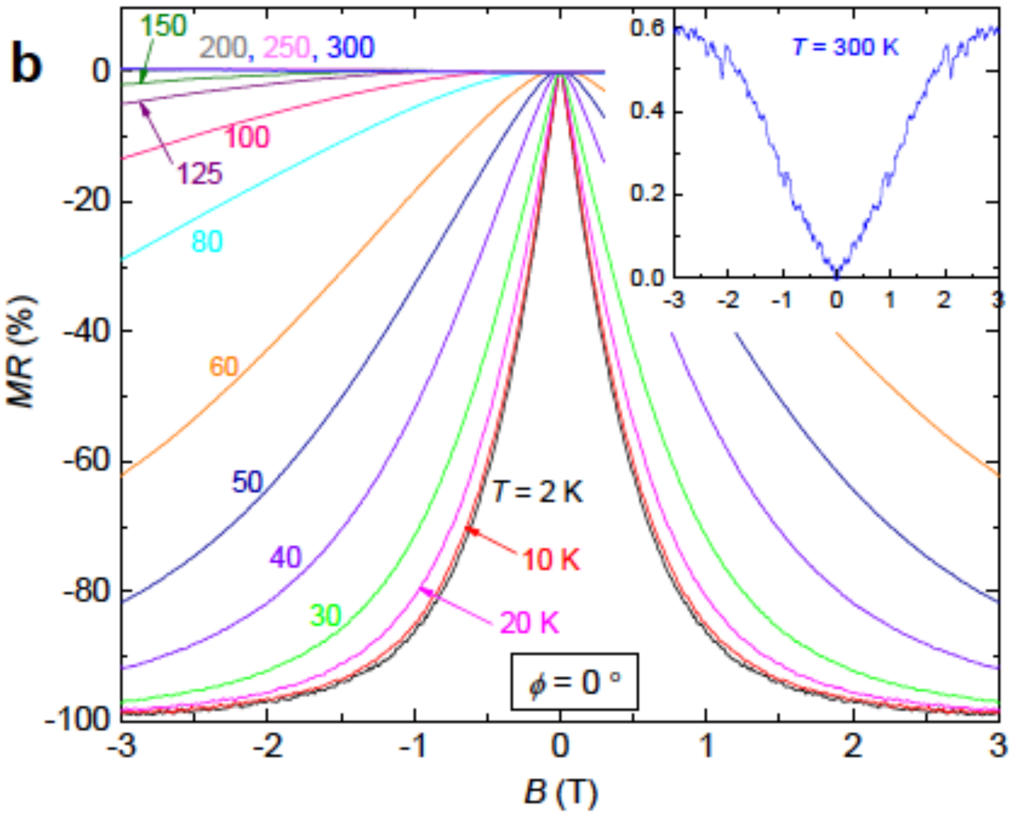
NbAs - X. Yang et al (Zhejiang Univ. China) arxiv:**1506.02283**

NbP - Z. Wang et al (Zhejiang Univ. China) arxiv:**1504.07398**

TaP - Shekhar, C. Felser, B. Yang et al (MPI-Dresden) arxiv:**1506.06577**

Bi$_{1-x}$Sb$_x$ at x ≈ 0.03 - Kim, et al. "Dirac versus Weyl Fermions in Topological Insulators: Adler-Bell-Jackiw Anomaly in Transport Phenomena. Phys. Rev. Lett., 111, 246603 (2013).

Negative MR in $TaAs_2$



Y.Luo et al, 1601.05524;
updated on June 8, 2016

Ta:
Z=73,
discovered
in 1802
in Uppsala,
Sweden

Anders Gustaf Ekeberg
(1767-1813)

"This metal I call *tantalum* ... partly in allusion to its incapacity, when immersed in acid, to absorb any and be saturated."

CME as a new type of superconductivity

London theory of superconductors, '35:

$$\vec{\mathbf{J}} = -\lambda^{-2}\vec{\mathbf{A}} \qquad \nabla \cdot \vec{\mathbf{A}} = 0$$


Fritz and Heinz London

$$\vec{\mathbf{E}} = -\dot{\vec{\mathbf{A}}}$$

$$\boxed{\vec{\mathbf{E}} = \lambda^2 \dot{\vec{\mathbf{J}}}}$$

assume that chirality is conserved:

$$\mu_5 \sim \vec{E}\vec{B}\ t$$

CME:

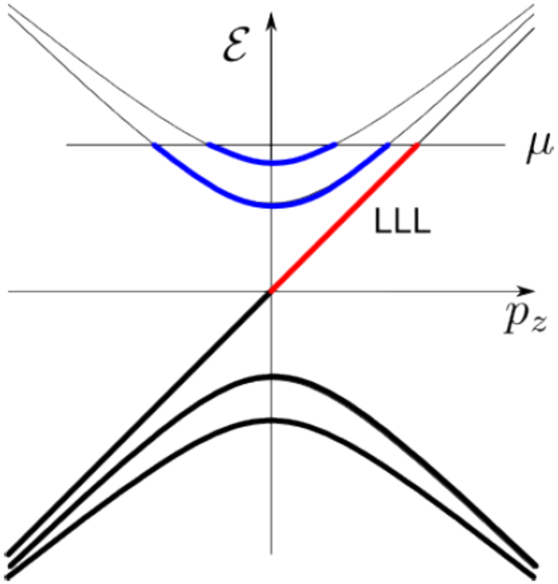
$$\vec{J} \sim \mu_5\ \vec{B}$$

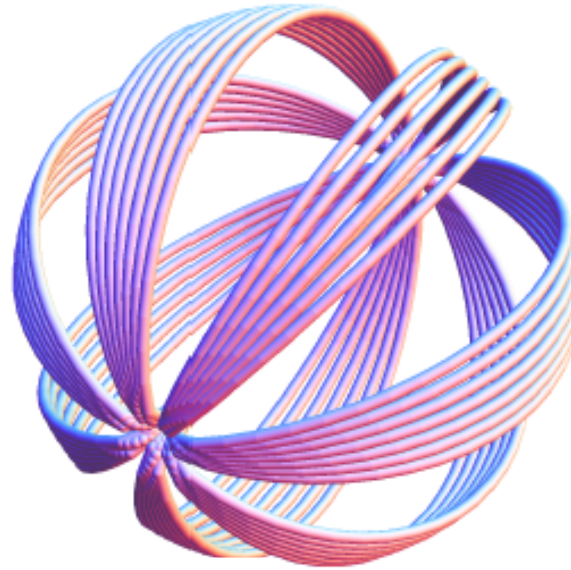
for $\vec{E} || \vec{B}$

$$\boxed{\vec{E} \sim B^{-2}\, \dot{\vec{J}}}$$

superconducting current, tunable by magnetic field!

Chirality transfer from fermions to magnetic helicity





$$\partial_\mu j^\mu_A = C_A \boldsymbol{E}\cdot\boldsymbol{B}$$

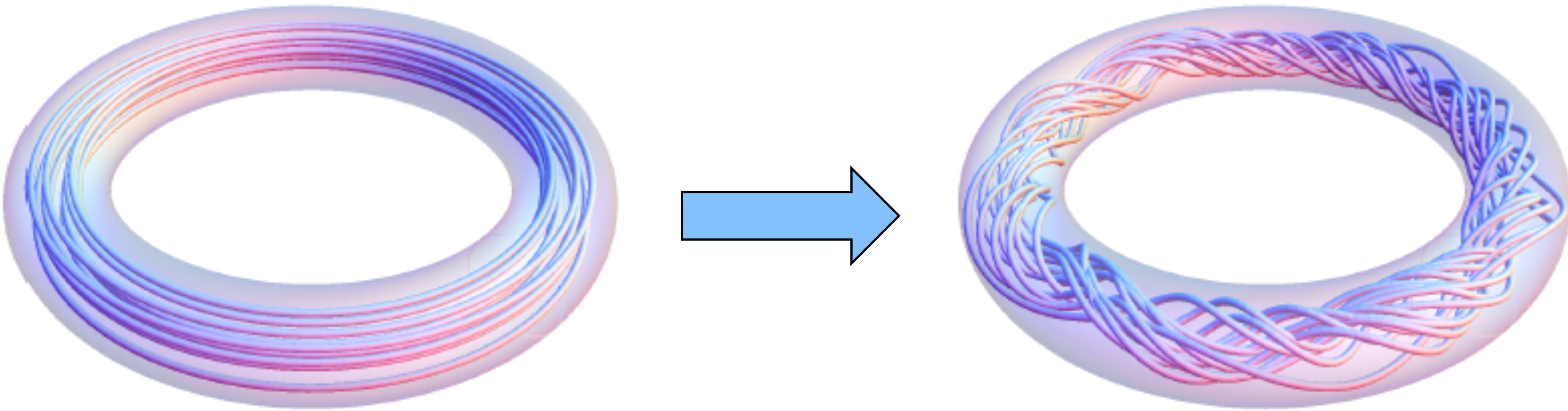
$$h_m \equiv \int d^3x\ \boldsymbol{A}\cdot\boldsymbol{B}$$

$$\int d^3x\ \boldsymbol{E}\cdot\boldsymbol{B} = -\frac{1}{2}\frac{\partial h_m}{\partial t}$$

$$h_0 \equiv h_m + h_F = \text{const}$$

Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031

Chirality transfer in axion electrodynamics



$$\nabla \times \boldsymbol{B} = \boldsymbol{j}$$

$$\boldsymbol{j}_{\rm CME} = C_{\rm A}\, \mu_A\, \boldsymbol{B} = \sigma_A \boldsymbol{B}$$

$\Rightarrow$

$$\nabla \times \boldsymbol{B} = \sigma_A\, \boldsymbol{B}$$

Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031

Chandrasekhar-Kendall states

Chandrasekhar-Kendall states minimize the energy at fixed helicity

$$h_m(\mathcal{K}) = \sum_{i=1}^{N} \phi_i^2 \, \mathcal{S}_i + 2 \sum_{i,j} \phi_i \phi_j \, \mathcal{L}_{ij}$$

$$\nabla \times \boldsymbol{B} = \sigma_A \, \boldsymbol{B}$$

H.K. Moffatt, R.Ricca '92,…

$$\nabla \times \boldsymbol{W}^{\pm}_{lm}(\boldsymbol{x};k) = \pm k \boldsymbol{W}^{\pm}_{lm}(\boldsymbol{x};k)\,, \qquad \nabla \cdot \boldsymbol{W}^{\pm}_{lm}(\boldsymbol{x};k) = 0\,,$$

$$\boldsymbol{B}(\boldsymbol{x},t) = \sum_{l,m} \int_0^\infty \frac{dk}{\pi} k^2 \left[\alpha^+_{lm}(k,t) \boldsymbol{W}^+_{lm}(\boldsymbol{x};k) + \alpha^-_{lm}(k,t) \boldsymbol{W}^-_{lm}(\boldsymbol{x};k)\right]$$

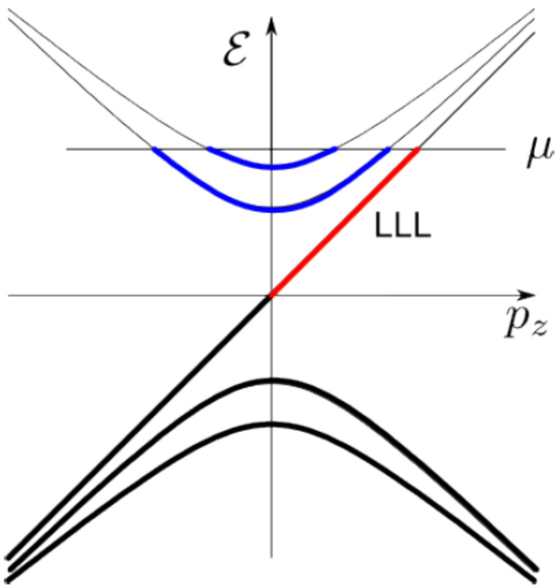
$$h_m(t) = \int_0^\infty \frac{dk}{\pi}\, k g(k,t)\,, \qquad g(k,t) \equiv g_+(k,t) - g_-(k,t)$$

Magnetic helicity spectrum:

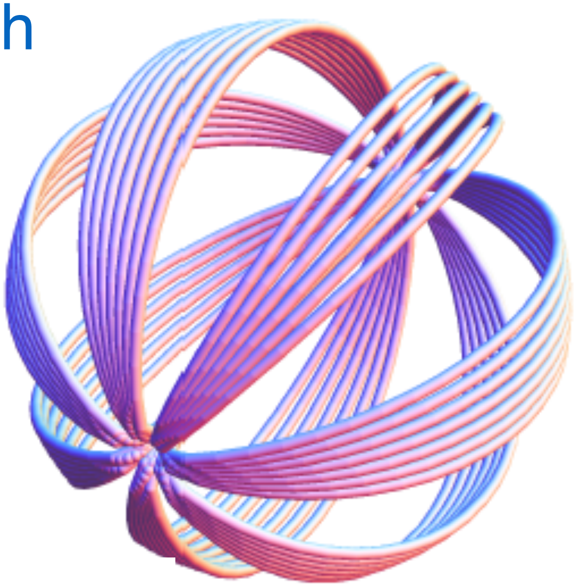
$$g_\pm(k,t) \equiv \sum_{l,m} |\alpha^\pm_{lm}(k,t)|^2$$

CK states in the QGP: M.Chernodub, arXiv:1002.1473

Self-similar inverse cascade



Instability at k< $c_A \mu_A$ leads to the growth of magnetic helicity

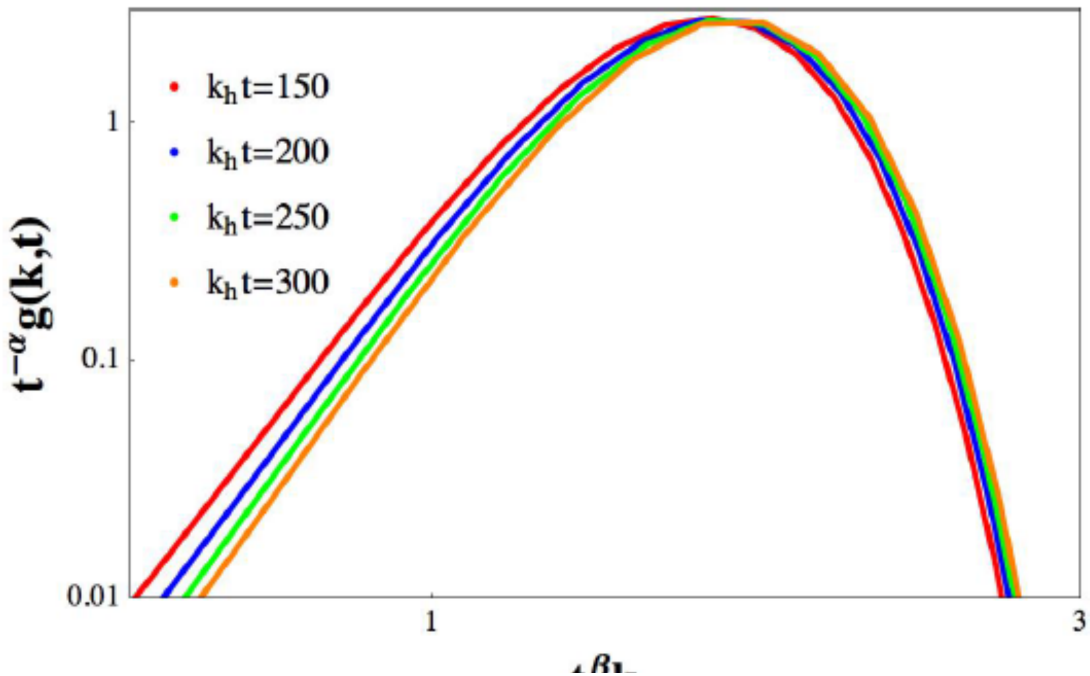


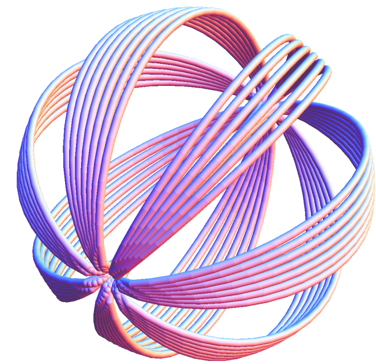
Increase of of magnetic helicity reduces μ_A

Inverse cascade itself was noted earlier, see refs in

Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031

Self-similar cascade of magnetic helicity driven by CME





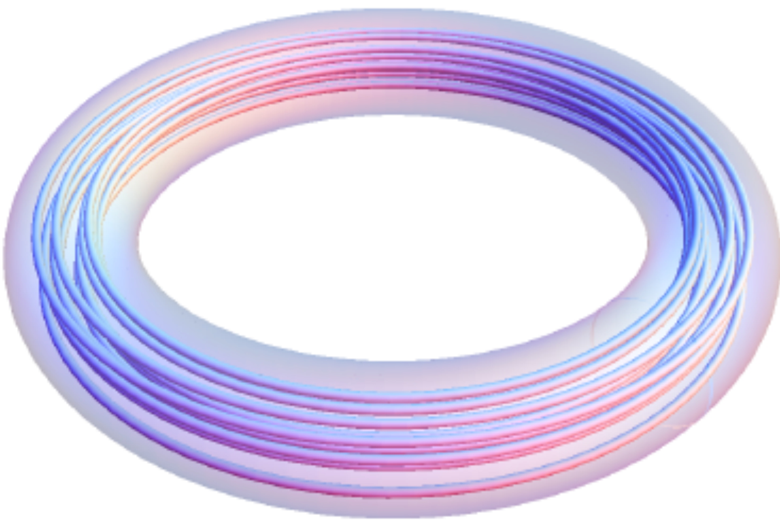
Hopfion states
e.g. L.Faddeev, A.Niemi, PRL 85(2010)3416
transform into CK ones

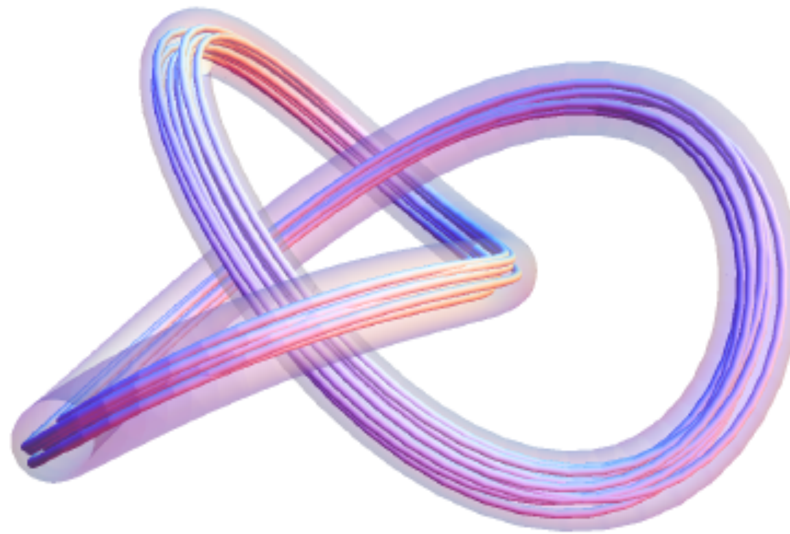
$$g(k,t) \sim t^{\alpha}\tilde{g}(t^{\beta}k) \qquad \alpha = 1\,, \qquad \beta = 1/2$$

Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031

Quantized CME from knot reconnections

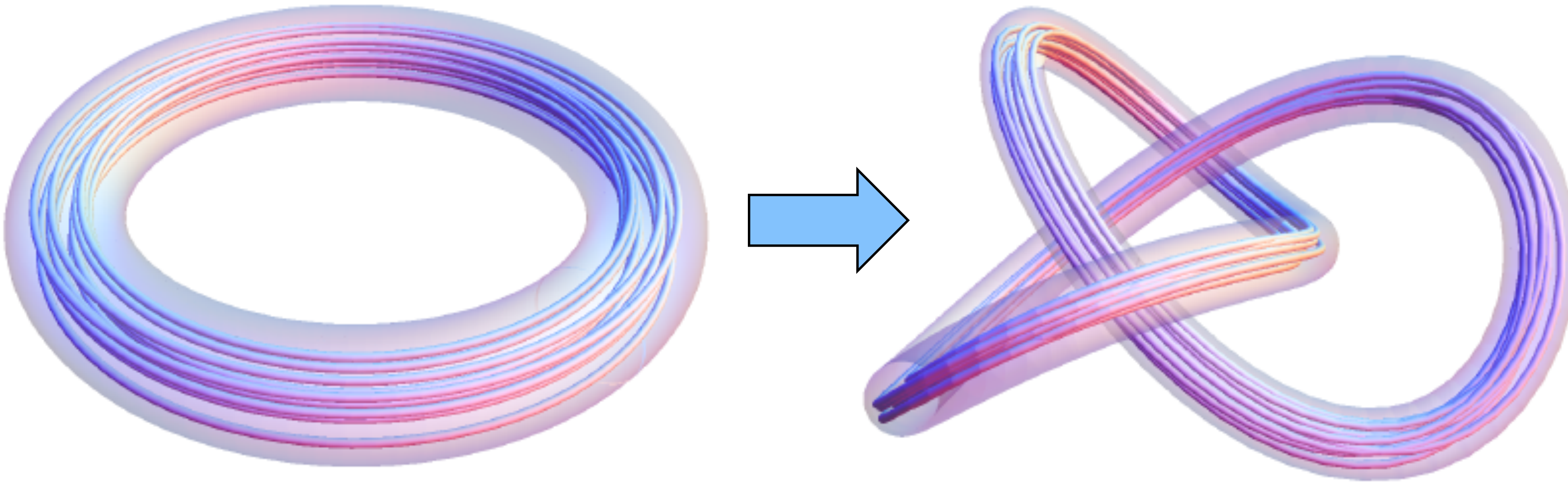
Y. Hirono, DK, Y. Yin, to appear





Consider a tube (unknot) of magnetic flux, with chiral fermions localized on it.
To turn it into a (chiral) knot, we need a magnetic reconnection.
What happens to the fermions during the reconnection?

Y. Hirono, DK, Y. Yin, to appear



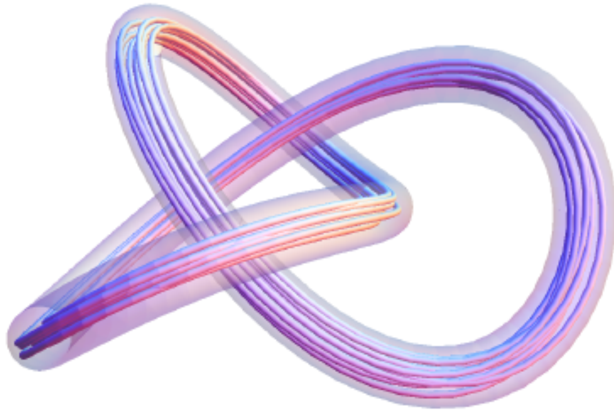
Changing magnetic flux through the area spanned by the tube will generate the electric field (Faraday's induction):

$$\frac{d}{dt}\Phi_B = -\oint_C \boldsymbol{E} \cdot d\boldsymbol{x}$$

The electric field will generate electric current of fermions (chiral anomaly in 1+1 D):

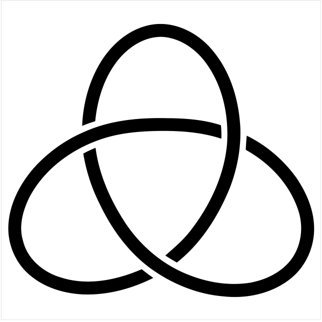
$$\Delta J = \Delta J_R + \Delta J_L = \frac{q^3\Phi^2}{2\pi^2 L}$$

Y. Hirono, DK, Y. Yin, to appear



Helicity change per magnetic reconnection is $\Delta\mathcal{H} = 2\Phi^2$.

Multiple magnetic reconnections leading to non-chiral knots do not induce net current (need to break left-right symmetry).



For N_+ positive and N_- negative crossings on a planar knot diagram, the total magnetic helicity is:

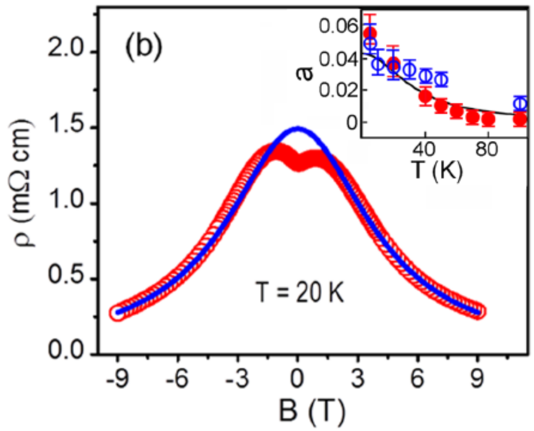
$$\mathcal{H} = 2(N_+ - N_-)\Phi^2$$

The total current induced by reconnections to a chiral knot:

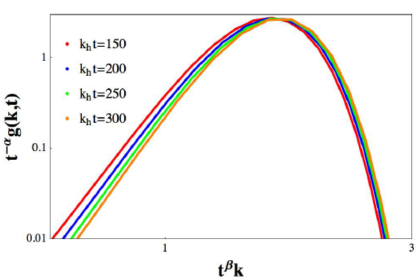
$$J = \frac{q^3\mathcal{H}}{4\pi^2 L}$$

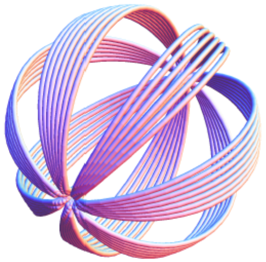
Summary-1

- Observation of CME in $ZrTe_5$



- CME and axion electrodynamics:
  self-similar inverse cascade of magnetic helicity towards the Chandrasekhar-Kendall state





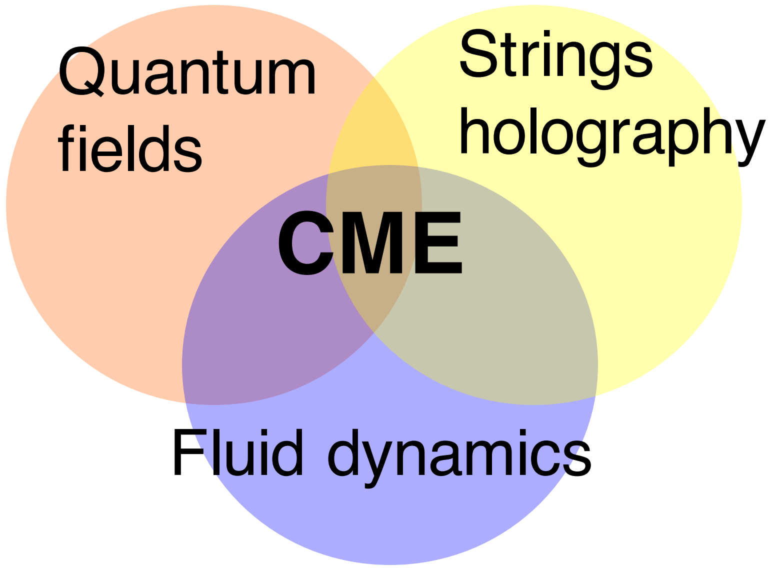
- Quantized CME from knot reconnections

$$J = \frac{q^3 \mathcal{H}}{4\pi^2 L}$$

Summary-2

Nuclear physics

Particle physics



Condensed matter physics

Real-world applications

Cosmology

Astrophysics