Arizona State University

Workshop in Tours
JUN 14, 2016
CESR

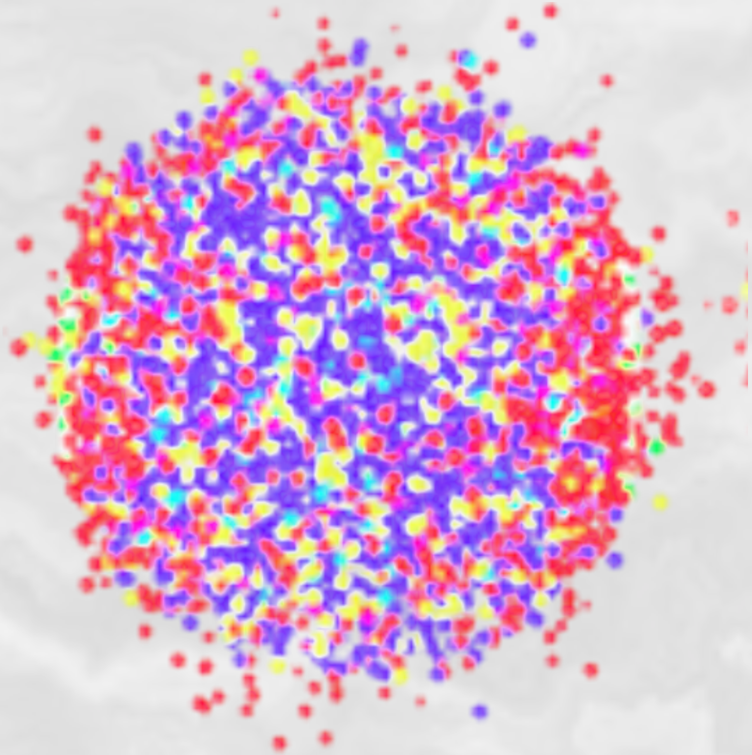
Anomalous chiral plasmas: from Dirac semimetals to cosmology

Igor Shovkovy
Arizona State University

WORKSHOP

JUNE 13, 2016 - JUNE 15, 2016

Condensed matter physics meets relativistic quantum field theory



CHIRAL PLASMAS

Chiral fermions

- *Massless* Dirac fermions:

$$(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{\mathbf{p}})\Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma}\cdot\vec{\mathbf{p}}}{|\vec{\mathbf{p}}|}\Psi = \text{sign}(p_0)\gamma^5\Psi$$

For particles $(p_0 > 0)$: chirality = helicity

For antiparticles $(p_0 < 0)$: chirality = - helicity

- Dirac fermions in *relativistic* regime
  - High temperature: $T >> m$
  - High density: $\mu >> m$

Chiral plasma

- Plasma of chiral fermions with $n_\mathrm{L} \neq n_\mathrm{R}$
- Note: Unlike electric charge (fermion number), chiral charge is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

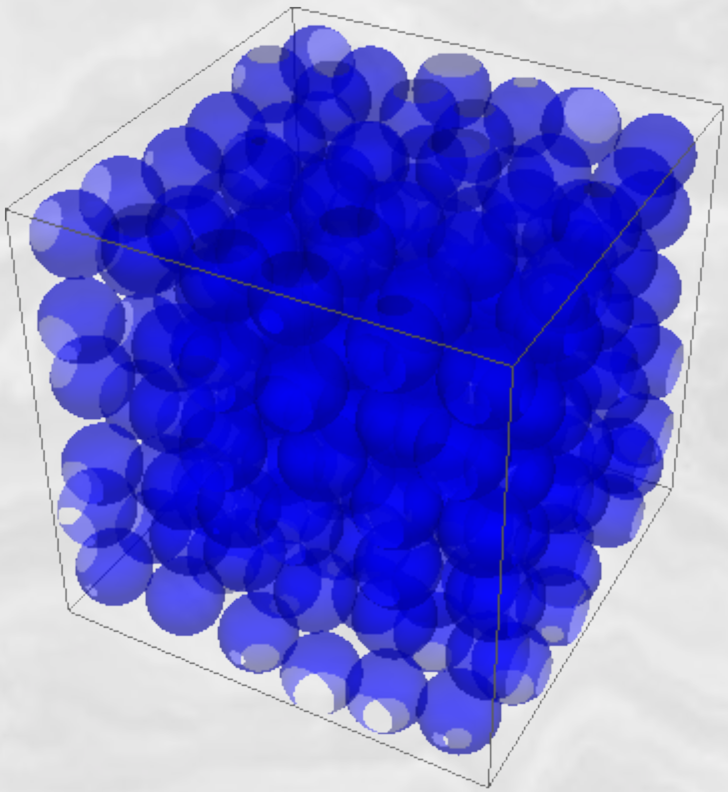
$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral symmetry is anomalous in quantum theory
- **Magnetic fields** often play critical role

Super-dense matter

- Electron/quark plasma inside compact stars

Pauli exclusion principle: fermions cannot occupy same quantum states (they end up filling out all states from $p_{\min} \approx 0$ to $p_{\max} \propto \hbar n^{1/3}$)



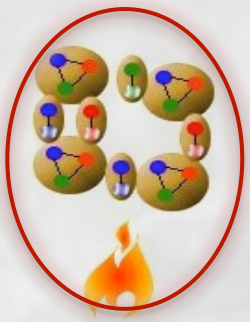
$$p_{\max} \propto 200 \left(\frac{n}{1\,\mathrm{fm}^3} \right)^{1/3} \mathrm{MeV/c}$$

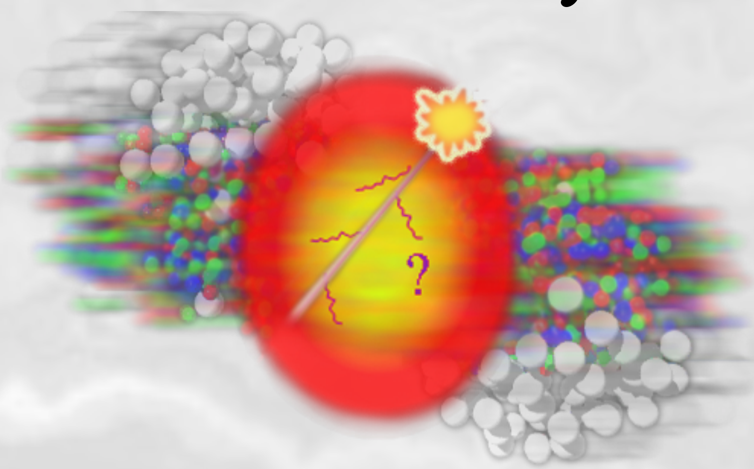
Magnetic fields in start

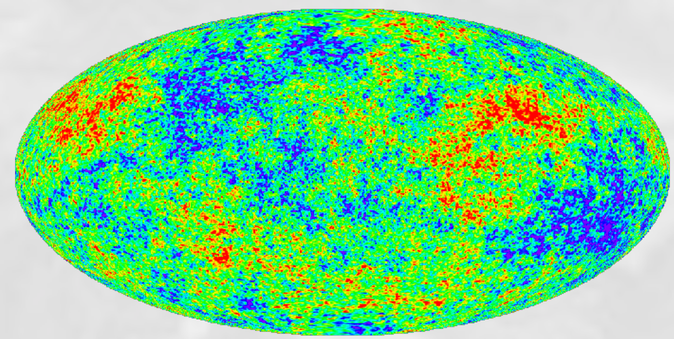
- Typical field strengths
  - **10^{10} to 10^{18} G (10 keV to 100 MeV)**
- Magnetic field may affect
  - Competition of ground state phases
  - EoS of dense baryonic matter
  - the M-R relation of compact stars
  - Transport and emission properties
  - Evolution of supernovas & protoneutron stars

Super-hot plasma

- **Quark gluon plasma** in heavy ion collisions
- **Hot matter** in the Early Universe





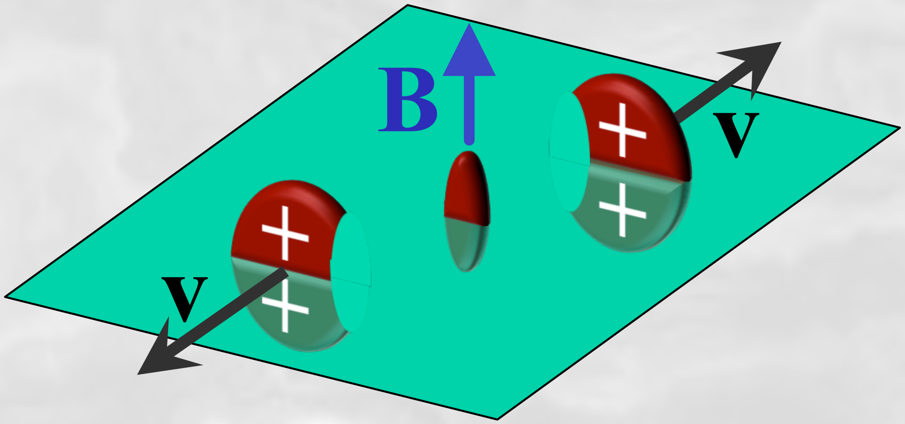


Heat is equivalent to **kinetic energy**: average kinetic energy of particles is proportional to temperature:

$$p \propto k_B T / c \sim 200 \left(\frac{k_B T}{200\ \text{MeV}} \right) \text{MeV}/\text{c} \quad (\text{assuming } p >> mc)$$

Magnetic fields in little Bangs

- Magnetized QGP at RHIC/LHC
  - **B ~ 10^{18} to 10^{19} G** (~ 100 MeV)



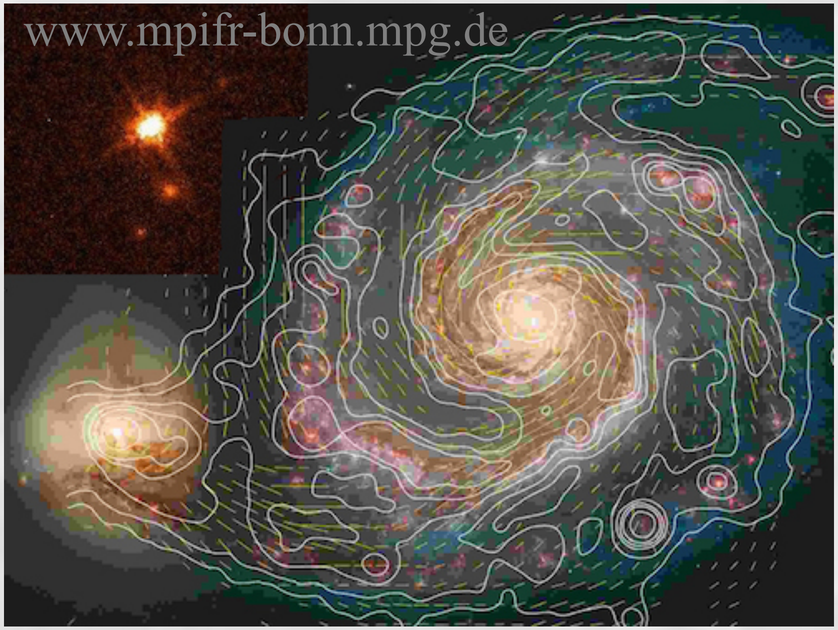
[Rafelski & Müller, PRL, 36, 517 (1976)],
[Kharzeev et al., arXiv:0711.0950],
[Skokov et al., arXiv:0907.1396],
[Voronyuk et al., arXiv:1103.4239],
[Bzdak &. Skokov, arXiv:1111.1949],
[Deng & Huang, arXiv:1201.5108]
…

- Using Lienard-Wiechert potentials,

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

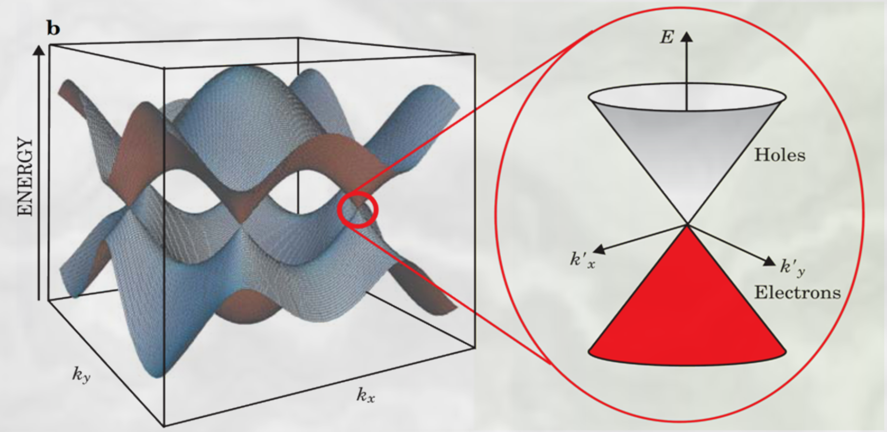
Magnetic fields in Universe



- Current galactic magnetic fields ~ 10^{-6} G
- Current magnetic fields in voids ~ 10^{-15} G
- Problem of magnetogenesis in Early Universe
- Perhaps during the electro-weak phase transition
  - **10^{20} to 10^{24} G** (~ 1 GeV to 100 GeV)

Dirac/Weyl materials

- 2D Dirac materials
  - Graphene



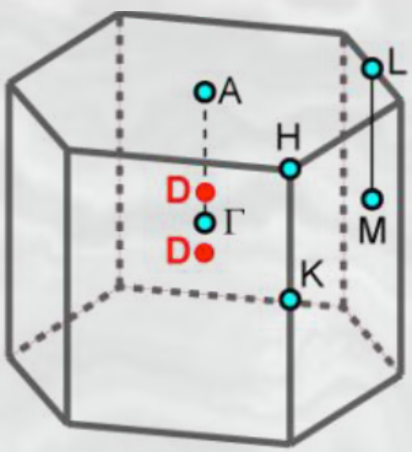
- 3D materials with Dirac/Weyl quasiparticles
  - $Bi_{1-x}Sb_x$ alloy (at $x \approx 4\%$)
  - Na_3Bi
  - Cd_3As_2
  - $ZrTe_5$
  - TaAs, NbAs, TaP, …

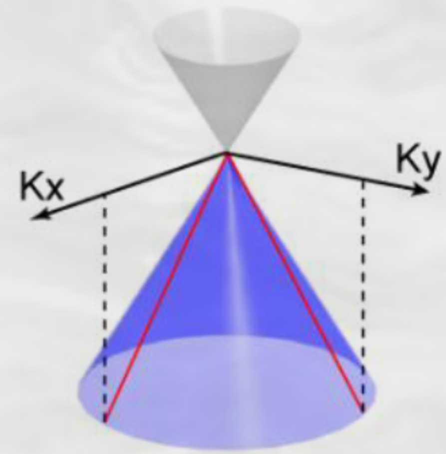
[Z. K. Liu et al., arXiv:1310.0391]

[M. Neupane et al., arXiv:1309.7892]

[S. Borisenko et al., arXiv:1309.7978]

[X. Li et al., arXiv:1412.6543]





- Magnetic fields ≤ **5×10^5 G**

CHIRAL SEPARATION EFFECT

$$\left\langle \vec{j}_5 \right\rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$

Chiral separation effect

- Slowly changing electric/chemical potential

$$\mu(z) = e\Phi(z) \quad \Rightarrow \quad eE_z = -\partial_z\left(e\Phi\right) = -\partial_z \mu$$

- From the anomaly relation,

$$\partial_z j_5^3 = \frac{e^2}{2\pi^2} B_z E_z = -\frac{e^2}{2\pi^2} B_z \partial_z \mu$$

- Suggesting that for massless fermions,

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

[Vilenkin, Phys. Rev. D 22 (1980) 3067]
[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]
[Newman & Son, Phys. Rev. D **73** (2006) 045006]

Landau spectrum at B≠0

- Dirac equation with massless fermions

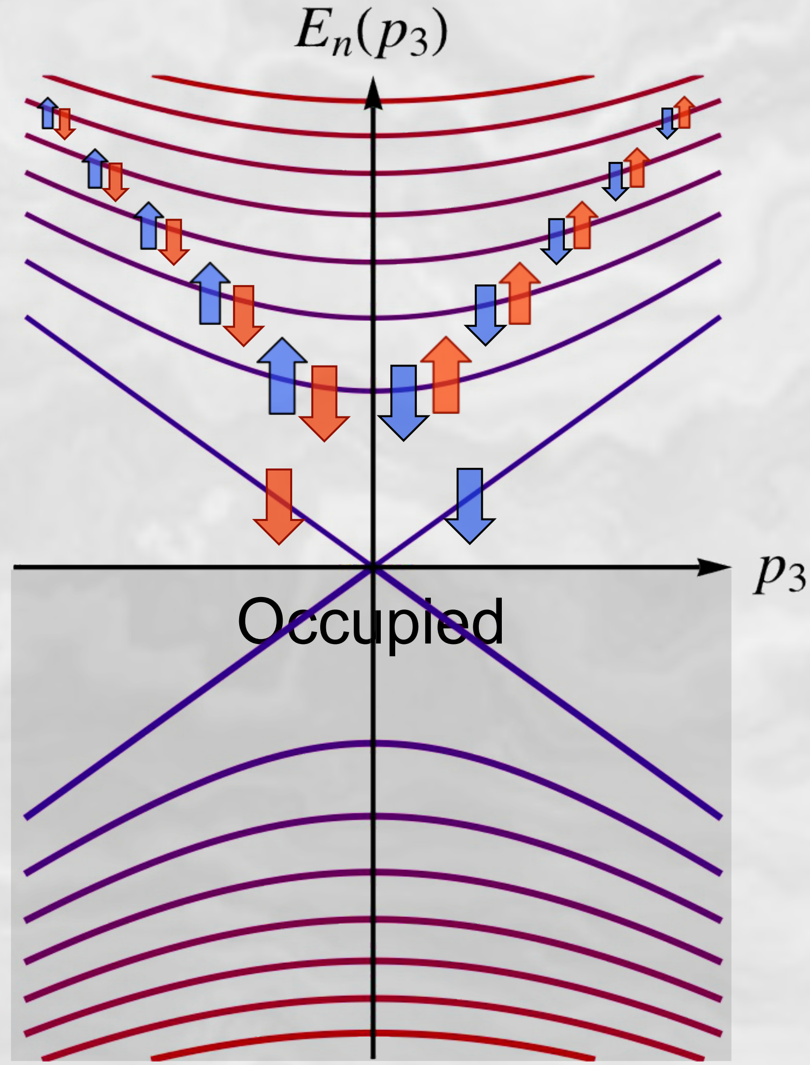
$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla} + ie\vec{A}\right)\right]\Psi = 0$$

- Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

where

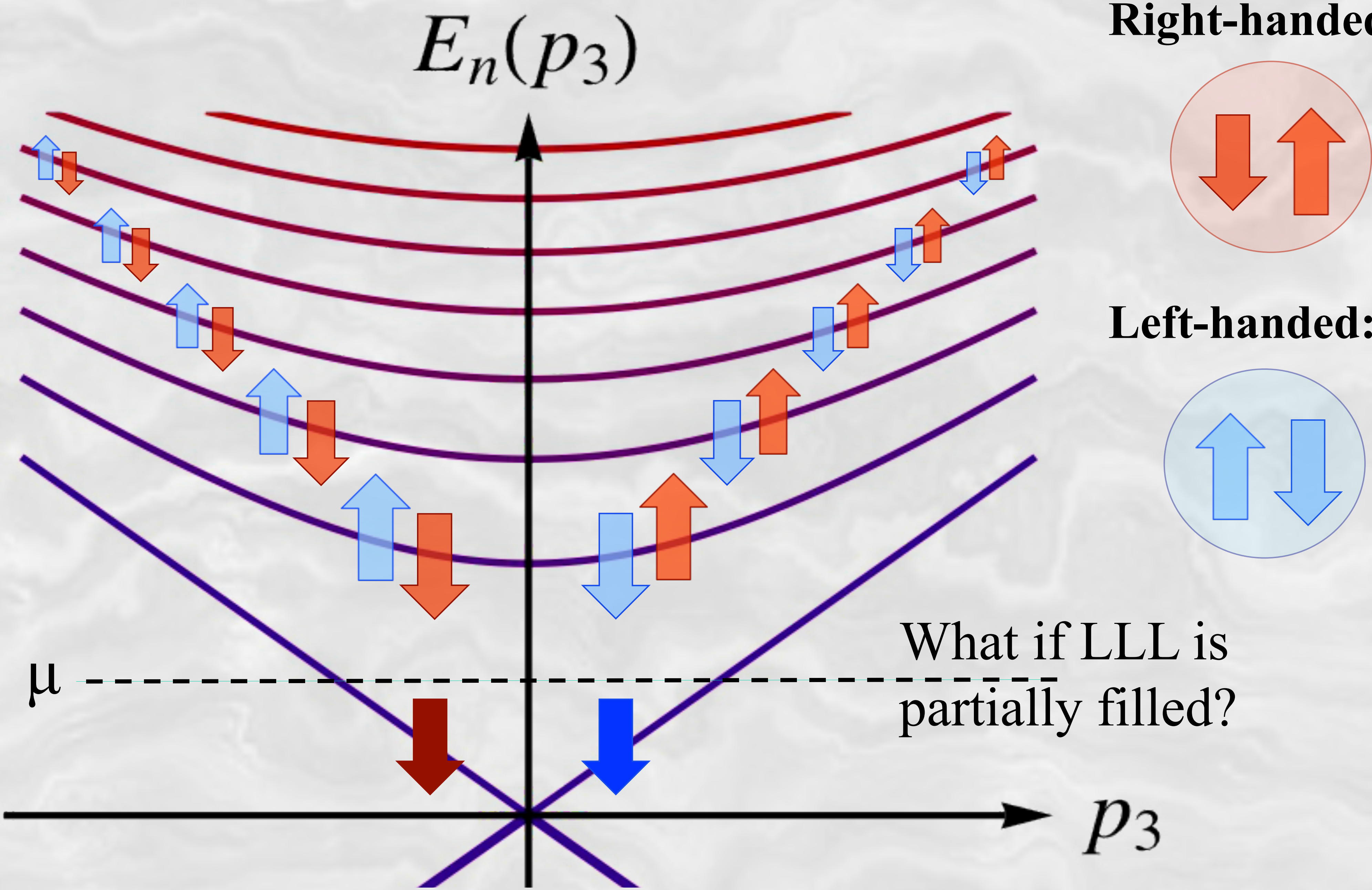
$$n = \overbrace{s}^{\boxed{s = \pm 1/2 \quad \text{(spin)}}} + \underbrace{k + \frac{1}{2}}_{\boxed{k = 0,\ 1,\ 2, \ldots \text{(orbital)}}}$$



Landau spectrum & μ≠0

Right-handed:

Left-handed:



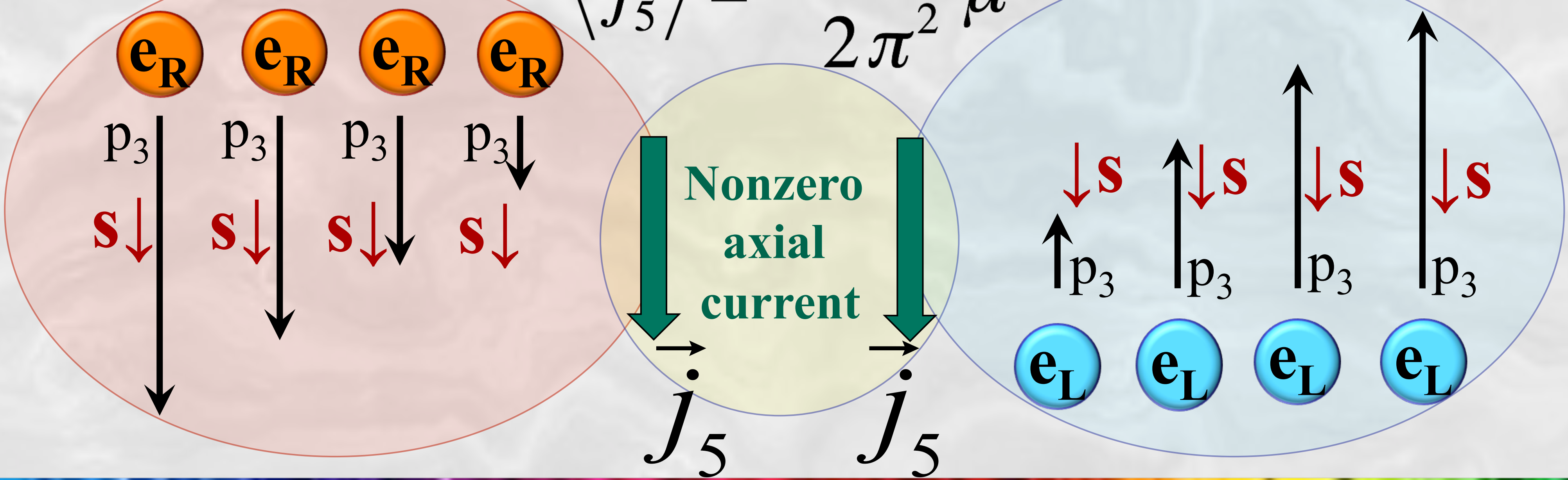
What if LLL is partially filled?

Partially filled LLL

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are R-handed
  - states with $p_3>0$ (and $s=\downarrow$) are L-handed

i.e., a nonzero **axial** current is induced

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2}\mu$$



CHIRAL MAGNETIC EFFECT

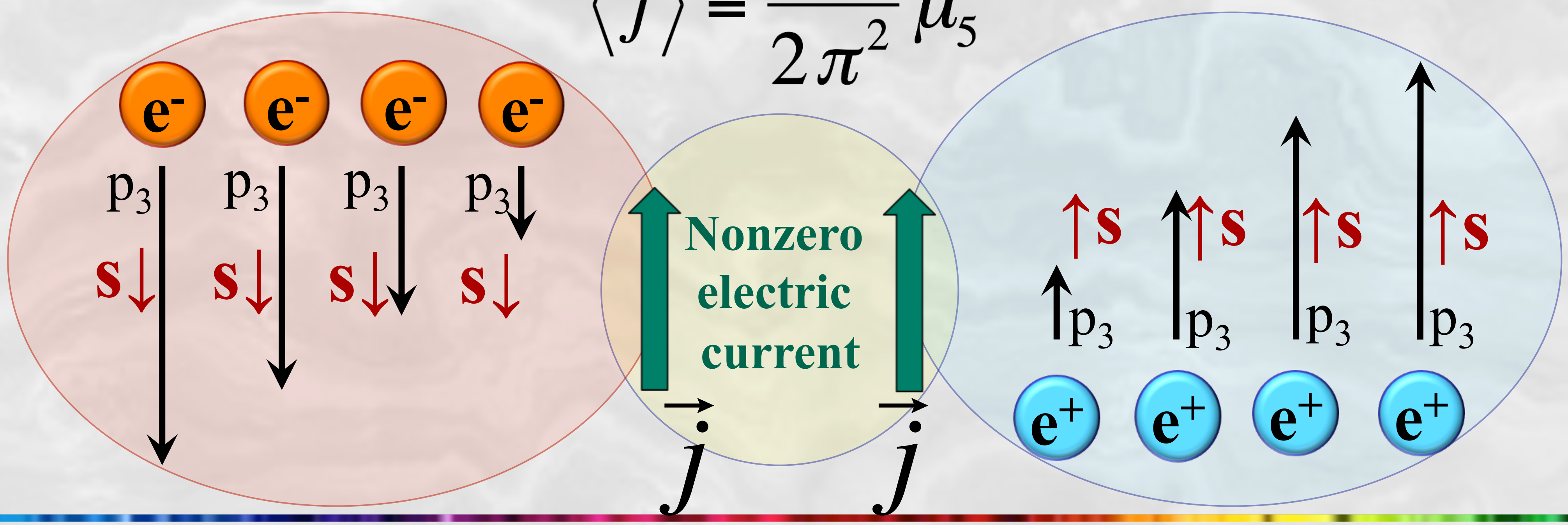
$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

Partially filled LLL @ $\mu_5 \neq 0$

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are R-handed **electrons**
  - states with $p_3>0$ (and $s=\downarrow$) are L-handed **positrons**

i.e., a nonzero **electric** current is induced

$$\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$



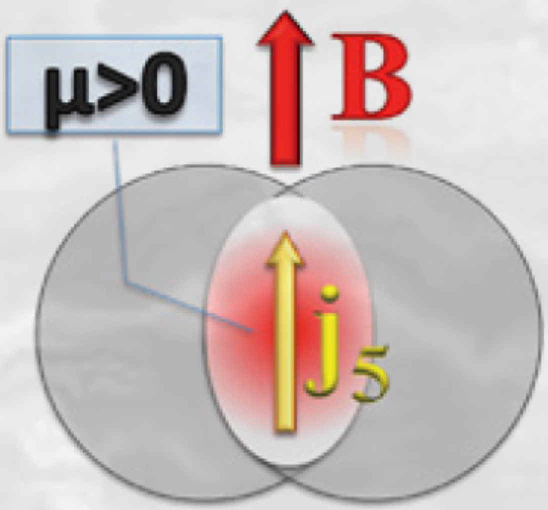
CHIRAL MAGNETIC WAVE

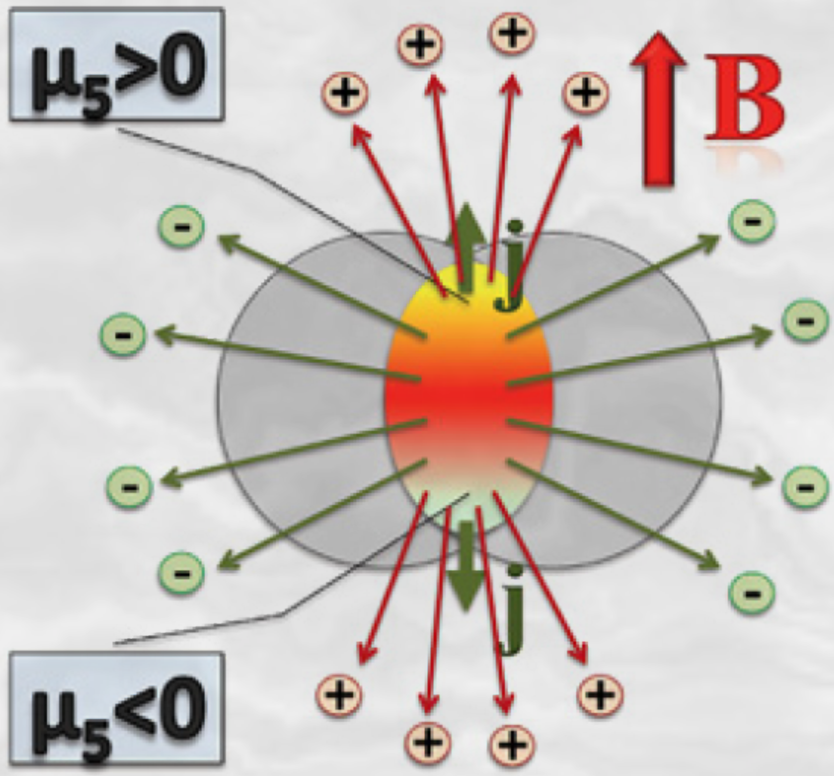
$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu \qquad \left\langle \vec{j} \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$

CMW/Quadrupole CME

- Start from a small baryon density and B≠0

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu \qquad \left\langle \vec{j} \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$





- Produce back-to-back electric currents

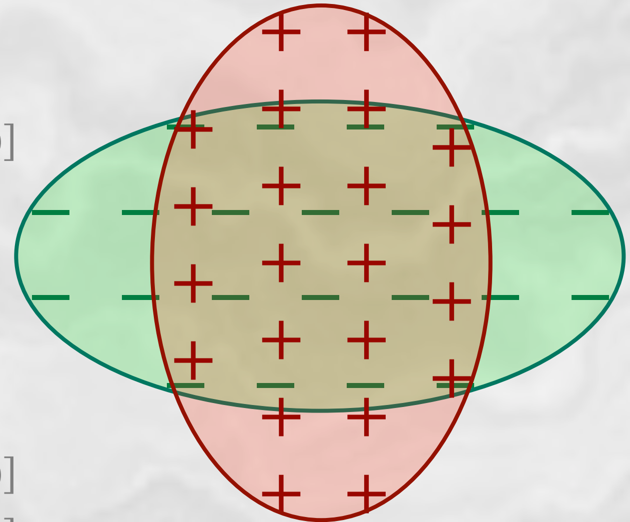
[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]
[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Experimental evidence

- Elliptic flows of π^+ and π^- depend on charge asymmetry:

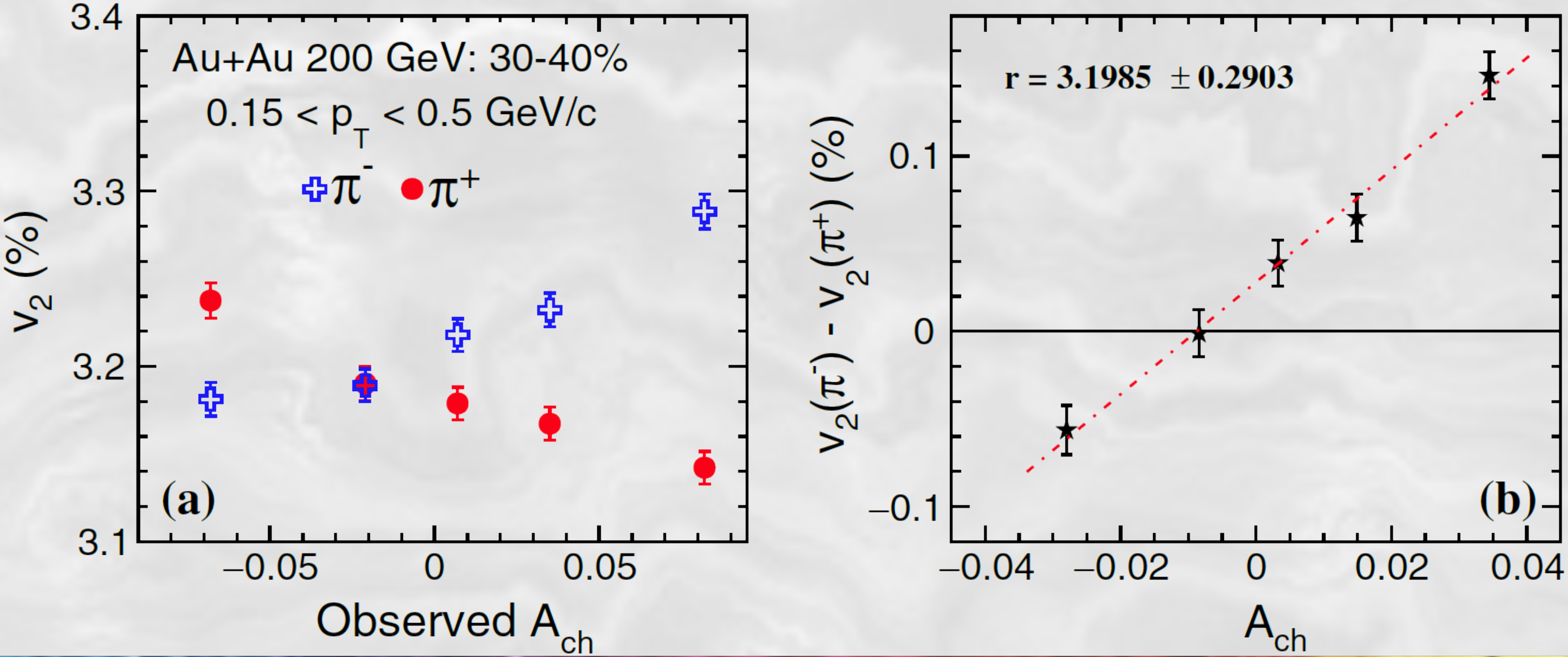
[Burnier, Kharzeev, Liao, Yee, PRL **107**, 052303 (2011)]

$$\frac{dN_\pm}{d\phi} \approx \overline{N}_\pm \left[1 + 2v_2 \cos(2\phi) \mp A_\pm r \cos(2\phi)\right]$$



[H. Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

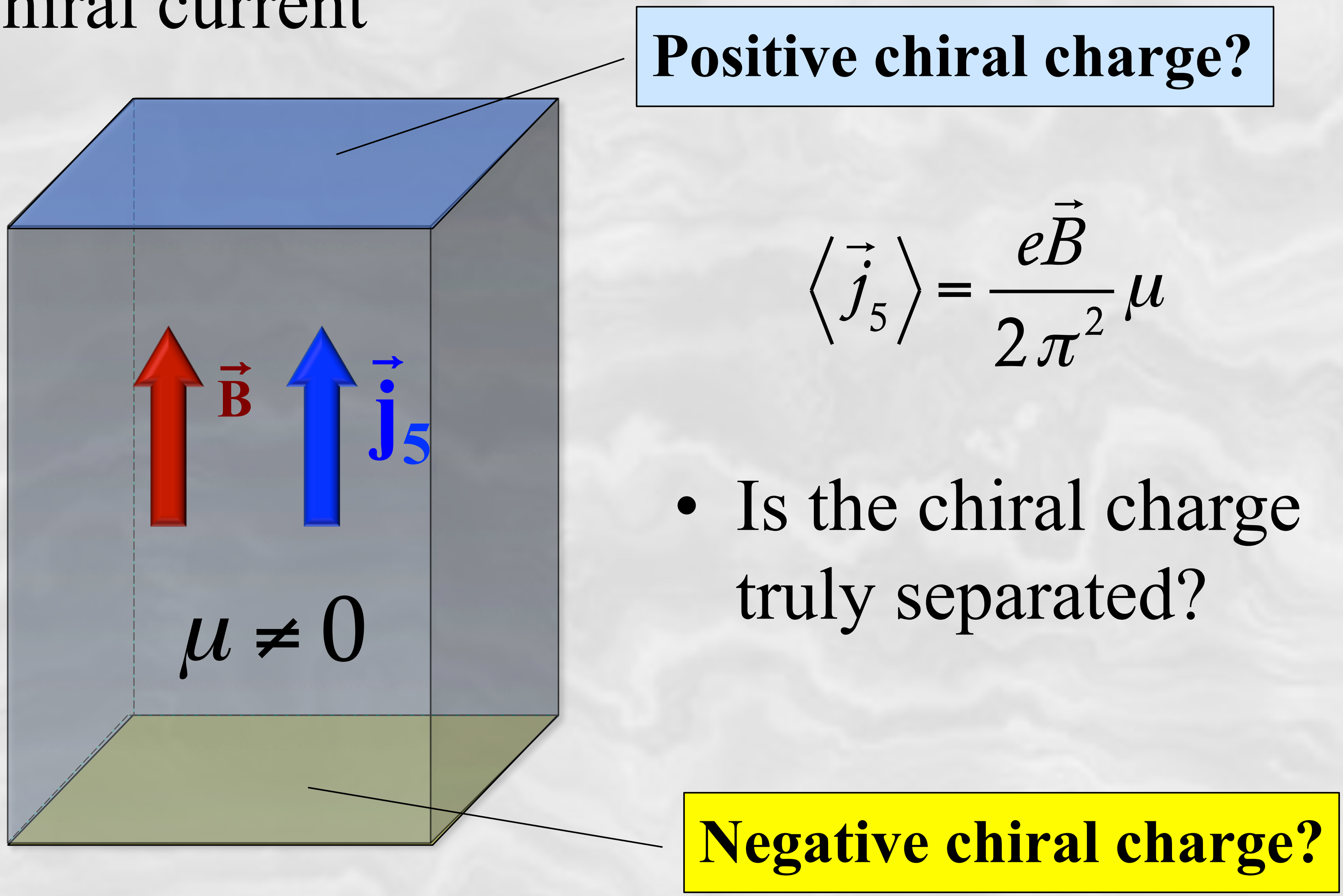


FURTHER DEVELOPMENTS

- How to account for a finite size?

Any effects of finite size?

- Magnetic field + electric chemical potential = chiral current



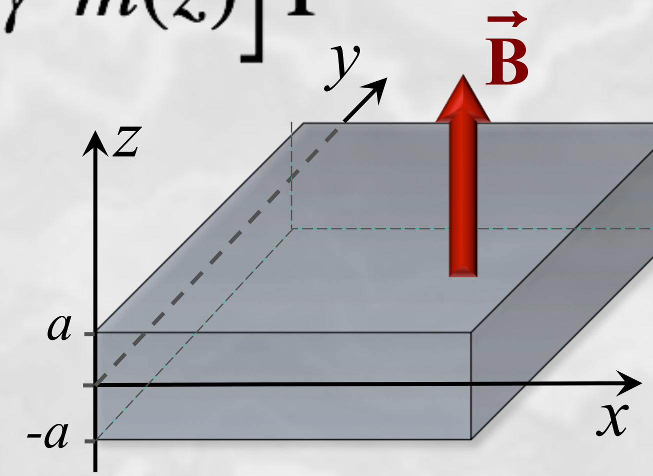
$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

- Is the chiral charge truly separated?

CSE in finite system

- Model of Dirac semimetal with a slab geometry

$$H = \int d^3 r \Psi^+ \left[v_F \vec{\alpha} \cdot \left(-i\vec{\nabla} + e\vec{A} \right) + \gamma^0 m(z) \right] \Psi$$



where $\vec{A} = (0, Bx, 0)$ and

$$m(z) = M\theta(z^2 - a^2) + m\theta(a^2 - z^2),$$

with vacuum band gap: $M \to \infty$ (broken chiral symmetry)

- Boundary conditions:

$$\Psi_{\text{bulk}}(\vec{r}_\perp, a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, a) \quad \text{and} \quad \Psi_{\text{bulk}}(\vec{r}_\perp, -a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, -a)$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

Wave functions

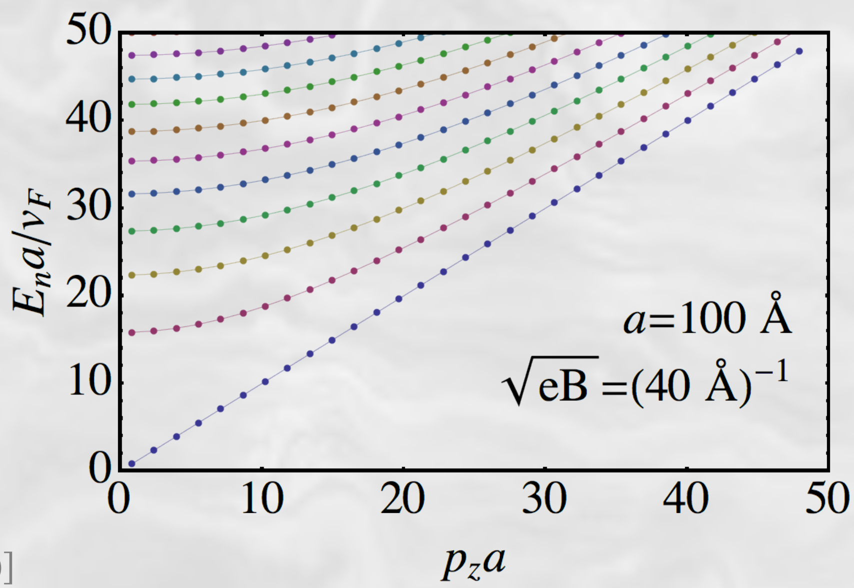
- Wave functions are standing waves, e.g.,

$$\text{LLL}: \quad \Psi_{\text{slab},n=0} = C_0 e^{-\frac{1}{2}(x/l+p_y l)^2} e^{i(p_y y + p_z a)} \begin{pmatrix} 0 \\ \dfrac{v_F p_z \cos\left(p_z(z-a)\right) - (m+iE_0)\sin\left(p_z(z-a)\right)}{im + v_F p_z - E_0} \\ 0 \\ -i\dfrac{v_F p_z \cos\left(p_z(z-a)\right) - (m-iE_0)\sin\left(p_z(z-a)\right)}{im + v_F p_z - E_0} \end{pmatrix}$$

where the wave vector p_z is determined by the spectral equation

$$v_F p_z \cos\left(2a\, p_z\right) + m \sin\left(2a\, p_z\right) = 0$$

$$\Rightarrow \quad p_{z,k}^{(m)} \approx \frac{\pi(2k-1)}{4a} + \frac{2m}{\pi v_F (2k-1)} + \ldots$$



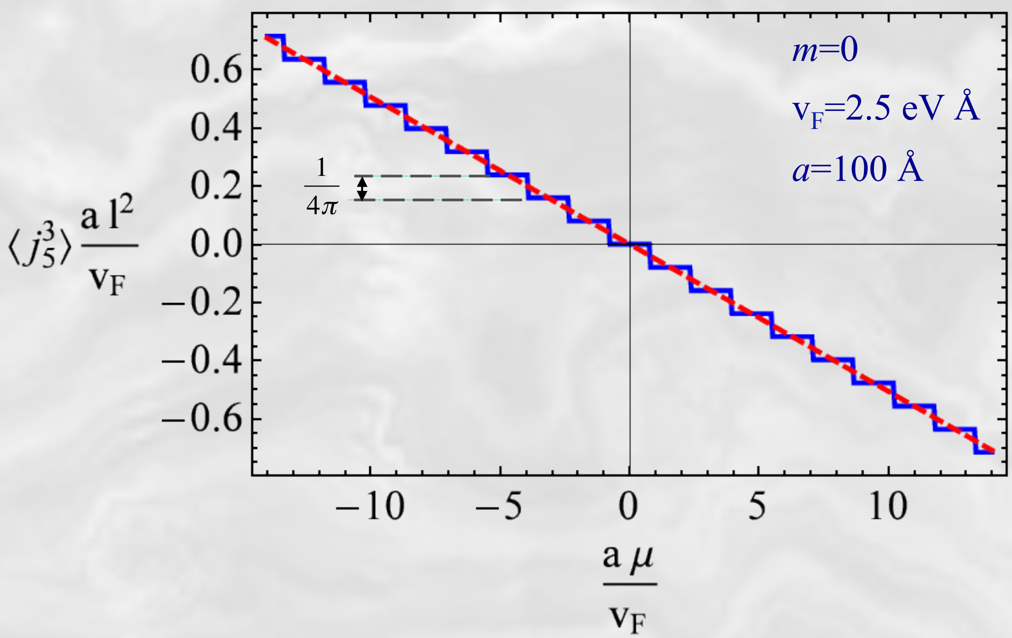
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

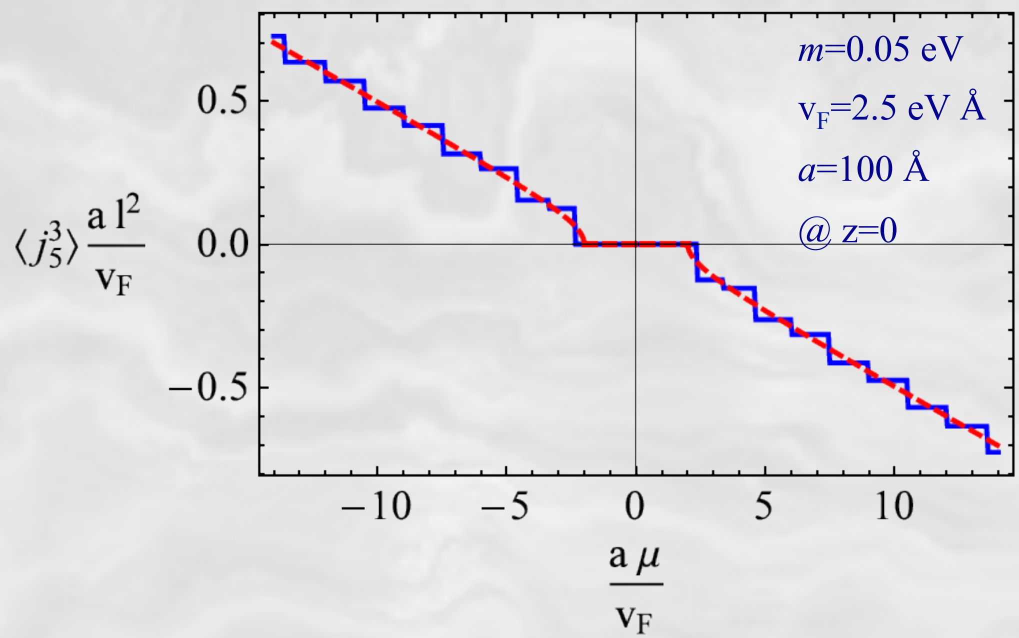
Discretized CSE

- Only LLL contributes

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B} v_F \,\mathrm{sign}(\mu)}{2\pi a} \sum_{p_z} \theta\left(\mu^2 - m^2 - v_z^2 p_z^2\right) \frac{\left(m^2 + v_z^2 p_z^2\right)\left[1 - \cos(2z\,p_z)\cos(2a\,p_z)\right]}{2\left(m^2 + v_z^2 p_z^2\right) + m v_F / a}$$

- For $m \to 0$: $\langle \vec{j}_5 \rangle = -\frac{e\vec{B} v_F \,\mathrm{sign}(\mu)}{4\pi a} k_{\max}$, where $k_{\max} = \left[\frac{2a|\mu|}{\pi v_F} + \frac{1}{2}\right]$

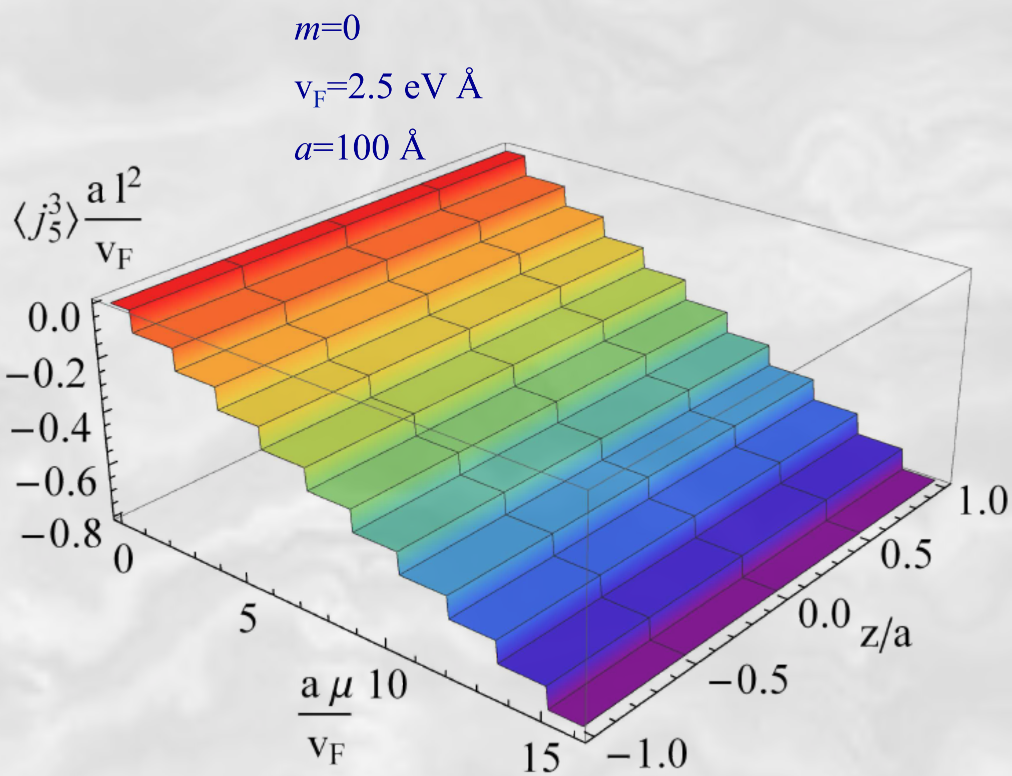


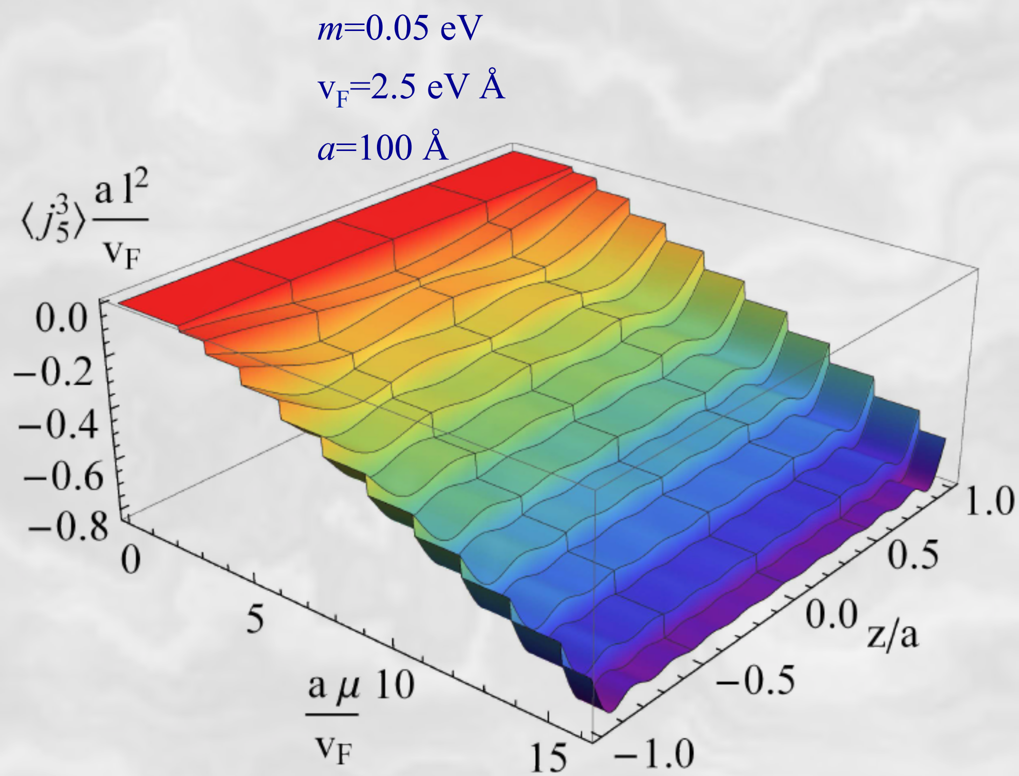


[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **92**, 245440 (2015)]

Quantization of axial current

- Axial current density is non-uniform when *m≠0*



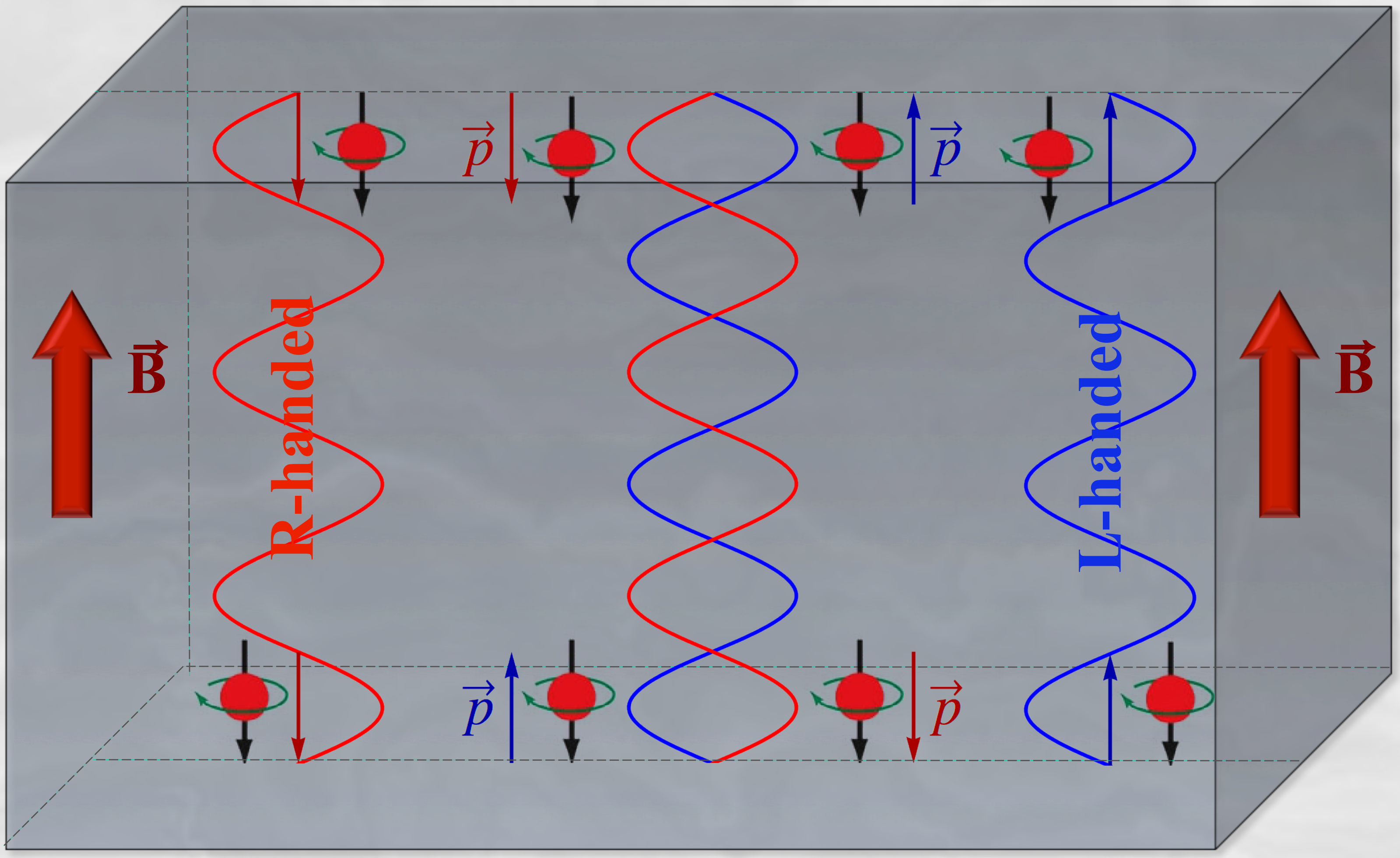


- Note that axial charge density vanishes: $\left\langle j_5^0 \right\rangle = 0$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 92, 245440 (2015)]

Axial current as a standing wave?

- Recall that LLL is spin polarized



- A perfect chirality flip at the boundary

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 92, 245440 (2015)]

Bottom line

- Chiral current in the CSE is discretized
- *$m \neq 0$:* chiral current density is non-uniform
- *$m=0$:* chiral current density is uniform
- Chiral current is **not** necessarily connected with a "flow" of chiral charge
- Chiral current need **not** lead to chiral charge accumulation on the boundary
- CME is qualitatively different from CSE

FURTHER DEVELOPMENTS

- How to account for inhomogeneities and time dependence?

[Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy, Phys. Rev. D93, 105028 (2016)]

Chiral kinetic theory

- Kinetic equation:

$$\frac{\partial f_\lambda}{\partial t}+\frac{1}{1+\vec{\Omega}_\lambda\cdot\vec{B}}\left[\left(\vec{E}+\vec{v}\times\vec{B}+(\vec{E}\cdot\vec{B})\vec{\Omega}_\lambda\right)\cdot\frac{\partial f_\lambda}{\partial\vec{p}}+\left(\vec{v}+\vec{E}\times\vec{\Omega}_\lambda+(\vec{v}\cdot\vec{\Omega}_\lambda)\vec{B}\right)\cdot\frac{\partial f_\lambda}{\partial\vec{x}}\right]=I_{\text{coll}}$$

- Definition of densities & currents:

$$n_\lambda=e\int\frac{d^3p}{(2\pi)^3}\left(1+\frac{e}{c}\vec{B}\cdot\vec{\Omega}_\lambda\right)f_\lambda$$

$$\vec{j}_\lambda=e\int\frac{d^3p}{(2\pi)^3}\left(\vec{v}+e\vec{E}\times\vec{\Omega}+\frac{e}{c}\vec{B}(\vec{v}\cdot\vec{\Omega}_\lambda)\right)f_\lambda+e\vec{\nabla}\times\int\frac{d^3p}{(2\pi)^3}f_\lambda\varepsilon_p\vec{\Omega}_\lambda$$

- Continuity equation:

$$\partial_t n_\lambda+\vec{\nabla}\cdot\vec{j}_\lambda=\frac{e^2\lambda}{4\pi^2c}(\vec{\boldsymbol{E}}\cdot\vec{\boldsymbol{B}})$$

Strategy

- Expand the solution in powers of e.m. fields & derivatives $(\vec{\nabla}\mu_\lambda, \partial_t \mu_\lambda, \ldots)$

$$f_\lambda = f_\lambda^{(0)} + f_\lambda^{(1)} + f_\lambda^{(2)} + \ldots$$

$$f_\lambda^{(0)} = \frac{1}{\exp\left(\frac{c\,p - \mu_\lambda}{T}\right) + 1}$$

- Additional equations for the evolution of μ_λ come from enforcing the continuity equation at each order

[Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy, Phys. Rev. D93, 105028 (2016)]

Results

- The resulting currents:

$$\underbrace{\vec{j} = \frac{e\vec{B}\mu_5}{2\pi^2 c}}_{\text{CME}} + \underbrace{\frac{\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2 T^2}\right)\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)}_{\text{drift \& diffusion}} + \underbrace{\frac{e\tau^2\mu}{3\pi^2}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B}}_{\text{Hall current}} + \vec{j}_{\text{new}}$$

$$\underbrace{\vec{j}_5 = \frac{e\mu\vec{B}}{2\pi^2 c}}_{\text{CSE}} - \underbrace{\frac{\tau T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2 T^2}\right)\frac{\partial\mu_5}{\partial\vec{x}}}_{\text{diffusion}} + \underbrace{\frac{2e\tau\mu_5\mu}{3\pi^2 c}}_{\text{CESE}}\vec{E} + \vec{j}_{5,\text{new}}$$

New terms in axial current

- New contribution to the electric current:

$$\vec{j}_{5,\text{new}} = \underbrace{\frac{e\tau^2\mu}{3\pi^2}\left(\vec{B}\times\frac{\partial\mu_5}{\partial\vec{x}}\right)}^{\text{Chiral Hall diffusion}} + \underbrace{\frac{e\tau^2\mu_5}{3\pi^2}\left(e\vec{E}-\frac{\partial\mu}{\partial\vec{x}}\right)\times\vec{B}}^{\text{Chiral Hall effect}} - \frac{e\tau\mu}{6\pi^2 c}\frac{\partial\vec{B}}{\partial t}$$

$$-\frac{2e\tau^2\mu\mu_5}{3\pi^2 c}\frac{\partial\vec{E}}{\partial t} - \frac{2\tau\mu\mu_5}{3\pi^2 c}\frac{\partial\mu}{\partial\vec{x}} - \underbrace{\frac{e\tau}{6\pi^2}\left(\vec{E}\times\frac{\partial\mu}{\partial\vec{x}}\right)}_{\text{anomalous chiral Hall effect}}$$

New terms in electric current

- New contribution to the electric current:

$$\vec{j}_{\text{new}} = \overbrace{\frac{e\tau^2\mu_5}{3\pi^2}\left(\vec{B}\times\frac{\partial\mu_5}{\partial\vec{x}}\right)}^{\text{Hall diffusion}} - \frac{e\tau^2T^2}{9c}\left(1+\frac{3\left(\mu^2+\mu_5^2\right)}{\pi^2T^2}\right)\frac{\partial\vec{E}}{\partial t}$$

$$\underbrace{-\frac{2\tau\mu\mu_5}{3\pi^2c}\frac{\partial\mu_5}{\partial\vec{x}}}_{\text{Chiral diffusion}} - \frac{e\tau\mu_5}{6\pi^2c}\frac{\partial\vec{B}}{\partial t} \underbrace{-\frac{e\tau}{6\pi^2}\vec{E}\times\frac{\partial\mu_5}{\partial\vec{x}}}_{\text{Anomalous Hall effect}}$$

Summary

- Chiral plasmas have widespread applications
  - Heavy-ion collisions
  - Cosmology
  - Dirac/Weyl semimetals
  - Neutron stars
- Anomaly plays a profound role in such plasmas
- Many interesting chiral/anomalous effects are triggered by a magnetic field
- Experimental search for signatures is underway