Holographic RG flows for Kondo-like impurities

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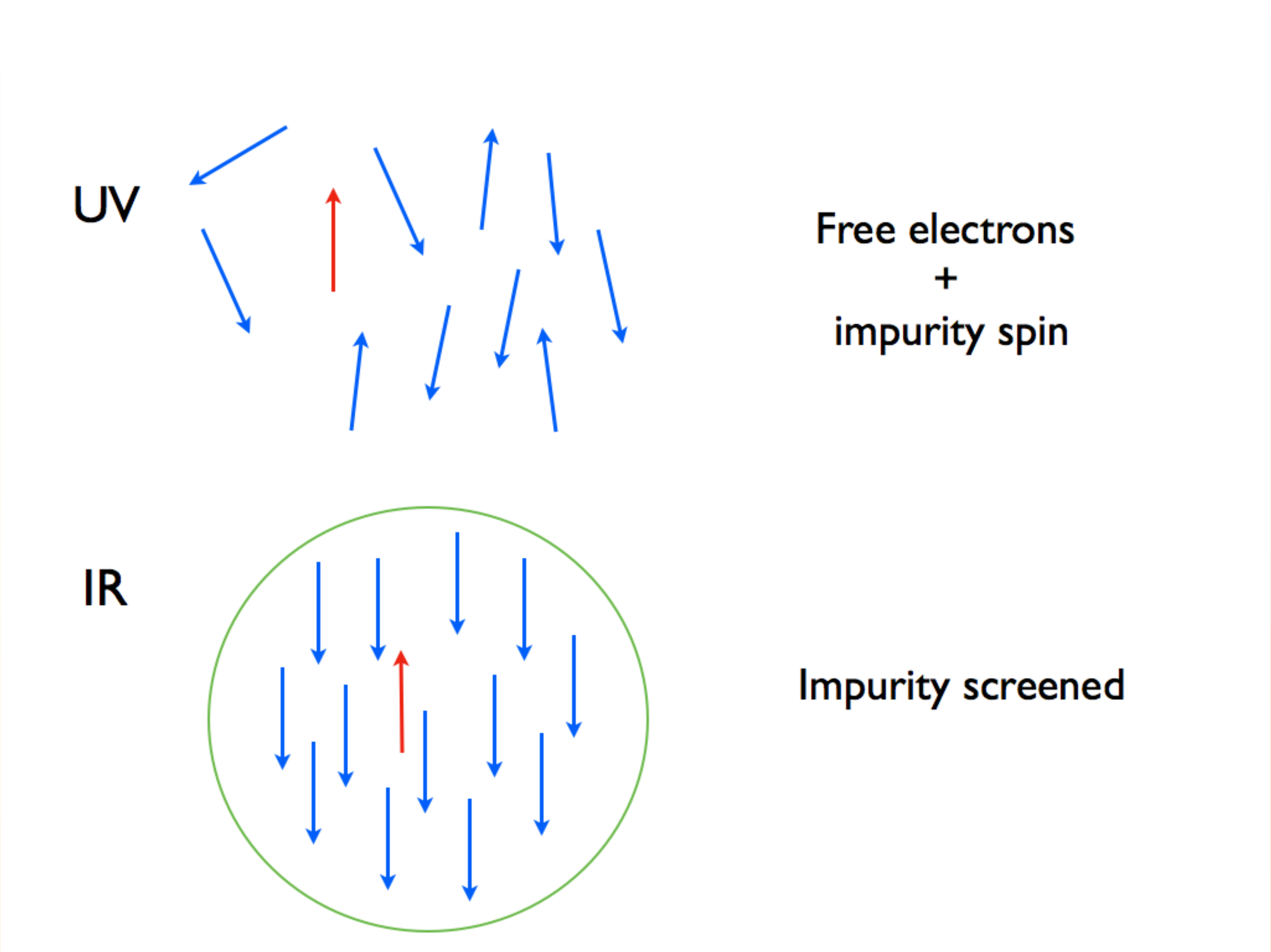
Based on 2001.04991 with Charles Melby-Thompson and Christian Northe

Kondo effect



Kondo model

Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

Impurity screened at low temperatures:
Logarithmic rise of **resistivity** at low temperatures

Dynamical scale generation

Due to symmetries: Model effectively $(1+1)$-dimensional

Hamiltonian:

$$H = \frac{v_F}{2\pi}\psi^\dagger i\partial_x\psi + \lambda_K v_F \delta(x)\vec{S}\cdot\vec{J}, \quad \vec{J} = \psi^\dagger \frac{1}{2}\vec{T}\psi$$

Decisive in development of renormalization group
IR fixed point, CFT approach Affleck, Ludwig '90's

Kondo models from gauge/gravity duality

Gauge/gravity requires large N: Spin group $SU(N)$

In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

Constraint: $\chi^\dagger \chi = q, \quad Q = q/N$

Interaction: $J^a S^a = (\psi^\dagger T^a \psi)(\chi^\dagger T^a \chi) = \mathcal{O}\mathcal{O}^\dagger$, where $\mathcal{O} = \psi^\dagger \chi$

Screened phase has condensate $\langle \mathcal{O} \rangle$

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192
Senthil, Sachdev, Vojta cond-mat/0209144

Kondo models from gauge/gravity duality

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

Results:

- RG flow from perturbation by 'double-trace' operator
- Dynamical scale generation
- AdS_2 holographic superconductor
- Power-law scaling of conductivity in IR with real exponent
- Screening, phase shift

Kondo models from gauge/gravity duality

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Top-down brane realization

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

- 3-7 strings: Chiral fermions ψ in 1+1 dimensions
- 3-5 strings: Slave fermions χ in 0+1 dimensions
- 5-7 strings: Scalar (tachyon)

Near-horizon limit and field-operator map

D3: $AdS_5 \times S^5$

D7: $AdS_3 \times S^5 \rightarrow$ Chern-Simons A_μ dual to $J^\mu = \psi^\dagger \sigma^\mu \psi$

D5: $AdS_2 \times S^4 \rightarrow \begin{cases} \text{YM } a_t \text{ dual to } \chi^\dagger\chi = q \\ \text{Scalar dual to } \psi^\dagger\chi \end{cases}$

Operator		Gravity field
Electron current J	$\Leftrightarrow$	Chern-Simons gauge field A in AdS_3
Charge $q = \chi^\dagger\chi$	$\Leftrightarrow$	2d gauge field a in AdS_2
Operator $\mathcal{O} = \psi^\dagger\chi$	$\Leftrightarrow$	2d complex scalar Φ

Bottom-up model

Action:

$$S = S_{CS} + S_{AdS_2}$$

$$S_{CS} = -\frac{N}{4\pi}\int \mathrm{Tr}\left(A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right),$$

$$S_{AdS_2} = -N\int d^3x\,\delta(x)\sqrt{-g}\left[\frac{1}{4}\mathrm{Tr} f^{mn} f_{mn} + g^{mn}\left(D_m\Phi\right)^{\dagger} D_n\Phi + V(\Phi^{\dagger}\Phi)\right],$$

$$D_\mu\Phi = \partial_m\Phi + iA_\mu\Phi - ia_\mu\Phi$$

Metric: BTZ black hole

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{1}{z^2}\left(\frac{dz^2}{h(z)} - h(z)\,dt^2 + dx^2\right), \qquad h(z) = 1 - z^2/z_H^2$$

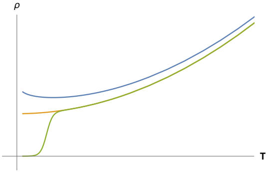
$$T = 1/(2\pi z_H)$$

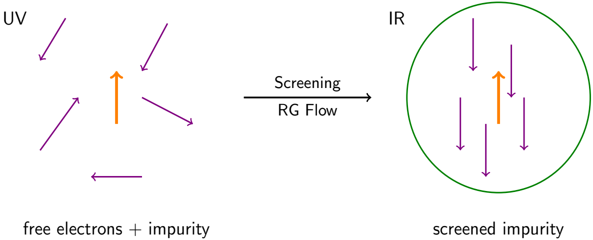
Contents

Defect and interface flows

Spin impurities and the Kondo effect

- Heavy magnetic impurity interacts with conduction electrons
- Ultraviolet: free electrons with mild antiferromagnetic coupling to spin
- Infrared: impurity is screened through binding with small number of conduction electrons





CFT description of the Kondo effect [Affleck+Ludwig '91]

Kondo Hamiltonian: couple magnetic impurity with spin j to free electrons,

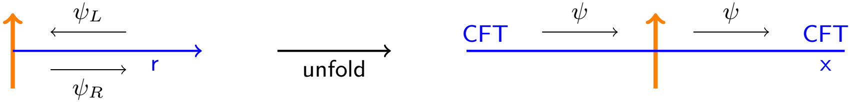
$$H = \sum_{\alpha,\vec{k}} \epsilon(k)\psi^{\dagger}_{\vec{k}\alpha}\psi^{\alpha}_{\vec{k}} + \frac{g}{2}\sum_{\vec{k},\vec{k}'} \vec{S}\cdot(\psi^{\dagger}_{\vec{k}}\vec{\sigma}\psi_{\vec{k}'})\,. \quad (1)$$

$\vec{S}$ acts on impurity Hilbert space of dimension $2j+1$.

S-wave modes near the Fermi surface scatter $\Longrightarrow$ 2d CFT description on half space. With boundary condition $\psi_L|_{x=0} = \psi_R|_{x=0}$,

$$S = \frac{v_F}{2\pi}\int_{x>0} dt\,dx\,(i\psi^{\dagger}_L\dot{\psi}_L - i\psi^{\dagger}_R\dot{\psi}_R) + \frac{g}{2}v_F\int dt\,\vec{S}\cdot(\psi^{\dagger}_L\vec{\sigma}\psi_L)\,. \quad (2)$$

Obtain a defect in 2d CFT by unfolding trick:

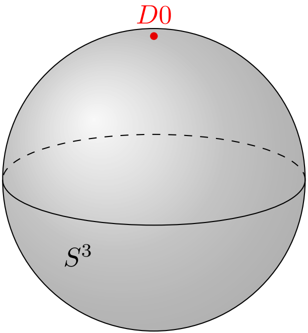


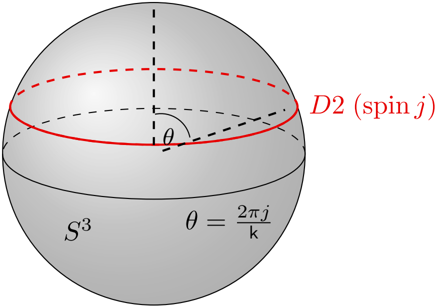
CFT description of the Kondo effect

Special case of boundary RG flow in $\widehat{\mathfrak{su}(2)}_k$ WZW model, a conformal non-linear sigma model on S^3 with k units of NS-NS three-form flux $H^{(3)}$.

Conformal boundary conditions preserving a $\widehat{\mathfrak{su}(2)}_k$ are classified by spin j, $0 \leq 2j \leq k$.

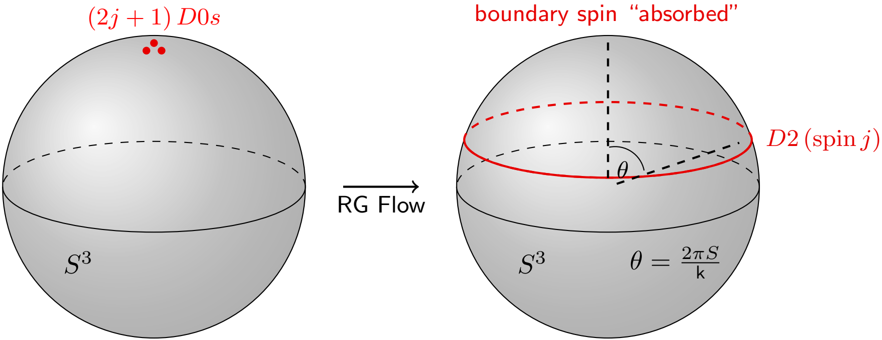
Boundary conditions: Neumann along the S^2 at $\theta = \frac{2\pi j}{k}$.





CFT description of the Kondo effect

- Exist RG flows between these two types of boundary condition.
- Adding the spin coupling results in a non-abelian polarization, causing the branes to puff up along the RG flow.
- In Kondo context: "absorption of boundary spin" principle [Affleck+Ludwig]



Kondo-like interface flows

- RG flow can be described as the fusion of a defect RG flow with a fixed boundary state [Gaberdiel+Bachas hep-th/0411067]
- Defects in $\widehat{\mathfrak{su}(2)}_k$ WZW model are Wilson-esque line operators:

$$\mathcal{D} = \mathrm{Tr}_{2j+1} \mathcal{P} \exp\left(ig \int dt\, \vec{S} \cdot \vec{J}(t) \right) ;$$

  $\vec{J}(z)$ is a chiral $\mathrm{SU}(2)$ current of the WZW model.
- g is marginally relevant and generates a defect flow.
- Chirality of J implies it can be fused with the D0 boundary without generating singularities. This generates D0→D2 RG flow.
- The flows generated by these defects are described by a similar non-abelian polarization process.

Goal: Use top-down holography to study defect flows exhibiting similar characteristics. Consider D1/D5 CFT for simplicity, and study flows characterized by non-abelian polarization.

Interfaces in the D1/D5 CFT

Brief review of D1/D5 CFT

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$ (with $M_4 =$ K3 or T^4):

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								

At low energies, described by a 2d gauge theory with $\mathcal{N} = (4,4)$ susy.

Gauge theory description. $\mathrm{U}(N_1) \times \mathrm{U}(N_5)$ gauge theory with bifundamental hypermultiplet. To get well-behaved CFT, turn on Fayet-Iliopoulos parameters to push us onto the Higgs branch. Gives:

Instanton description. D5 brane has a coupling $\int C^{(2)} \wedge \mathrm{Tr}(F \wedge F)$. This means that D1 branes can be dissolved as $\mathrm{U}(N_5)$ gauge instantons on M_4.

Low energy dynamics described by an $\mathcal{N} = (4,4)$ non-linear sigma model on (a deformation of) the moduli space of instantons on M_4. [Strominger+Vafa '96]

Aside: some basic facts about $\mathcal{N} = (4,4)$ SCFT

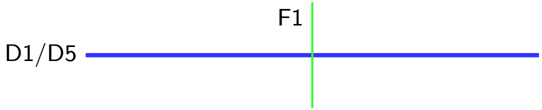
- $\mathcal{N} = 2$ supersymmetry in $d = 4$ has 8 real supercharges: Q^I_α and $\overline{Q}_{I\dot{\alpha}} = (Q^I_\alpha)^\dagger$.
- α and $\dot{\alpha}$ are indices for the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of $\mathrm{Spin}(1,3)$.
- $I = 1, 2$ is an R-symmetry index and transforms as a doublet under an $\mathrm{SU}(2)$ R-symmetry group.
- 4d $\mathcal{N} = 2$ supersymmetry can be considered the dimensional reduction of 6d $\mathcal{N} = (1,0)$ supersymmetry.
- 2d $\mathcal{N} = (4,4)$ supersymmetry is the dimensional reduction of minimal 6d $\mathcal{N} = (1,0)$ supersymmetry, *i.e.* the same algebra but with all fields independent of x^2 through x^5.
- The left and right supercharges anticommute to translations along x^- and x^+ respectively ($x^\pm = \frac{1}{2}(x^0 \pm x^1)$). They live in doublets of a global $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry. This comes from the $\mathrm{Spin}(4)$ symmetry in the $x^{2\cdots5}$ directions.
- At a conformal fixed point, the two $\mathrm{SU}(2)$ global R-symmetries are enhanced to algebras of Kac-Moody type acting on the left- and right-moving fermions respectively. The 6d $\mathrm{SU}(2)$ symmetry is absent.

Interfaces in D1/D5 CFT

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$:

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								
F1 (p)	•		•							

- p fundamental strings intersect the D1/D5 system
- preserves $\mathcal{N} = 4,\ d = 1$ supersymmetry
- when string ends on D1/D5 system, realized in gauge theory description as a Wilson line. Sources jump in background electric field, changing the CFT on one side while preserving the central charge. This case is an interface, not a defect.



Wilson line interfaces in D1/D5 CFT

- Particularly interested in a Wilson line corresponding to a long string connecting a distant D3 brane to the D1/D5 system.
- Two types of modes: D3-D5 strings and D3-D1 strings. D5-D3 strings give one complex Grassmann mode. D1-D3 strings give a hypermultiplet.
- In CFT, fields corresponding to D1-D5 strings have expectation values, allowing D1-D3 strings to turn into D5-D3 strings and vice versa. As a result they mix and most of them increase in energy.
- After mixing, lowest-lying fermions have Lagrangian [Tong+Wong '14]

$$L_\eta = \eta^\dagger (i\partial_0 + \Omega_A \partial_t Z^A)\eta \tag{3}$$

where η is in the fundamental of $\mathrm{U}(N)$, Z^A is the coordinate on $\mathcal{M}$, and Ω_A is a $\mathrm{U}(N)$ connection on $M_4 \times \mathcal{M}$.
- This can be rewritten as the insertion of

$$W = \mathrm{Tr}_F \mathcal{P} \exp\left(i \int dt\, \partial_t Z^A \Omega_A(y_0, Z) \right) \tag{4}$$

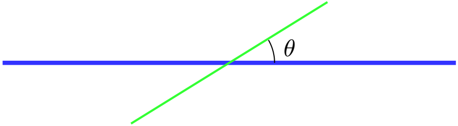
with y_0 the location of the Wilson line in M_4.

Interfaces in D1/D5 CFT

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$:

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								
(p,q)	•	-	•							

- More generally: intersect D1/D5 with (p,q) strings – bound states of p fundamental strings and q D1-branes.
- Preserves same supersymmetries as D1/D5+F1 system provided (p,q) string runs at angle $\theta = \tan^{-1}(q/g_s p)$ in x^{12} plane. (We set $C^{(0)} = 0$.)
- If it ends on D1/D5, N_1 jumps across the interface. This changes the central charge. In gauge theory it corresponds to a jump in the rank of the gauge group $\mathrm{U}(N_1)$. In the sigma model it corresponds to an embedding of one instanton moduli space inside another.



Interface RG flows in holography

D1/D5 holography

Gravitational dual of D1/D5 system.

- D1/D5 theory flows to a CFT at low energies
- Dual gravitational description: type IIB string theory on near-horizon limit of D1/D5 geometry
- Geometry: $\mathrm{AdS}_3 \times S^3 \times M_4$ supported by $F^{(3)}$ flux on AdS_3 and S^3
- Symmetries: $\mathcal{N} = (4,4)$ small superconformal algebra.
  Bosonic part: $\mathfrak{so}(2,2) \times \mathfrak{so}(4)$.

Add (p,q) defect. Defect is conformal $\implies$ preserve $\mathfrak{so}(2,1) \times \mathfrak{su}(2)$ $+$ (8 superconformal) subalgebra of vacuum symmetries

Geometry: $\mathrm{AdS}_3 \times S^3 \times M_4$. Brane embedding:

	t	x	z	θ	ϕ	χ	6	7	8	9
(p,q)	$\bullet$	$-$	$\bullet$							

Probe brane solutions

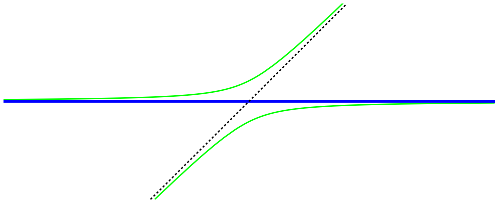
- (p, q) string interface dual to (p, q) strings in near-horizon geometry
- Action: when D1 fields are abelian, behavior determined by DBI-CS action

$$I = qT_{\mathrm{D1}} \int d^2\xi e^{-\Phi} \sqrt{-\det(\hat{g} + F)} + qT_{\mathrm{D1}} \int (C^{(2)} + F)$$

F1 charge p encoded in electric field F_{tx}.

- Solutions are the near-horizon limit of:

$$x_1(x_2) = \frac{qT_{\mathrm{D1}}}{pT_{\mathrm{F1}}}(x_2 - \frac{g_s \alpha' N_5}{x_2})\,.$$



Flows from brane polarization

- Deform branes by non-abelian polarization. Coordinates on S^3 become non-commuting matrices $\implies$ extended on fuzzy S^2 inside S^3.
- Brane polarization in RR background: (p,q) strings puff up into D3 branes [Myers hep-th/9910053]. Can be described using DBI-CS action either for nonabelian D1-branes or an abelian D3-brane.
- BPS solutions for general (p,q) obtained from κ symmetry projector (e.g. along lines of [Gomis &al. hep-th/9907022]).
- Polarization in S-dual NS background: NS background is S^3 with N_5 units of $H^{(3)}$ flux — the $\widehat{\mathfrak{su}(2)}_{N_5}$ WZW model.
  So dual interface flow is controlled by generalized Kondo RG flow.

Geometry: $\mathrm{AdS}_3 \times S^3 \times M_4$

	t	x	z	θ	ϕ	χ	6	7	8	9
D3 (p,q)	$\bullet$	$(-)$	$\bullet$		$\bullet$	$\bullet$				

Flows from brane polarization

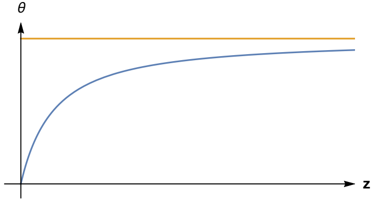
- In D3-brane description,

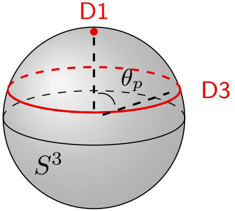
$$I = T_{\text{D3}} \int d^4\xi e^{-\Phi}\sqrt{-\det(\hat{g}+F)} + T_{\text{D3}} \int (C^{(2)} \wedge F + \frac{1}{2} F \wedge F)$$

- Simplest case: when D3 branes carry no D1 charge, solution is given by

$$z = z_0 \frac{\sin\theta}{\theta_p - \theta} \qquad \theta_p = \pi \frac{p}{N_5}$$

where z is the radial coordinate in Poincaré patch.





(For $\mathcal{N}=4$, $d=4$ SYM: c.f. D5 brane realization of Wilson lines in antisymmetric representation [Gomis+Passerini '06].)

Backreacted supergravity dual of defect fixed points

Conformal junction solutions in D1/D5 theory [Chiodaroli+Gutperle+Krym '09]

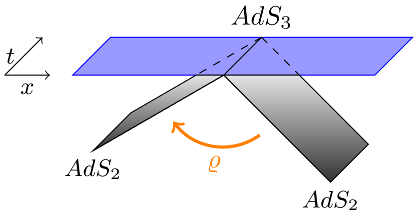
- Require symmetries at fixed point: $\mathfrak{so}(2,1) \times \mathfrak{so}(3)$, 8 supersymmetries
- Geometry is fibration of $\mathrm{AdS}_2 \times S^2 \times M_4$ over an open Riemann surface:

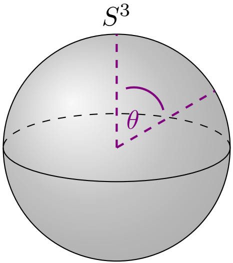
$$ds_{10}^2 = f_1^2 ds_{\mathrm{AdS}_2}^2 + f_2^2 ds_{S^2}^2 + f_3^2 ds_{M_4}^2 + \rho^2 dw\, d\bar{w} \quad (5)$$

  Functions depend only on $(w, \bar{w})$. Preserve 8 supersymmetries.

- Example: vacuum AdS

$$ds_{10}^2 = \cosh^2 \varrho\, ds_{AdS_2}^2 + \sin^2 \theta\, ds_{S^2}^2 + 2ds_{M_4}^2 + d\varrho^2 + d\theta^2$$

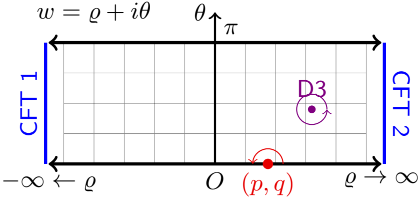




Adding (p, q) branes

Solutions depend on harmonic functions $a,\ b,\ u,\ v$ and their duals $\tilde{a},\ \tilde{b},\ \tilde{u},\ \tilde{v}$.

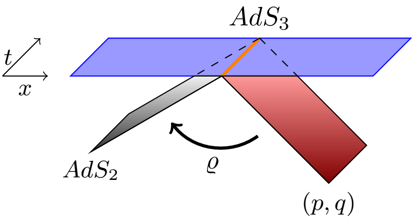
Brane solutions have local singularities. Identify them by matching charges and singularities to flat space brane solutions.

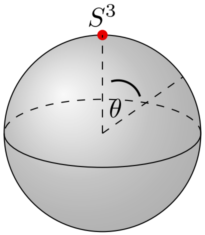


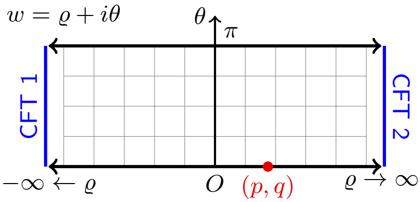
$$Q_{D1} = 4\pi\left(\int_{\mathcal{C}} \frac{4u}{a}\frac{au - b^2}{au + \tilde{b}^2} i(\partial_w c^{(1)} - \chi\partial_w b^{(1)})dw + \int_{\mathcal{C}} 4C_{T^4}dw\right) + c.c.$$

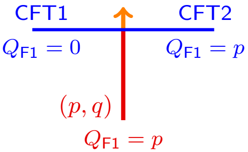
$$Q_{D3} = \int_{\mathcal{C}} dC^{(4)} = C_{T^4}|_{cycle} \qquad C_{T^4} = \frac{1}{2}\left(\frac{b\tilde{b}}{a} - \tilde{u}\right)$$

Adding (p, q) branes

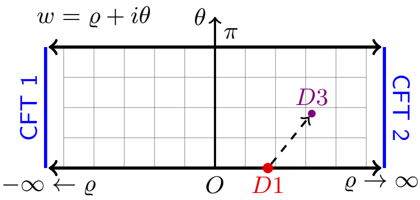


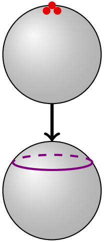






(p,q) defect flows





- Boundary RG flow $\Rightarrow$ CFTs remain unchanged
- Invariance of charges
  $\Rightarrow$ location of $D3$ in terms of (p,q) string data

Interface entropy (g factor)

- Boundary entropy $s = \log g$ is a useful property of boundaries/interfaces
- In BCFT language, describes overlap of boundary state with vacuum:

$$g = \langle 0 | \mathcal{B} \rangle\rangle$$

- s also equals the interface contribution to entanglement entropy when the interface lies in the center of the entangling interval I
- When gravitational dual is semi-classical, entanglement entropy is computed by the Ryu-Takayanagi formula
- Fix a point $p \in \mathrm{AdS}_2$. Defines a symmetric entangling interval.
- Corresponding RT surface is the 8d manifold whose AdS_2 coordinate equals p. [Chiodaroli+Gutperle+Hung 1005.4433]

Can compute in closed form for our interfaces. The expression is complicated but indeed decreases across flow (as required by g theorem).

Crucially, it contains contributions from the bulk CFT not computable in the probe brane limit.

Summary and future work

Summary.

- Studied holographic duals of interface RG flows in the D1/D5 theory
- Probe brane limit: BPS RG flows for general (p, q) string defects
- Supergravity description: used work of [CGK] to find exact solutions to classical IIB supergravity representing backreaction
- g-factor, including CFT contributions, in semi-classical limit of gravity
- Not discussed in the talk: interfaces carrying D5/NS5 charges

In progress/future work.

- Finite temperature effects
- More detailed study from CFT point of view
- Generalizations to other top-down theories, especially AdS$_3 \times S^3 \times S^3 \times S^1$

Thank you for your attention!