Solitons and Black-Hole Microstate Structure





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<u>Outline</u>

- Motivation and Challenges
- Microstate Geometries:

The only mechanism to avoid horizons

- Microstate Geometries as Holographic Phases
 <u>* New scales for Black Hole Physics</u>
- Building BPS Microstate Geometries

 Scaling Microstate geometries
- Holography and Fluctuating Microstate Geometries
- Microstate Geometries as Backgrounds for New Physics

★ W-branes ★ Tunneling

Motivation for Microstate Geometries I

General Relativity + Quantum mechanics ⇒ Non-unitary evolution arising from Hawking evaporation of black holes

An old conceit: Black hole evaporation time- scales are incredibly long: Fix with small corrections to GR?

e.g. via stringy or quantum gravity $((Riemann)^n)$ corrections to radiation?

New urgency

Mathur (2009): Using strong subadditivity of quantum information

 \Rightarrow There must be O(1) corrections to the Hawking states at the horizon

Microstate geometries/Fuzzballs

Find new structures and mechanisms in string theory that can address this issue by avoiding the formation of horizons

<u>Microstate Geometry</u> = Smooth, horizonless solutions to the bosonic sector of supergravity with the same asymptotic structure as a given black hole/ring

The Challenges for Horizon-Scale Structure

Old Story I: Black-Hole Uniqueness (1960-1980's)

In General Relativity coupled to electromagnetism in 3+1 dimensions: Black holes are almost completely featureless

 \Rightarrow Microstate structure is invisible from outside the black hole

Once a black hole forms, matter (including a naive *firewall*) is swept away from horizon region in the light-crossing time of black hole ...

Old Story II: Solitons in GR

GR non-linearities:

Could there be interesting solitons = Smooth, classical lumps What could these represent in Nature?

Macroscopic/large scale, smooth, end-states for stars:

★ Time independent ★ Massless fields

In General Relativity coupled to massless fields:

Time-independent solutions with time-independent matter necessarily have horizons ⇒ They must have singularities "No solitons without horizons"

Old Story III: The Fate of Weak-Coupling Resolutions

Set $G_{Newton} = 0$ and understand the microstate structure of material that will form a black hole at finite G_{Newton} ...

e.g. String theory: Strominger and Vafa: hep-th/9601029

Increase G_{Newton}, (or string coupling, g_s)

★ Matter/microstate structures shrink

* Horizon areas grow: $R_S = \frac{2 G_{Newton} M}{c^2}$



The Horowitz-Polchinski Correspondence Principle

As G_{Newton} or g_s increases, whatever microstates you have found disappear behind a horizon: Microstates are Planck scale fuzz deep inside the black hole

The Many Faces of the Same Underlying Problem

Black holes

Uniqueness theorems \Leftrightarrow All matter swept from horizon region

Massive Matter: Consequences of Tolman-Oppenmeir-Volkov equation

Massive fields cannot produce a resolution at the horizon scale: No massive field is stiff enough to prevent collapse to black hole

- Speed of sound > Speed of light
- * Buchdahl's Theorem: Central pressure/density infinite unless R_{matter} > 9/4 M

Weak/vanishing coupling resolutions

Will not survive at strong coupling:

Configurations of massive fields shrink as G_{Newton} (or g_s) increases

Massless fields

Appears hopeless: Massless fields travel at the speed of light ... only a "dark star" or black hole can hold such things into a star.

"No solitons without horizons"

Old Story IV: The Holographic Meta-Argument (~2000)

N=4 Yang-Mills on D3 branes is dual to String Theory in $AdS_5 \times S^5$



N=4 Yang-Mills is a unitary quantum field theory

⇒ Quantum gravity must be unitary within String Theory

In particular, string theory, via holography, must provide a unitary description of blackhole evaporation in asymptotically AdS spaces ..



Motivation for Microstate Geometries II

How does all this work in practice at strong coupling?

- What are the strong coupling phases of matter underlying stringy black-hole physics?
- How do you describe these phases holographically?
- What are the states underlying stringy black-hole physics?
- How do you see the microstates holographically?
- How does this relate to microstate descriptions at vanishing string coupling, for example, Strominger/Vafa?

What are the holographic duals of the Strominger-Vafa microstates?

• How do you count the microstates at strong coupling?

The study of Microstate Geometries is a systematic program that addresses these issues... at non-vanishing string coupling

The Microstate Geometry Program:

Find mechanisms and structures that resolve singularities and prevent the formation of horizons in Supergravity

Finite G_{Newton} (or g_s): Stringy resolution at/grow with horizon scale \Rightarrow Very long-range effects \Rightarrow Massless limit of string theory: Supergravity

<u>Microstate Geometry</u> \equiv Smooth, horizonless solutions to the bosonic sector of supergravity with the same asymptotic structure as a given black hole/ring

How to evade: "No solitons without horizons theorems?"

Solitons and Topology

Assume time-invariance: and there is a time-like Killing vector, K.





Unlike black-hole space-times, solitonic solutions are sectioned by smooth, space-like hypersurfaces, Σ .

Mass/energy is conserved and can be defined through a smooth integration over a regular surface: $M = \int_{\Sigma} T_{00} d\Sigma$

No solitons without horizons:

Equations of motion for massless field theory + Time-independent matter $\Rightarrow \int_{\Sigma} T_{00} d\Sigma \sim \int_{\Sigma} d[F \wedge B] \quad \text{where} \quad i_{K}(*F - F \wedge A) = dB$

In general: $T_{00} = \text{total derivative in } \Sigma \implies M \equiv 0$

 \Rightarrow If asymptotic to $\mathbb{R}^{D-1,1}$, then space-time must be globally flat, $\mathbb{R}^{D-1,1}$

<u>The Error</u>: This argument neglects topology T_{00} = total derivative in Σ only locally ...

Correct calculation \Rightarrow

Gibbons and NPW

$$M \sim \sum_{p} \int_{\Sigma} \mathbf{F}^{(p)} \wedge H^{(D-p-1)} \quad \text{where} \quad H^{(D-p-1)} \equiv harm(i_{K}(*_{D} \mathbf{F}^{(p)} + \dots))$$

Correct conclusion: No solitons without topology



A phase transition driven by the Chern-Simons interaction

Subtleties in boundary conditions: Yet more solitons ...

$$K^{\mu}\frac{\partial}{\partial x^{\mu}} = \frac{\partial}{\partial t} \longrightarrow T_{00} \equiv K^{\mu}K^{\nu}T_{\mu\nu} \longrightarrow M = \int_{\Sigma} T_{00} d\Sigma$$

This is just the Komar mass:

$$M = -\frac{1}{16\pi G_D} \frac{(D-2)}{(D-3)} \int_{S^{D-2}} *dM$$



$$K^{\mu}K^{\nu}g_{\mu\nu} = g_{00} \approx -1 + \frac{16\pi G_D}{(D-2)A_{D-2}}\frac{M}{\rho^{D-3}} + \dots$$
 at infinity

Subtlety: If space-time is asymptotically Minkowskian then Komar mass and ADM mass are the same and $M \equiv 0 \Rightarrow$ Entire space-time be globally flat, $\mathbb{R}^{D-1,1}$

If space-time is asymptotic to $R^{d-1,1} \times (S^1)^n$ then Komar mass \neq ADM mass $M_{Komar} \equiv 0 \Rightarrow$ space-time is globally $R^{d-1,1} \times (S^1)^n$

Example: Euclidean Schwarzschild bolt + time

$$ds_{4,1}^2 = -dt^2 + \left(1 - \frac{2m}{r}\right)d\tau^2 + \left(1 - \frac{2m}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

Asymptotic to, but not globally, $\mathbb{R}^{3,1} \times S^1$ $M_{Komar} = 0$ but $M_{ADM} = m$

<u>More solitons:</u> Classify solitons asymptotic to $R^{d-1,1} \times (S^1)^n$ or $AdS^{d-1,1} \times S^n$?

Conclusions

- 1) Solitons CAN be supported by cohomological magnetic fluxes
- 2) This is the only way to support solitons/microstate geometries for asymptotically Minkowskian geometries
- <u>Correct mantra:</u> "No solitons without topology"

Is this also true for space-times asymptotic to $\mathbb{R}^{d-1,1} \times (\mathbb{S}^1)^n$?

Geometric transitions to Microstate Geometries provide the only possible

Supergravity mechanism that can support horizon-scale structure!

What is the holographic interpretation of this result?

Some General Principles of Holographic Field Theory

- ★ Radial behavior in gravity ↔ RG flows in the holographic field theory
- ★ Normalizable fluxes in gravity ↔ States in the holographic field theory
- ★ Core of the gravity solution ↔ Deep Infra-red Limit of Field Theory

 - Scales of gravitational core structures IR scales of field theory
 - Fluxes in the gravitational core structures
 Order parameters of the infra-red physics
- ★ Families of phase transitions represented by brane sources, as seen from infinity/UV, replaced by smooth cohomological fluxes in the IR
 - Many important examples Gopakumar-Vafa; Dijkgraaf-Vafa; Lin-Lunin-Maldacena; Klebanov-Strassler
 - ★ Physically meaningful IR limits for holography:
 - Smooth supergravity solutions
 - Singular sources that can be identified in terms of branes
 - Stringy excitations around supergravity/brane backgrounds



- * IR geometry: Branes undergo phase transition to "bubbled geometry"
- ★ Singular IR Geometry: wrong physics/ wrong IR phase of the gauge theory
- ★ Confining phase of field theory ⇔ Fluxed IR geometry: Fluxes dual to gaugino condensates = order parameter of confining phase
- * Scale of the bubble $\sim \lambda_{SQCD}$

Apply exactly the same holographic principles to black holes ...

Microstate Geometries represent new phases of black-hole physics:

Back-reacted Brane Geometry



Features of the new phase

- ★ Size, λ_T , of a typical cycle \leftrightarrow Scale of phase transition
- ★ Fluxes through cycles ↔ Order parameters of new phase
- * "Physical Depth" defined by z_{max} = maximum redshift between infinity and the bubbles at the bottom that resolve the black hole

Traditional black-hole physics misses all the rich complexity of truly stringy black holes ... with new emergent phases of matter



Stringy Black Holes

New physics, new phases, new order parameters and two new scales (at least)

***** Scale of a typical cycle, λ_T ***** "Depth" of the "throat," Z_{max}

Traditional black holes: $\lambda_T = 0$, $z_{max} = \infty$

No solitons without topology \leftrightarrow Only these phases of stringy matter can give structure at the horizon scale

<u>Two distinct and independent ideas from microstate geometries</u>

- 1) A string theory *mechanism* to support structure at the horizon scale
 - The bubbled geometry provides a background to study other string phenomena, like fluctuations and brane wrapping
 - Holography: Such geometries describe a phases of the black-hole physics
- 2) Microstates: Fluctuations/Configurations in these phases
 - Supergravity fluctuations/moduli of microstate geometries
 = Coherent semi-classical description of detailed microstate structure
 - Other structures in microstate geometries (like W-branes)
 = description of other microstate structures (like Higgs branch states)
 - Holography: Understand the Microstates that are being captured

Note: Discussion so far is general ... not just BPS/supersymmetric

BPS Microstate Geometries

Phase structure and the supergravity mechanism

Building BPS Microstate Geometries

IIB Supergravity on T^4 : Supergravity in six-dimensions + BPS \Rightarrow Six dimensional metric ansatz: *Gutowski, Martelli and Reall*

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}} \frac{(dv+\beta)(du+\omega-\frac{1}{2}Z_3(dv+\beta))}{(du+\omega-\frac{1}{2}Z_3(dv+\beta))} + \sqrt{\mathcal{P}}V^{-1}(d\psi+A)^2 + \sqrt{\mathcal{P}}V\,d\vec{y}\cdot d\vec{y}$$

u = null time; $(v, \overline{\psi})$ define a double S¹ fibration over a flat R³ base with coordinates, y.



The non-trivial homology cycles are defined through the pinching off of the $S^1 \times S^1$ fibration at special points in the R^3 base.

Cycles support non-trivial cohomological fluxes ...

The scale of everything is set by the "warp factors:" V, P and Z_3

Scaling solutions



Fix charges and fluxes:

One can tune the orientation of the homology cycles so that the configuration remains *smooth* but *scales to an arbitrarily small size* in the R³ base ...

In the full six-dimensional geometry this scaling process:



- The bubbles descend and AdS throat
- ★ The bubbles retain their physical size
- The diameter of the throat limits to a fixed size determined by the charges and angular momentum of the configuration

End result: Looks almost exactly like a BPS black hole to as close to the horizon as one likes ... but then it caps off in a smooth microstate geometry.

A Decade of **BPS** Microstate Geometries

- There are vast families of smooth, horizonless BPS microstate geometries
- ★ New physics at the horizon scale
 - ⇒ The cap-off and the non-trivial topology, "bubbles," arise at the original horizon scale



- * Scaling microstate geometries with AdS throats that can be made arbitrarily long but cap off smoothly
 - Look exactly like a BPS black hole as close as one likes to the horizon Length/depth is classically free parameter
 - Long AdS throat: One can do holography in the AdS throat
 - \Rightarrow BPS black-hole physics/microstate structure described by a CFT
- ★ Six-Dimensional Microsite Geometries ↔ D1-D5 CFT
 - <u>Goal: construct precise map:</u>

Bubbled geometry ↔ Phase of CFT Geometric fluctuations ↔ Coherent combinations of microstates

The Energy Gap

 λ_{gap} = maximally redshifted wavelength, at infinity of lowest collective mode of bubbles at the bottom of the throat.

 $E_{gap} \sim (\lambda_{gap})^{-1}$

Egap determines where microstate geometries begin to differ from black holes



BPS: Semi-classical quantization of the moduli of the geometry:

- \star The throat depth, or z_{max} , is not a free parameter
- $\star E_{gap}$ is determined by the flux structure of the geometry
- * E_{gap} Longest possible scaling throat: $E_{gap} \sim (C_{cft})^{-1}$

Bena, Wang and Warner, arXiv:hep-th/0608217 de Boer, El-Showk, Messamah, Van den Bleeken, arXiv:0807.4556

Exactly matches Egap for the stringy excitations underlying the original state counting of Strominger and Vafa

⇒ Scaling microstate geometries are phases/representatives of states in the "typical sector" that provides the dominant contribution to the entropy ... Complete realization of geometric transition to new phases



Field theory: Scale of new phase ~ vevs of order parameters

Classically: Freely choosable geometry and scale parameter. Can have $\lambda_T >> \ell_p$ Field theory: Phase structure and order-parameter vevs are freely choosable

Open issues:

- Are the phases selected by dynamics/phase space volume?
- What sets λ_T ? Is large λ_T be entropically favored?

BPS Microstate Geometries Semi-Classical Microstate structure

<u>The D1-D5 System: <u>14</u> BPS = 8 supersymmetries</u>



IIB Superstring on T⁴ × S¹(z) × R^{4,1}

- * N₅ D5 branes wrap a $T^4 \times S^1(z)$
- ★ N₁ D1 branes wrap S¹(Z)

★ (1+1)-dimensional CFT on S¹(z) from open strings stretched between D1's and D5's $X_{(r)}^{\dot{A}A}(z,\bar{z}) \quad \psi_{(r)}^{\alpha\dot{A}}(z) \quad \widetilde{\psi}_{(r)}^{\dot{\alpha}\dot{A}}(\bar{z})$ *Chan-Paton labels:* $r = 1,..., N = N_1 N_5$

*(4,4) supersymmetry and $c = 6N = 6 N_1 N_5$

Holographic Dual: IIB Supergravity on $T^4 \times S^1(z) \times R^{4,1}$ \Rightarrow Supergravity in six-dimensions Geometric transition: \star D1/D5 replaced by RR 3-form flux \star Conformal vacuum: global $AdS_3 \times S^3$

Rich 1/4 BPS ground state structure: The ~N1 N5 RR ground states 1/4 BPS Holographic Dictionary: Lunin and Mathur; Kanitscheider, Skenderis and Taylor

<u>The D1-D5-P System: <u>18</u> BPS = 4 supersymmetries</u>

<u>1/8</u> BPS States: Add Left-moving momentum

<u>Right-movers</u>: RR vacua (4 susies)

<u>Left-movers</u>: Any CFT excitation, $L_0 = N_P =$ momentum charge

Count states: Count partitions of N_P in a CFT with $c = 6 N_1 N_5$ \Rightarrow Black-hole entropy

$$S = 2\pi \sqrt{\frac{c}{6}L_0} = 2\pi \sqrt{N_1 N_5 N_P}$$

Strominger and Vafa: hep-th/9601029

Holographic Dual of the Strominger-Vafa (Index) States?



 \mathbb{R}^4

D1

D5

 $\frac{1}{8}$ BPS Shape modes on AdS₃ × S³

<u>Goals:</u>

Ζ

- Construct fully-back-reacted fluctuating microstate geometries
- Holographic dictionary for these ½ BPS states
- Determine the stringy microstate structure visible within supergravity?

Status of BPS Microstate Geometries

BPS Microstate geometries define strongly interacting phases of the D1-D5 CFT

• Microstate geometries accessing correct phase and typical black-hole microstates with correct $E_{gap} \sim (C_{cft})^{-1}$

Where we are computationally with classical microstate geometries:

- Single Bubble/Two-centered geometries: Difficult but manageable
- Holographic dictionary for these 1/8 BPS states is now quite well-developed
- Constructing the holographic duals of the complete supergraviton gas on single bubble is within reach

Semi-classical geometries cannot see all string states, however:

- Arguments suggest that semi-classical structure can see enough black-hole microstates to obtain correct entropy growth $~S~\sim~\sqrt{N_1\,N_5\,N_P}$
- Broader goal: Identify semi-classical moduli spaces whose quantization can describe all the microstates of the black hole

Where we need to go next

- Strominger-Vafa: Most of the black-hole entropy comes from the highly-twisted sectors of the underlying CFT
- Deep, scaling geometries access these highly-twisted sectors (Egap ~(N1N5)⁻¹)
 ★ Necessarily involve multi-center, multi-bubble solutions
- Most general fluctuating geometries (superstrata) only constructed for shallow, single-bubble/two-centered geometries
 - Understand the links and holographic dictionary between deep, scaling geometries and twisted sectors of CFT
 - Construct generic, multi-centered families of superstrata in scaling geometries (Egap ~(N1N5)⁻¹)

← Holographic duals of twisted sector states

New ideas:

W-Branes and Supergravity Hypermultiplets Quantum Tunneling and Microstate Geometries

Old Story V: Heterotic-Type II duality

Abelian Maxwell fields arise directly in compactification of perturbative type II strings/supergravity: $F_{(p)} = \sum_{J} F_{(2),space-time}^{J} \wedge \omega_{(p-2),internal}^{J}$

Where can one get the *non-Abelian* Maxwell fields, "W-bosons," of $E_8 \times E_8$?

Hull and Townsend: Brane-wrapping of homology cycles.

The harmonic forms, ω , are dual to (p-2)-cycles in the compactification manifold and wrapped branes carry vector-multiplets in supergravity ...

- Mass of vector multiplets ~ Volumes of dual (p-2)-cycles
- Interactions of vector multiplets ~ Intersection form of cycles

When the size of cycles in the compactification manifold shrink to the string scale then the vector bosons become massless \rightarrow massless W-bosons

For K_3 there is a point in the moduli space where 16 2-cycles shrink to zero size and where the intersection form of these cycles is given by the Dynkin diagram of $E_8 \times E_8 \rightarrow non-Abelian$ Maxwell fields, "W-bosons," of $E_8 \times E_8$

<u>NB</u>: Number of BPS brane wrappings > size of homology basis

<u>W-branes in Scaling Microstate Geometries</u> Topological Structure Microstate Geometry + New Field Theory Phases

New low-mass excitations in these phases



Base geometry

Cycles shrink to zero size



Which geometry governs the masses of these W-brane states?

W-branes = branes wrapped around non-trivial cycles.

D-p brane wrap p-cycles can yield new BPS states of the system

These solitonic branes look like particles in remaining dimensions.

Duals of "Higgs Branch states"

<u>Complete space-time geometry</u> Cycles retain finite size but descend AdS throat



Massless W-branes

<u>DBI action</u>: Mass of W-brane states ~ Scale in base geometry

- Scale in full geometry
 (Red Shift from scaling BPS throat)
- ⇒ Deep scaling geometries have new classes of low mass/massless states

How many such states?

Naive count: One per cycle. A brane can wrap each non-trivial cycle

Actual count: Vastly larger number.

<u>Crucial insight:</u> Solitonic W-branes look like particles on the T^4 but this T^4 is threaded by magnetic fluxes and so each W-brane wrapping cycles in the space-time actually occupies distinct Landau levels on the T^4 .

Martinez and Niehoff: arXiv:1509.00044







Every distinct node sequence = Independent W-brane

Martinec and Niehoff: arXiv:1509.00044

Distinct W-branes ⇔ Higgs branch states of quiver quantum mechanics Denef This gives a semi-classical, solitonic description of the Higgs branch states

Count strings of sausages:

Number of such W-branes \Leftrightarrow Number of 3-derangements

3-derangements count Higgs Branch states in quiver quantum mechanics Bena, Berkooz, de Boer, El-Showk, Van den Bleeken: arXiv:1205.5023

The numbers of such states have the right growth with total charge to get the correct parametric entropy growth of the black hole ...

The Invisible Quantum Elephant of Black-Hole Physics

Curvature at horizon: $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}|_{horizon} = \frac{3}{16}\frac{G^2}{M^4} \Rightarrow Large black hole is classical at horizon scale$ Fermi Golden Rule: $\mathcal{T}_{i \to f} = \frac{2\pi}{\hbar} < \psi_f |\mathcal{H}_{int}|\psi_i > |^2 \rho \checkmark \text{density of states}$ Number of states inside black hole $\sim e^{+16\pi M^2/m_P^2}$ Number of states in the black hole in the middle of Milky Way: $e^{10^{90}}$

It is the extreme density of states that makes an apparently classical black hole behave as a quantum object Mathur: 0805.3716; 0905.4483 Mathur and Turton: 1306.5488

Macroscopic tunneling probabilities ~ O(1)?

Iosif Bena, Daniel R. Mayerson, Andrea Puhm, Bert Vercnocke, arXiv:1512.05376

Final thought...

Maybe in spite of its macroscopic size, the near-horizon properties of black holes are dominated by quantum effects ... and this is what makes the O(1) changes to horizon-scale physics

So what good is all this classical supergravity analysis?

Obvious, boring answer:

Microstate Geometries are the semi-classical limit of these quantum effects: The gravitational expression of coherent sets of black-hole quantum states ...

Much more interesting "holographic" answer:

Supergravity identifies the long-range, large scale degrees of freedom that control physics at the horizon scale ...

- Supergravity controls the phase structure of the new black-hole physics
- Maybe we only have to perform the semi-classical quantization of all these relatively simple degrees of freedom to get a good picture of what is really happening at the horizon of a black hole ..

Supergravity is the emergent description of horizon-scale physics.

Conclusions

- Solving the information problem requires O(1) changes to the physics at the horizon scale
- Large scale resolutions must be based on microstate geometries with non-trivial topology and fluxes
- Microstate geometries define, holographically, new phases of black-hole physics
- Solving the information paradox via holography in supergravity requires the new black-hole phases described by microstate geometries
- New scales in black-hole physics: Transition scale, λ_T , and maximum red-shift, Z_{max} ; related to E_{gap} of fluctuation spectrum
- BPS solutions:
 - * Vast families of explicit examples
 - * Remarkably good laboratory for developing all these ideas
 - ★ Holographic dictionary for geometric fluctuations and CFT states is becoming well-developed
 - * Holographic duals of "Strominger-Vafa" states under construction
 - ***** Semi-classical description of entropy with $S \sim \sqrt{N_1 N_5 N_P}$ is within reach
 - Lots of exciting new ideas: W-branes and Higgs branches; tunneling calculations; non-BPS; Investigations of infall ...