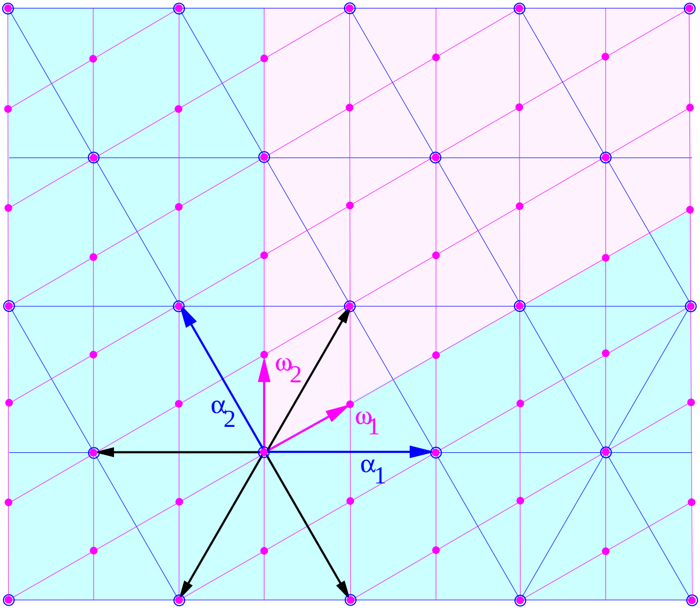


ω2
α2
ω1
α1

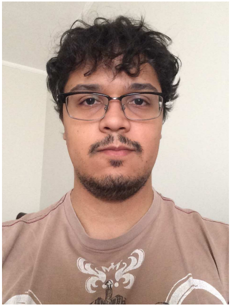
Effective Actions for Superconducting Chiral Fermions

Michael Stone

Institute for Condensed Matter Theory
University of Illinois

Tours LE STUDIUM, June 13th 2016

Work done in collaboration with Pedro Lopes



Talk based on:

- X-L. Qi, E. Witten, S-C. Zhang, *Axion topological field theory of topological superconductors*, Phys. Rev. B **87** 134519 1-10 (2013).
- MS, Pedro Lopes, *Effective action and electromagnetic response of topological superconductors and Majorana-mass Weyl fermions,* Phys. Rev. **B93** 174501 (2016); arXiv:1601.07869.

Outline

1. Motivating paradox

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2. Weyl superconductors
   - Majorana mass
   - Vector *vs* axial-vector gauge fields
   - Anomalies — and a second paradox?

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   - 3+1 dimensions
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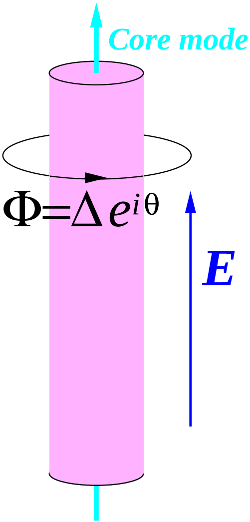
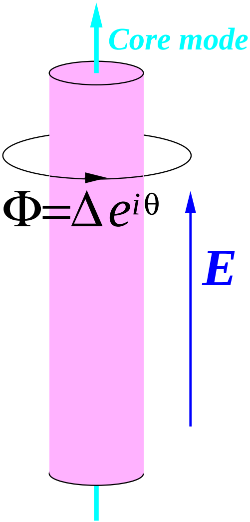
Motivating paradox

- Dirac electron with chiral mass



Motivating paradox

- Dirac electron with chiral mass
- Mass winds around vortex



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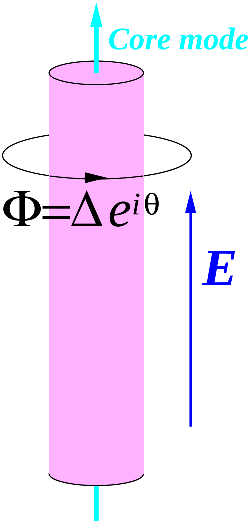

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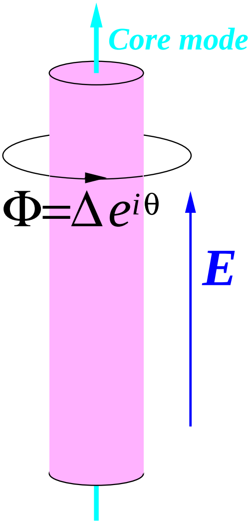

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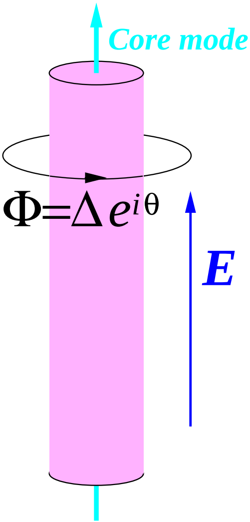

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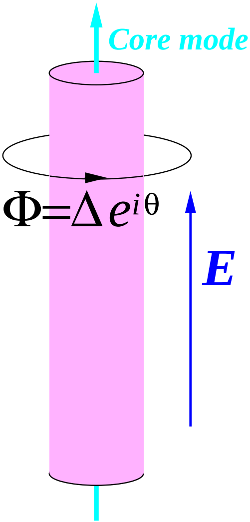
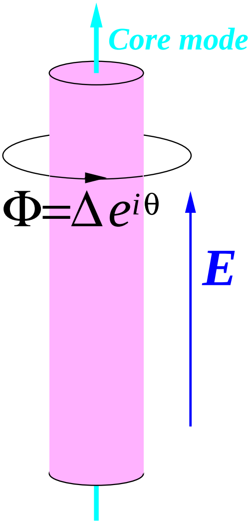


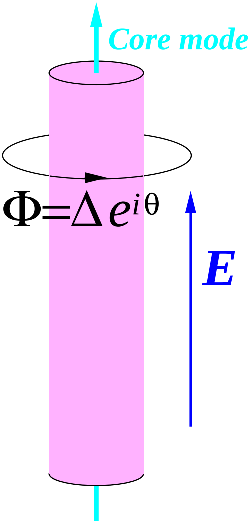
Motivating paradox

- Dirac electron with chiral mass
- Mass winds around vortex
- Chiral vortex-core mode
- Electric field parallel to vortex
- Anomaly in core current
- Charge flows in from the outside
- Classic Callan-Harvey effect.



Motivating paradox

- Weyl fermion with chiral Majorana mass



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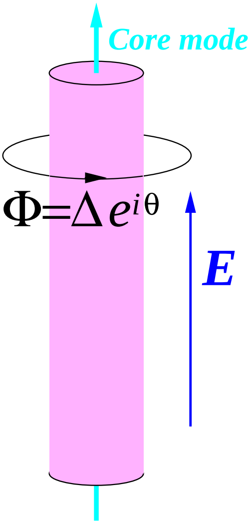

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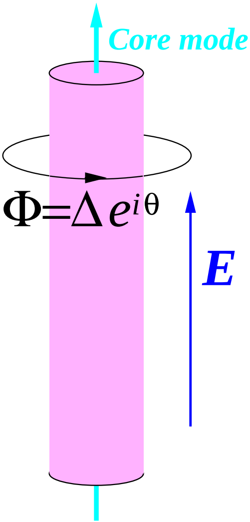

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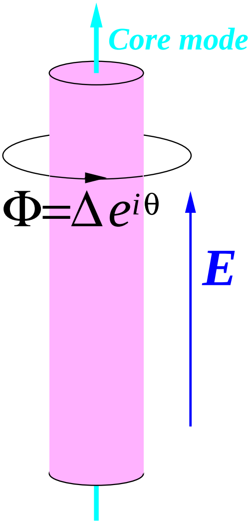

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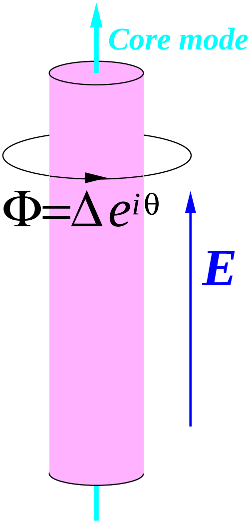

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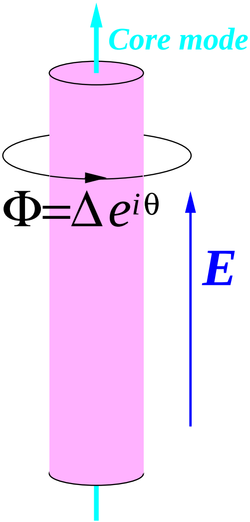

Motivating paradox

- Weyl fermion with chiral Majorana mass
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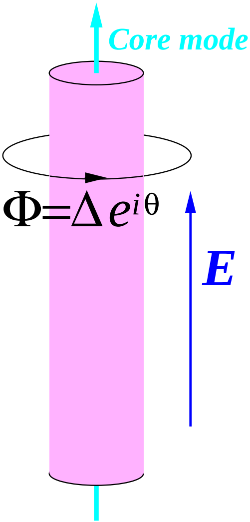
- Electric field parallel to vortex
- QZW claim charge flows in from the outside
- — but core mode is neutral!
- Where does the charge go?



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Weyl Hamiltonian

- Quantum Weyl Hamiltonian

$$
\begin{aligned}
\hat{H}_{\rm Weyl}[A] &= \int \psi^\dagger \left\{ \boldsymbol{\sigma}\cdot(\mathbf{p}-e\mathbf{A}) + e\phi \right\} \psi \, d^3x \\
&= \int \psi_{\rm c}^\dagger \left\{ -\boldsymbol{\sigma}\cdot(\mathbf{p}+e\mathbf{A}) - e\phi \right\} \psi_{\rm c} \, d^3x.
\end{aligned}
$$

- Here $\psi_{\rm c} = i\sigma_2\psi^*$
- Right handed particle charge e. Left handed antiparticle charge $-e$.

Majorana mass

- Add chiral Majorana mass coupling

$$\hat{H}_1 = \frac{1}{2}(\Phi\psi^\dagger\psi_{\rm c} + \Phi^*\psi_{\rm c}^\dagger\psi) = \frac{1}{2}(\Phi\epsilon_{\alpha\beta}\psi^*_\alpha\psi^*_\beta - \Phi^*\epsilon_{\alpha\beta}\psi_\alpha\psi_\beta)$$

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- Put together to get $\hat{H}_{\rm BdG}[A] =$

$$\frac{1}{2}\int d^3x \begin{pmatrix}\psi^\dagger & \psi_{\rm c}^\dagger\end{pmatrix} \begin{bmatrix}\boldsymbol{\sigma}\cdot(\mathbf{p}-e\mathbf{A})+e\phi & \Phi \\ \Phi^* & -\boldsymbol{\sigma}\cdot(\mathbf{p}+e\mathbf{A})-e\phi\end{bmatrix} \begin{pmatrix}\psi \\ \psi_{\rm c}\end{pmatrix}$$

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- Weyl fermion + Majorana mass = BdG Superconductor.

Dirac Hamiltonians

Usual Dirac hamiltonian + vector gauge field:

$$H_{\text{Dirac}} = \begin{bmatrix} \boldsymbol{\sigma}\cdot(\mathbf{p}-e\mathbf{A})+e\phi & \Phi \\ \Phi^* & -\boldsymbol{\sigma}\cdot(\mathbf{p}-e\mathbf{A})+e\phi \end{bmatrix}$$

Dirac Hamiltonians

Our BdG has axial vector gauge field:

$$H_{\text{Dirac}} = \begin{bmatrix} \boldsymbol{\sigma}\cdot(\mathbf{p}-e\mathbf{A})+e\phi & \Phi \\ \Phi^* & -\boldsymbol{\sigma}\cdot(\mathbf{p}+e\mathbf{A})-e\phi \end{bmatrix}$$

Anomalies

- Right-handed Weyl fermion + right-handed gauge field

$$\partial_\mu j^\mu_{\rm R} = \frac{e^2}{32\pi^2}\epsilon^{\mu\nu\sigma\tau}\,{\rm tr}\left\{F^{\rm R}_{\mu\nu}F^{\rm R}_{\sigma\tau}\right\}$$

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- Dirac fermion + vector gauge field

$$\partial_\mu j^\mu_{5} = \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\sigma\tau}{\rm tr}\left\{F^{\rm V}_{\mu\nu}F^{\rm V}_{\sigma\tau}\right\}$$

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- Dirac fermion + axial-vector gauge field

$$\partial_\mu j^\mu_5 = \frac{1}{3}\cdot\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\sigma\tau}\,{\rm tr}\left\{F^{\rm A}_{\mu\nu}F^{\rm A}_{\sigma\tau}\right\}$$

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WZW effective action

- Couple Weyl fermions to gauge fields

$$\nabla_\mu \psi_R = (\partial_\mu + R_\mu)\psi_R, \quad \nabla_\mu \psi_L = (\partial_\mu + L_\mu)\psi_L$$

- Gap with mass term

$$H_{\rm mass} = \Delta(\psi^*_{L,i} U_{ij} \psi_{R,j} + \psi^*_{R,i} U^\dagger_{ij} \psi_{L,j})$$

- Classical gauge invariance

$$\begin{aligned} L &\to L^{h_L} = h_L^{-1} L h_L + h_L^{-1} d h_L \\ R &\to R^{h_R} = h_R^{-1} R h_R + h_R^{-1} d h_R \\ U &\to h_L^{-1} U h_R. \end{aligned}$$

- Everything slaved to external R, L, U fields. Seek effective action.

Mañes construction of WZW effective action

- Chern-Simons form ω_{2n-1} such that $d\omega_{2n-1} = \mathrm{tr}\left\{F^n\right\}$.
- Usual 5-form choice $\omega_5(A) = \mathrm{tr}\left\{AF^2 - \frac{1}{2}FA^3 + \frac{1}{10}A^5\right\}$.
- Select alternative 5-form $\tilde{\omega}_5(R, L)$ such that

$$d\tilde{\omega}_5(R, L) = \mathrm{tr}\left\{(F^R)^3\right\} - \mathrm{tr}\left\{(F^L)^3\right\} \text{ and } \tilde{\omega}_5(R^h, L^h) = \tilde{\omega}_5(R, L)$$

- Then $\tilde{\omega}_5(R^{h_R}, L^{h_L}) = \tilde{\omega}_5(R, L^U)$ where $U = h_L h_R^{-1}$.
- Can write $\tilde{\omega}_5(R, L) = \omega_5(R) - \omega_5(L) + dS_4$
- $S_4(R, L)$ is the Bardeen counterterm

Wess-Zumino-Witten action

- Set

$$\tilde{C}[L,R] = \frac{-i}{24\pi^2}\int_{M_5}\tilde{\omega}_5(L,R)$$

- Define 4-d Wess-Zumino functional $W[L,R,U]$ by

$$\tilde{C}[L,R^U] = C[L,R] + W[L,R,U]$$

- Then $\tilde{C}[L,R^U]$ is invariant under

$$\begin{aligned} R &\rightarrow R^{h_R} = h_R^{-1}Rh_R + h_R^{-1}dh_R,\\ L &\rightarrow L^{h_L} = h_L^{-1}Lh_L + h_L^{-1}dh_L,\\ U &\rightarrow h_L^{-1}Uh_R. \end{aligned}$$

- But $C[L,R]$ and $W[L,R,U]$ not separately invariant

Witten's formula

- Witten 1983; Kaymakcalan *et al*. 1984; Mañes 1985:

$$W[R,L,U] = -\frac{i}{240\pi^2}\int_M \mathrm{tr}\,\{(U^{-1}dU)^5\} - \frac{i}{48\pi^2}\int_{\partial M} Z(L,R,U)$$

- With $Z(R,L,U) =$

$$\begin{aligned}
&-\mathrm{tr}\,\{U_L(LdL+dLL+L^3) - U_L^3 L\} - \mathrm{tr}\,\{R \leftrightarrow L\}\\
&+\tfrac{1}{2}\mathrm{tr}\,\{U_L L U_L L\} - \tfrac{1}{2}\{R \leftrightarrow L\}\\
&-\mathrm{tr}\,\{U^{-1}LUR^3\} + \mathrm{tr}\,\{URU^{-1}L^3\}\\
&-\mathrm{tr}\,\{U^{-1}LU(RdR+dRR)\} + \mathrm{tr}\,\{URU^{-1}(LdL+dLL)\}\\
&-\mathrm{tr}\,\{URU^{-1}LU_L L\} - \mathrm{tr}\,\{U^{-1}LURU_R R\}\\
&+\mathrm{tr}\,\{LdUU_R RU^{-1}\} + \mathrm{tr}\,\{RdU^{-1}U_L LU\}\\
&-\mathrm{tr}\,\{dLdURU^{-1}\} + \mathrm{tr}\,\{dRdU^{-1}LU\}\\
&+\tfrac{1}{2}\mathrm{tr}\,\{RU^{-1}LURU^{-1}LU\}.
\end{aligned}$$

- Here $U_L = dUU^{-1}$, and $U_R = U^{-1}dU$.

Action for topological insulator

- Abelian vector-gauge field: $U = e^{-i\theta}$, $L = R \to A$

$$Z \to 6i\, d\theta\, \mathrm{tr}\, \left\{ AdA + \frac{2}{3}A^3 \right\},$$

- Hence

$$\begin{aligned} W[A,\theta] &= \frac{1}{8\pi^2}\int_{\partial M} d\theta\, \mathrm{tr}\, \left\{ AdA + \frac{2}{3}A^3 \right\} \\ &= -\frac{1}{8\pi^2}\int_{\partial M} \theta\, \mathrm{tr}\, \{F^2\}. \end{aligned}$$

- This is the usual "θ" term that appears in topological insulators.

Action for our Majorana-mass Weyl fermion

- Set $L = -R$ and divide by two because $\psi_L = (\psi_R)_c$
- Include non-topological superconducting kinetic term.
- Find:

$$\begin{aligned} S[\theta, A] &= \frac{1}{2}\int_{M_4=\partial M_5} d^4x \left\{ \frac{f^2}{2}(\partial_\mu\theta + 2eA_\mu)(\partial^\mu\theta + 2eA^\mu) \right. \\ &\qquad \left. - \frac{\theta}{96\pi^2}\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}F_{\sigma\tau} \right\} \\ &\quad - \frac{1}{96\pi^2}\int_{M_5} d^5x \epsilon^{\mu\nu\rho\sigma\tau}A_\mu F_{\nu\rho}F_{\sigma\tau}, \end{aligned}$$

- S invariant under the gauge transformation

$$\begin{aligned} \theta &\to \theta - 2\alpha e, \\ A_\mu &\to A_\mu + \partial_\mu\alpha, \end{aligned}$$

Current and Anomaly

- 4-d Boundary current:

$$J^{\mu}_{\rm Num} = f^2(\partial^\mu\theta + 2eA^\mu) + \frac{e}{48\pi^2}\epsilon^{\mu\nu\sigma\tau}(\partial_\nu\theta + 2eA_\nu)F_{\sigma\tau}$$

- Equation of motion for θ:

$$-f^2\partial_\mu(\partial^\mu\theta + 2eA^\mu) = \frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}F_{\sigma\tau}$$

- Chiral anomaly:

$$\partial_\mu J^{\mu}_{\rm Num} = \frac{e^2}{32\pi^2}\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}F_{\sigma\tau}$$

- <u>One third</u> of anomaly comes from the topological term; <u>two-thirds</u> from first term

Whence the 1/3?

Keep L and R distinct and look at

- Currents on boundary

$$
\begin{aligned}
J_R^\mu &= -f^2(\partial^\mu\theta - L^\mu + R^\mu) + \frac{1}{24\pi^2}\epsilon^{\mu\nu\sigma\tau}(\partial_\nu\theta - L_\nu + R_\nu)\left(F^R_{\sigma\tau} + \frac{1}{2}F^L_{\sigma\tau}\right) \\
J_L^\mu &= +f^2(\partial^\mu\theta - L^\mu + R^\mu) + \frac{1}{24\pi^2}\epsilon^{\mu\nu\sigma\tau}(\partial_\nu\theta - L_\nu + R_\nu)\left(F^L_{\sigma\tau} + \frac{1}{2}F^R_{\sigma\tau}\right)
\end{aligned}
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$$J_L^\mu = +f^2(\partial^\mu\theta - L^\mu + R^\mu) + \frac{1}{24\pi^2}\epsilon^{\mu\nu\sigma\tau}(\partial_\nu\theta - L_\nu + R_\nu)\left(F_{\sigma\tau}^L + \frac{1}{2}F_{\sigma\tau}^R\right)$$

- Equation of motion for the θ field

$$-f^2\partial_\mu(\partial^\mu\theta - L^\mu + R^\mu) = \frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\tau}(F_{\mu\nu}^L F_{\sigma\tau}^L + F_{\sigma\tau}^R F_{\mu\nu}^R + F_{\mu\nu}^R F_{\sigma\tau}^L)$$

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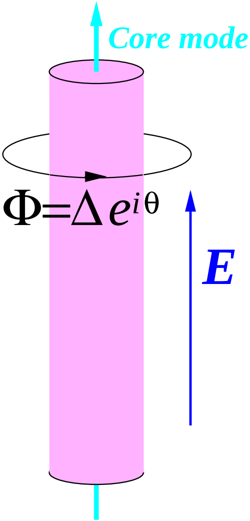
Paradox still?

- Usual Callan Harvey inflow

$$j^\mu = \frac{e}{8\pi^2}\epsilon^{\mu\nu\sigma\tau}\partial_\nu\theta F_{\sigma\tau},$$



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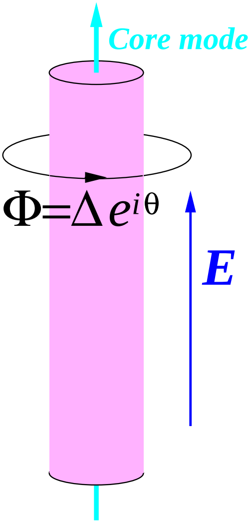
- QZW inflow

$$j^{\mu} = \frac{e}{16\pi^2}\epsilon^{\mu\nu\sigma\tau}\partial_{\nu}\theta F_{\sigma\tau},$$



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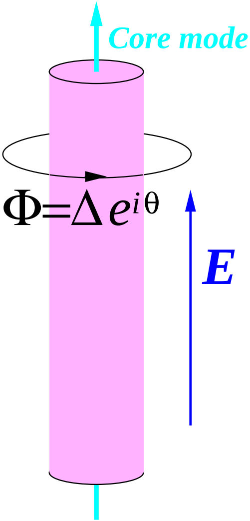
$$j^\mu = \frac{e}{16\pi^2}\epsilon^{\mu\nu\sigma\tau}\partial_\nu\theta F_{\sigma\tau},$$

- Our inflow

$$j^\mu = \frac{e}{48\pi^2}\epsilon^{\mu\nu\sigma\tau}(\partial_\nu\theta+2eA_\nu)F_{\sigma\tau},$$



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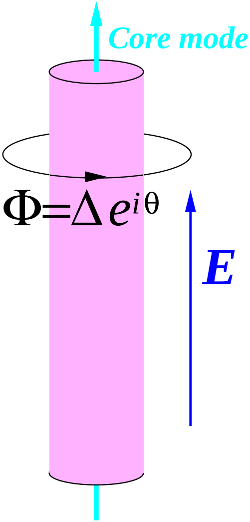
- Not zero!



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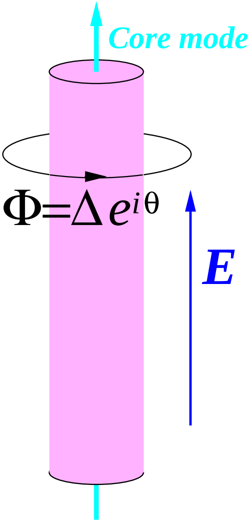
- Not zero!
- But ....



Chiral Magnetic effect

- With suitable $\mathbf{B} = (0, 0, B_3 = F_{12})$ anomaly eq becomes

$$\partial_0 J^0_{\rm Num} + \partial_3 J^3_{\rm Num} = \frac{1}{4\pi^2}(\partial_0 A_3 - \partial_3 A_0) B_3$$

- Satisfied by

$$\begin{aligned} J^0_{\rm Num} &= \rho_0 + \frac{e^2}{4\pi^2} A_3 B_3, \\ J^1_{\rm Num} &= J^2_{\rm Num} = 0, \\ J^3_{\rm Num} &= -\frac{e^2}{4\pi^2} A_0 B_3 \end{aligned}$$

- $\Rightarrow$ Usual CME:

$$\mathbf{J}_{\rm CME} = \frac{e\mu_R}{4\pi^2}\mathbf{B}$$

where $-eA_0 \to \mu_R$.

Superfluid Chiral magnetic effect

- Because of Meissner condition we get different result

$$\begin{aligned} J^0_{\rm Num} &= -2ef^2A^0, \\ J^1_{\rm Num} &= -2ef^2A^1, \\ J^2_{\rm Num} &= -2ef^2A^2, \\ J^3_{\rm Num} &= -\frac{e^2}{12\pi^2}A^0B_3 = \frac{\mu_R e}{12\pi^2}B_3 \end{aligned}$$

- Fluid is rotating

$$\mathbf{\Omega} = -\left(\frac{e}{2\mu_R}\right)\mathbf{B}$$

- Reduced CME current

$$J^3_{\rm Num} = \frac{\mu_R e}{12\pi^2}B_3$$

- Only 1/3 of the usual value.

Is the factor 1/3 correct?

From:Amado, Landsteiner, Peña-Benítez; JHEP 05 (2011) 081

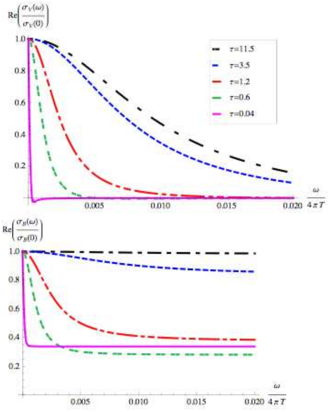


Figure 2: Chiral vortical (up) and magnetic (bottom) conductivities as function of the frequency close to $\omega = 0$. Real (left) and imaginary (right) part of the normalized conductivity for different values of the dimensionless temperature.

Conclusions

- Somewhat different formulæ from QZW.
- Some interesting paradoxes raised and resolved
- Possible applications to understanding issues around CME and CVE