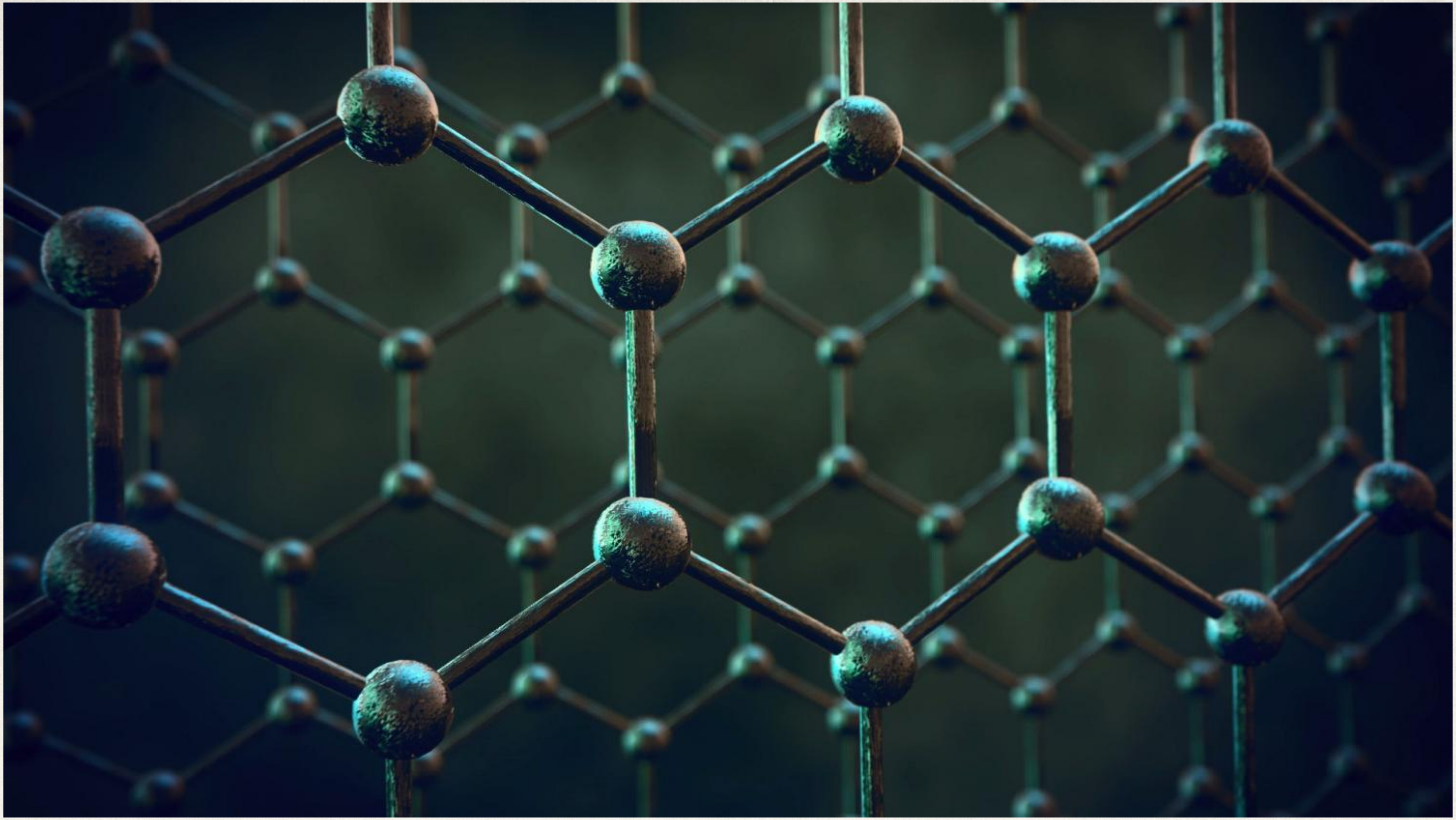


Graphene and Boundary Conformal Field Theory

Christopher Herzog (King's College London)

June 5, 2020

1707.06224, 1709.07431, 1807.01700, 1912.09225

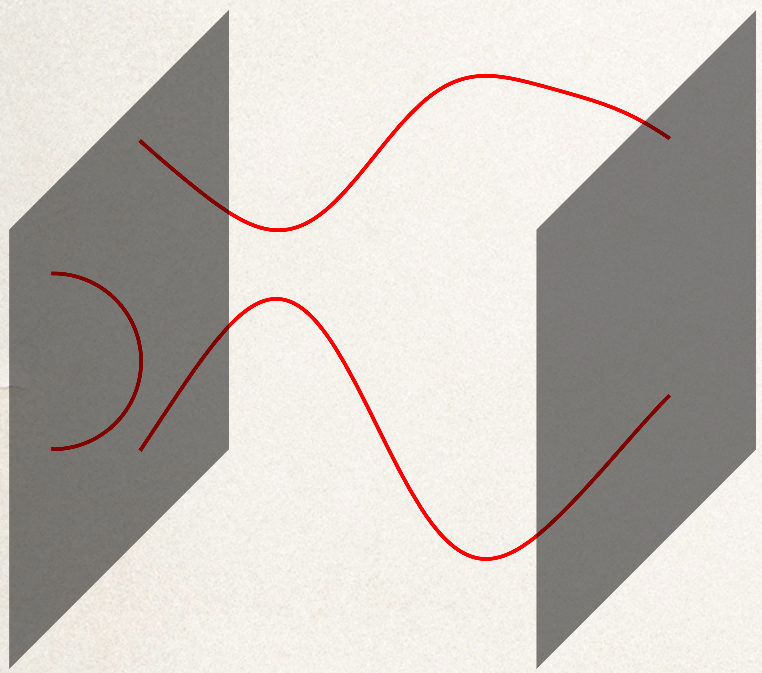
(Gupta, Huang, Jensen, Jeon, Shamir, Virrueta)

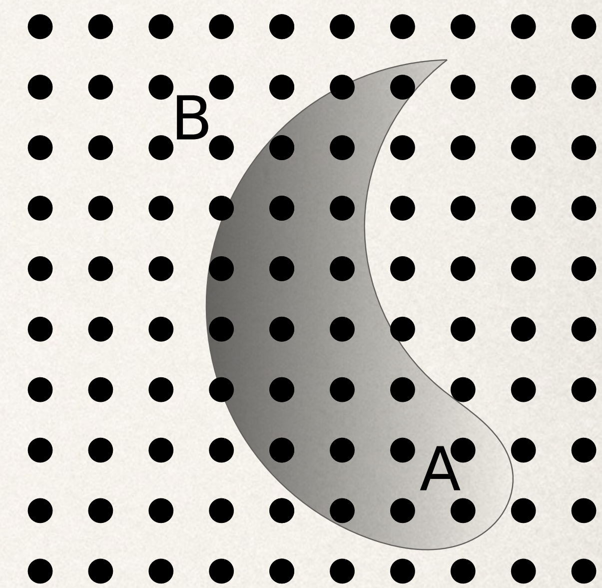
Outline

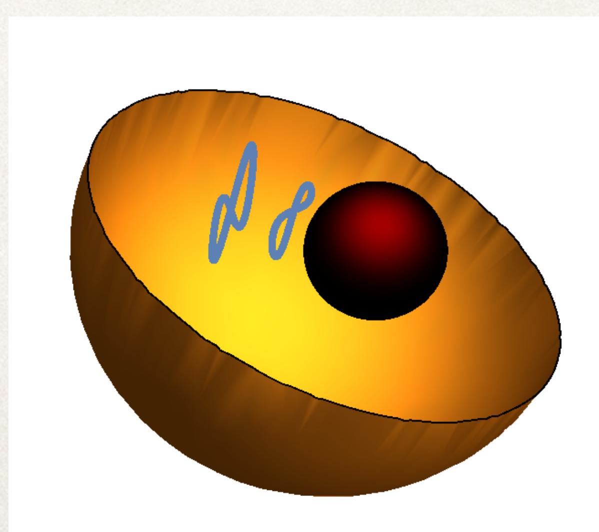
- Convince you that a system with a 3+1d photon and 2+1d electron (relativistic cousin of graphene) is extremely interesting.
- Use that system to compute conductivities exactly, at any value of the interaction strength.

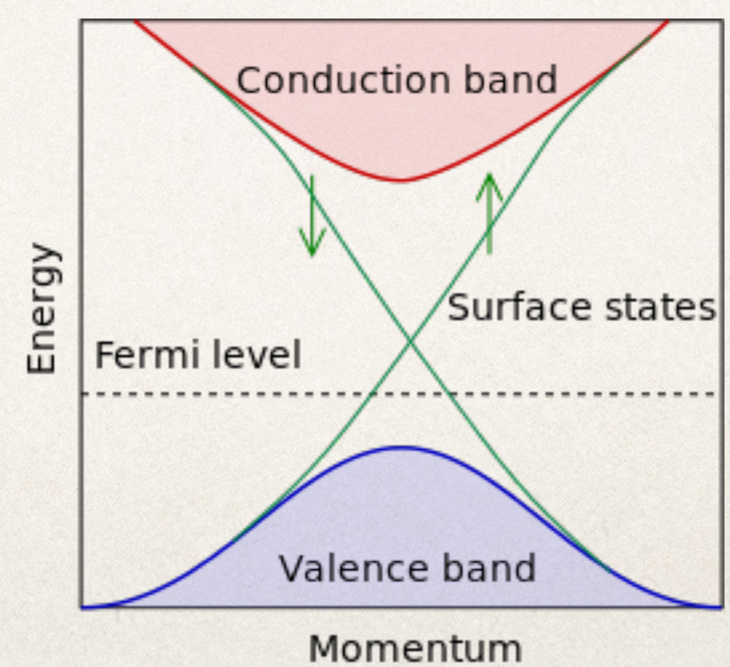
personal take on boundaries and defects:

Many of the most significant developments in high energy theory (and beyond) over the last thirty years involve boundaries and defects.

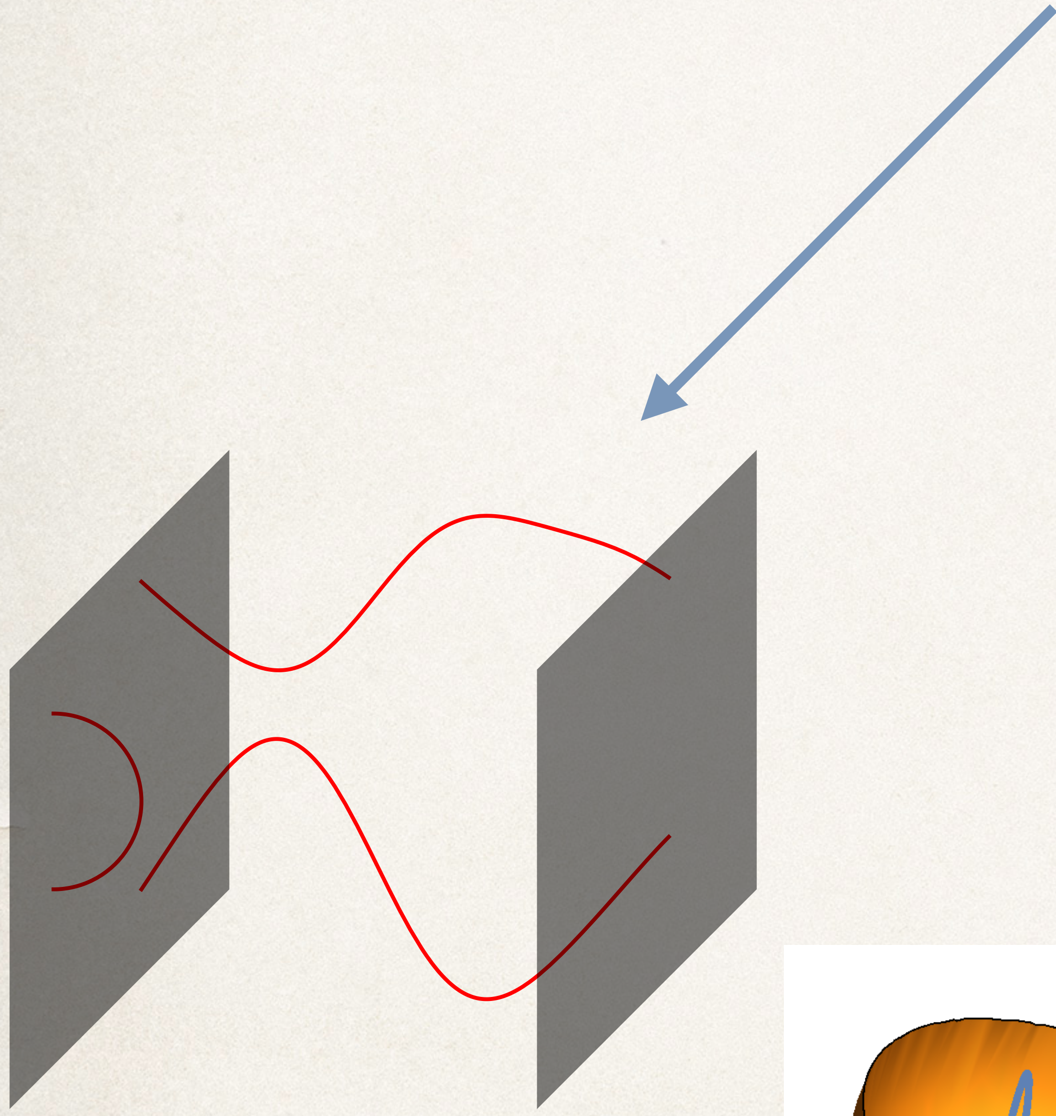


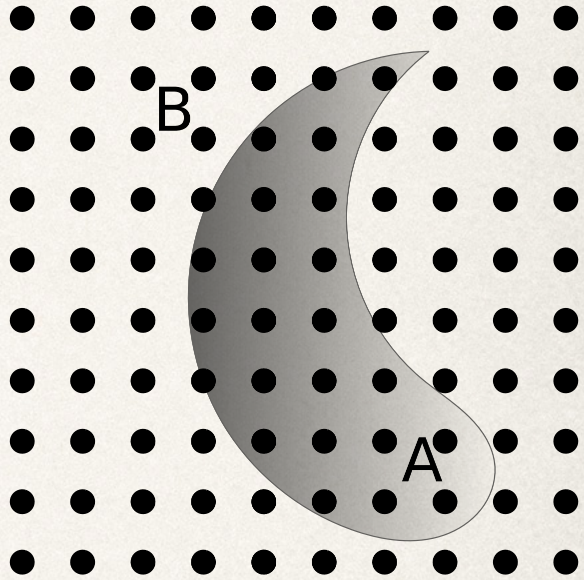


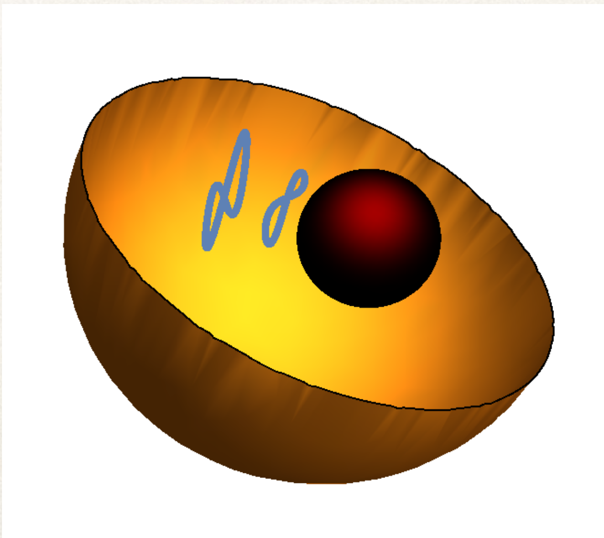


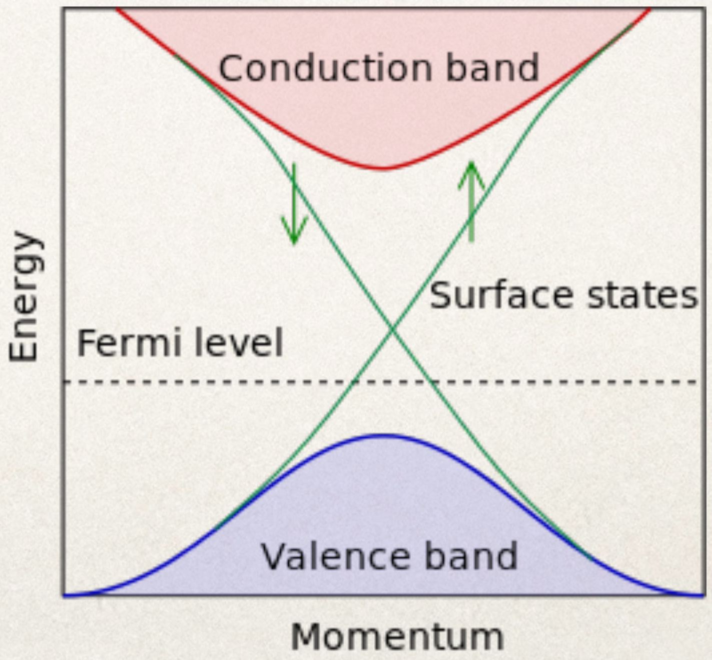


D-branes as boundary conditions for open strings

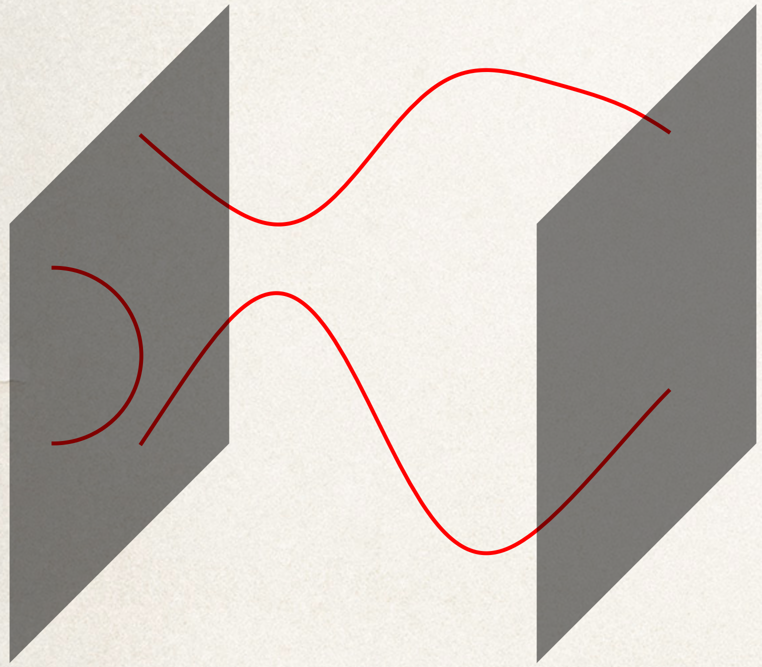


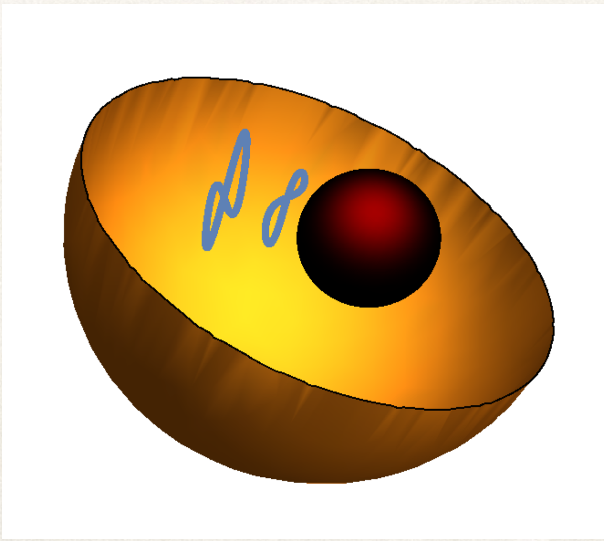


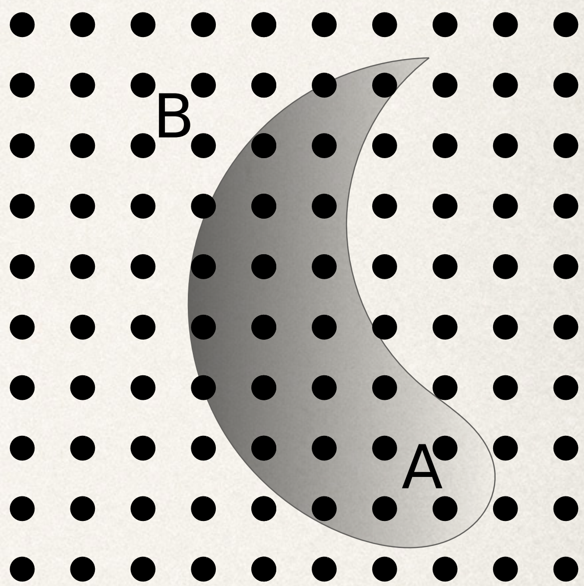


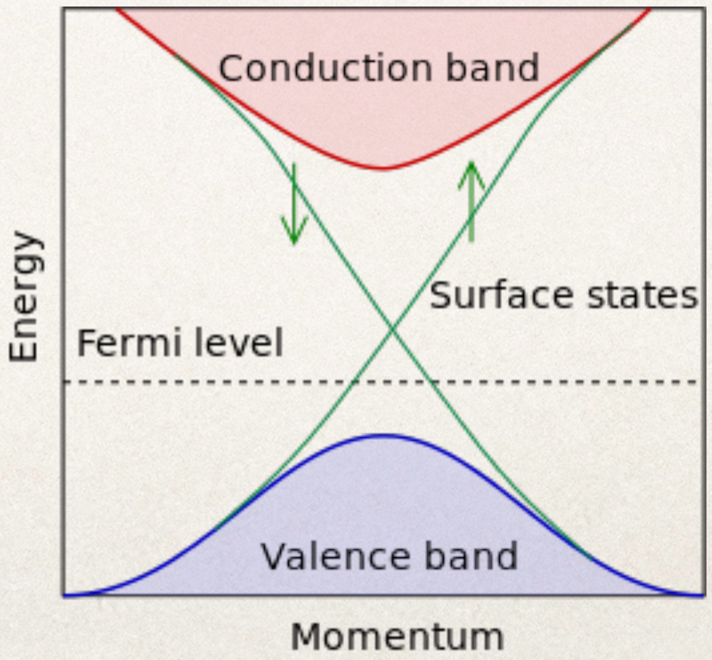


In AdS/CFT, conformal boundary of anti-de Sitter space is where the conformal field theory "lives".

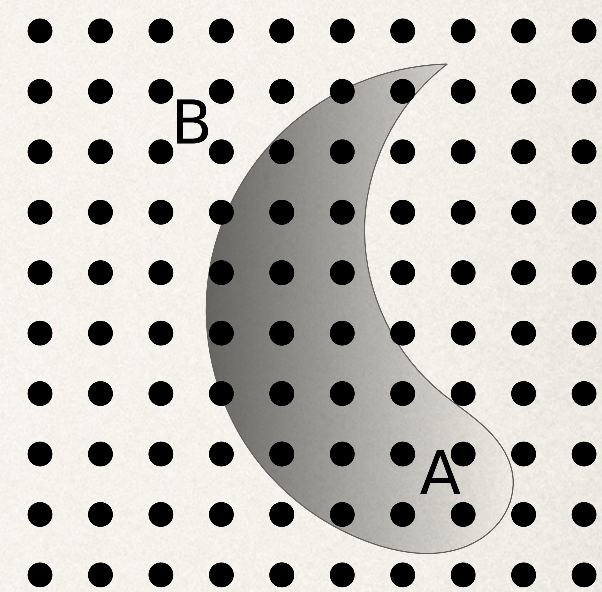


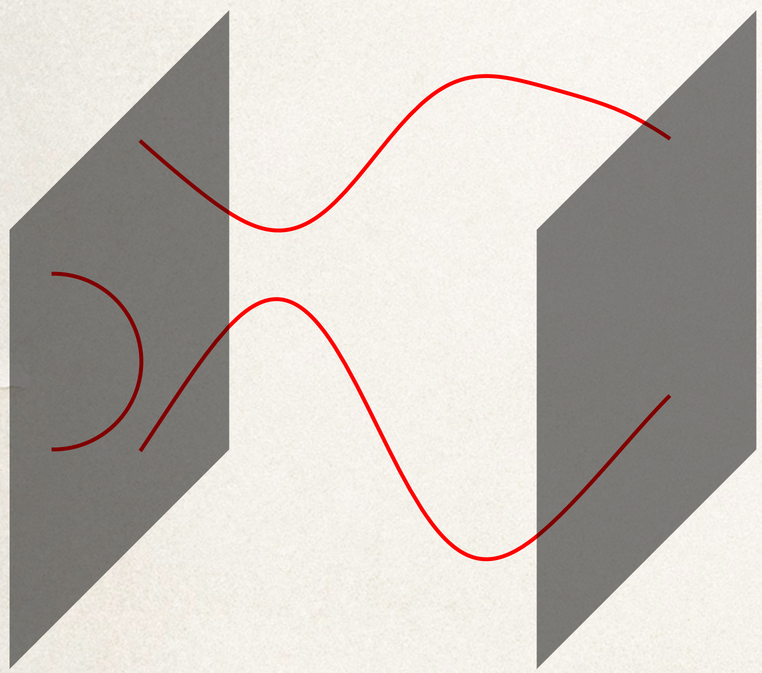


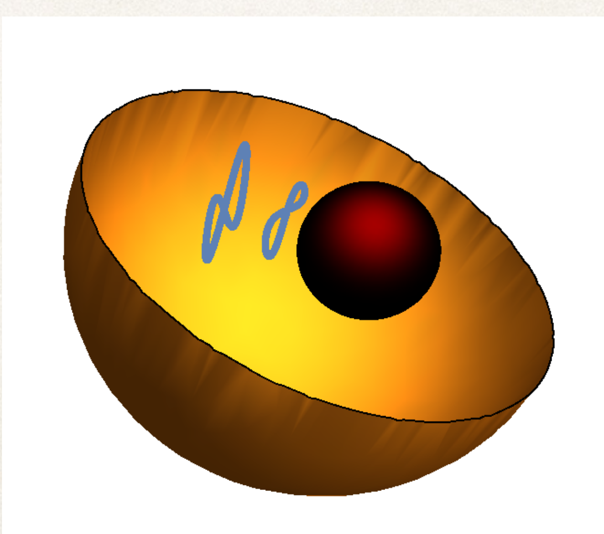


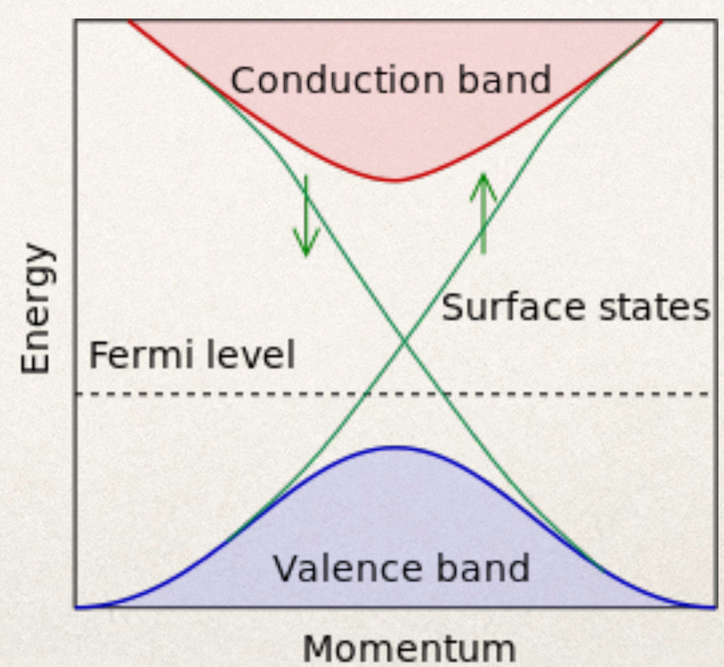


For topological insulators, the insulating bulk material has conducting (massless) surface states that are protected by symmetry.

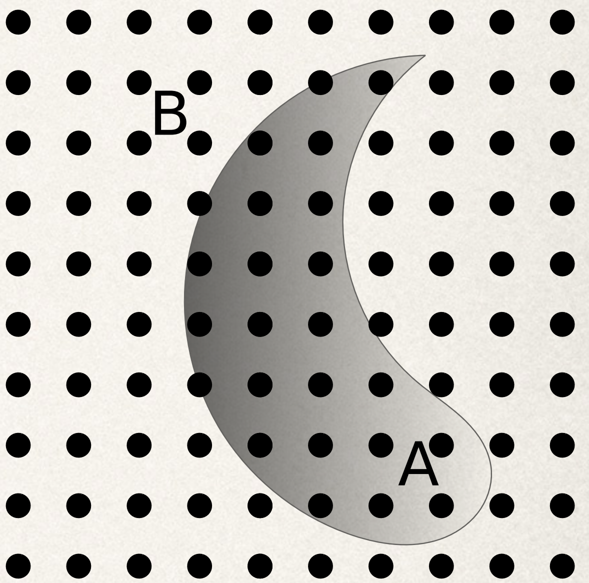


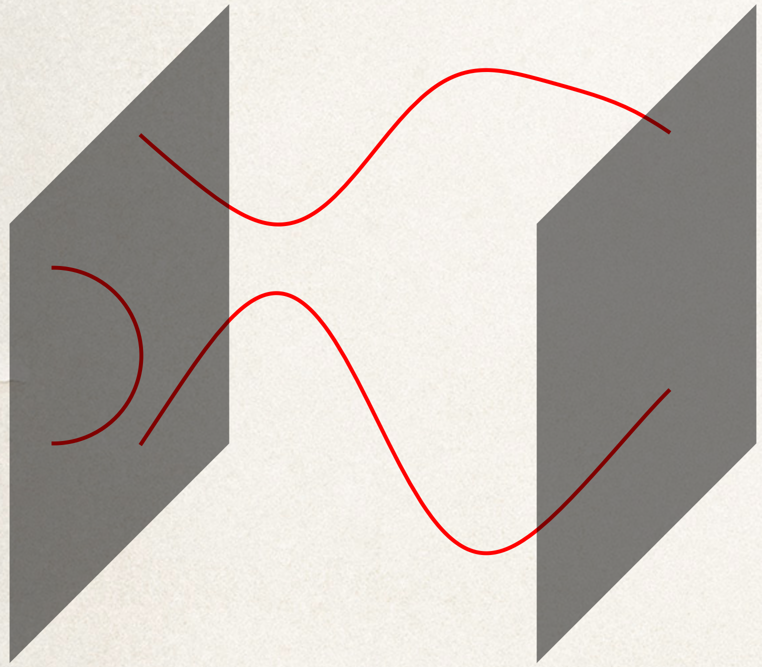


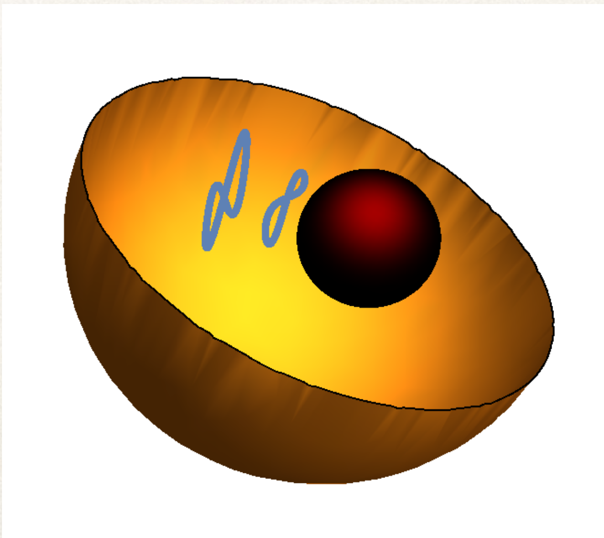


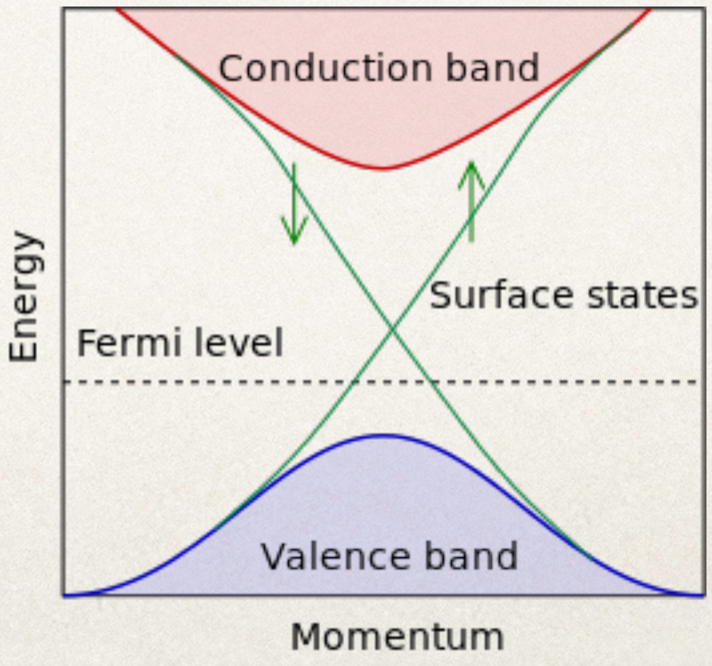


In field theory, entanglement is often measured with respect to spatial regions, leading to the importance of the “entangling surface”.









Would all of these developments have been “obvious” if we just understood quantum field theory in the presence of a boundary a little better to begin with?

I would like to convince you that studying

$$S = -\frac{1}{4}\int_{\mathcal{M}} \mathrm{d}^4x\, F^{\mu\nu}F_{\mu\nu} + \int_{\partial\mathcal{M}} \mathrm{d}^3x (i\bar{\psi}\not{D}\psi)$$

instead of the textbook

$$\boxed{D_\mu = \nabla_\mu - igA_\mu}$$

$$S = \int_{\mathcal{M}} \mathrm{d}^4x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}\not{D}\psi\right]$$

or

$$S = \int_{\mathcal{M}} \mathrm{d}^3x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}\not{D}\psi\right]$$

is a very interesting thing to do.

Mixed dimensional QED has something for everyone

$$S = -\frac{1}{4}\int_{\mathcal{M}} \mathrm{d}^4x\, F^{\mu\nu}F_{\mu\nu} + \int_{\partial\mathcal{M}} \mathrm{d}^3x (i\bar{\psi}\not{D}\psi)$$

where $D_\mu = \nabla_\mu - igA_\mu$ boundary conditions: $F_{nA} = g\bar{\psi}\gamma_A\psi$

- relation to graphene
- relation to large N_f QED$_3$ (Kotikov-Teber '13; Gaiotto '14)
- behavior under electric-magnetic duality (Son '17)
- example of a bCFT with an exactly marginal coupling
- conformal symmetry anomalies
- supersymmetric versions $\longrightarrow$
  - localization, exact results for transport

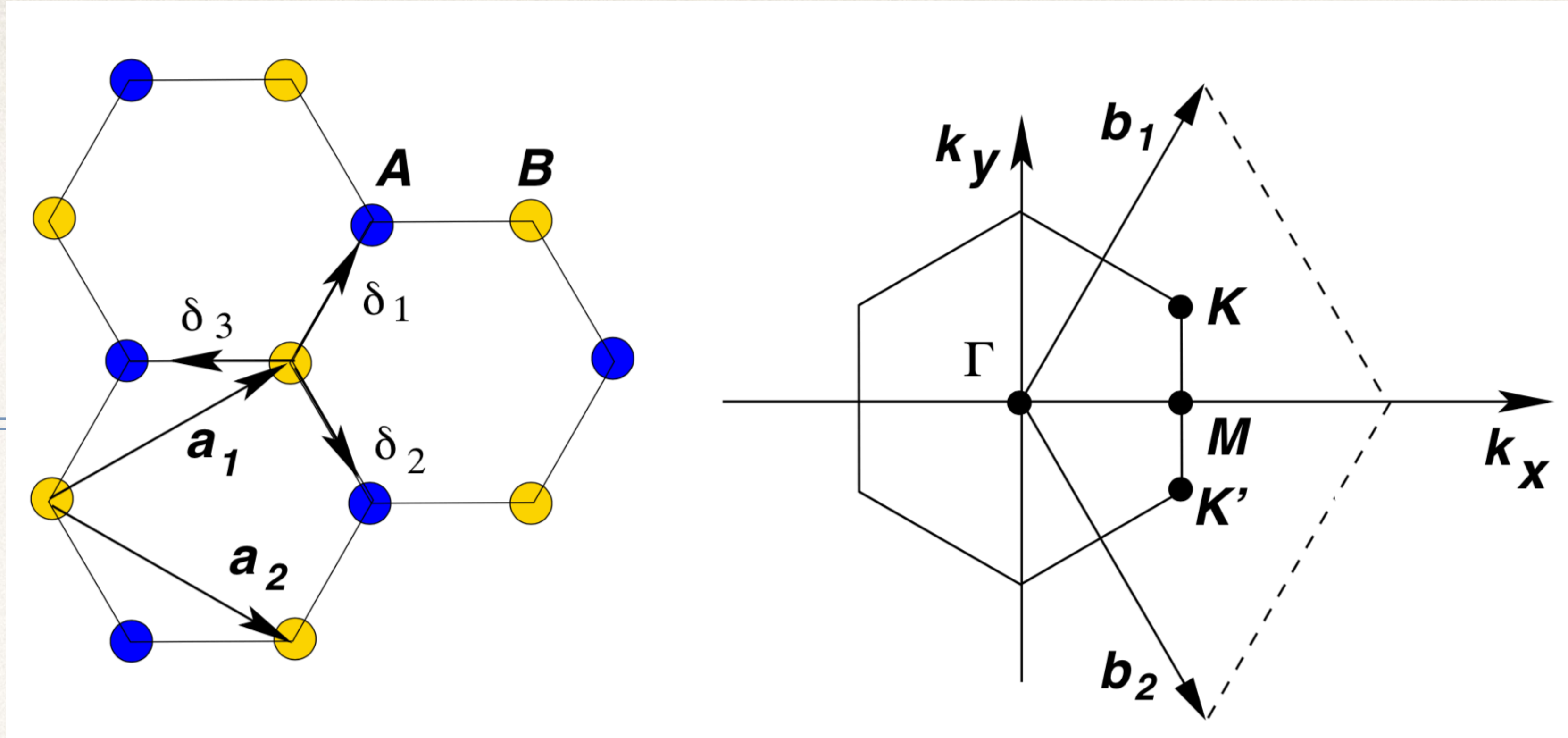
our work

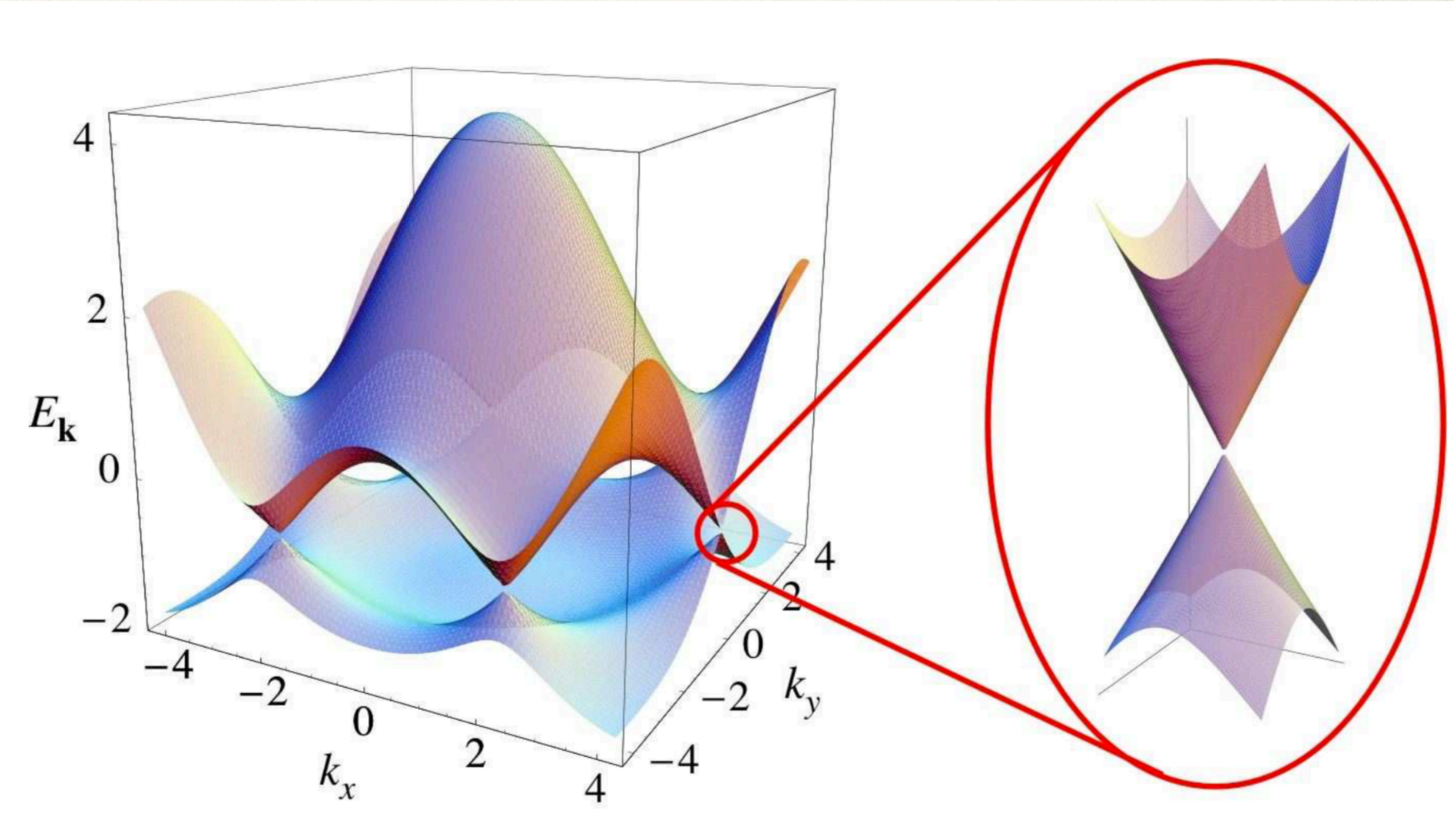
Graphene

A tight binding model with nearest neighbor hopping gives a nodal Fermi surface with linear dispersion relation.

$$E_{\pm}(\mathbf{q}) \approx v_F|\mathbf{q}| + O(q/K)^2$$

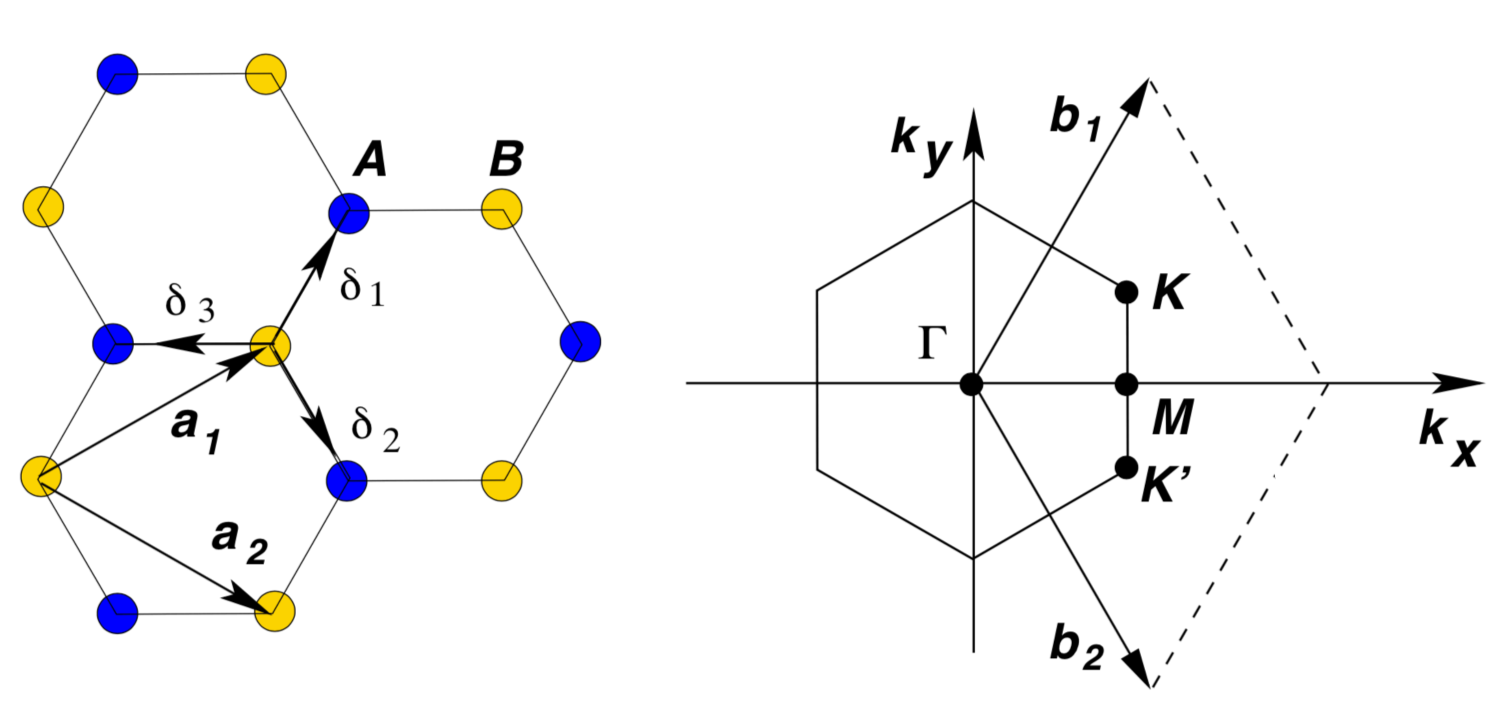
$$v_F \approx \frac{c}{300}$$





from Castro Neto et al. 2008

From Graphene to a Dirac Fermion



- There is an electron with momentum close to *K* that hops from the A lattice to the B lattice. This is one two component "relativistic fermion".

- Ditto for K' $\longrightarrow$ a second fermion

- The two electrons are related by time reversal and together assemble to make a four component Dirac fermion, but where spin has acquired a new peculiar meaning.

Interactions in Graphene

- Via phonons: Can be modeled by a gauge field that couples oppositely to the two two-component fermions. Like QED3 with a funny coupling to the photon.
- Via photons: Can be modeled by an essentially instantaneous Coulombic interaction. Magnetic effects are suppressed because v_F is so small.

Son's model of graphene

cond-mat/0701501

$$-\sum_{a=1}^{N}\int dt\, d^2x(\bar{\psi}_a\gamma^0\partial_0\psi_a + v\bar{\psi}_a\gamma^i\partial_i\psi_a + iA_0\bar{\psi}_a\gamma^0\psi_a) + \frac{1}{2g^2}\int dt\, d^3x(\partial_i A_0)^2$$

things to note

- only electric interactions
- electrons travel at speed $v \approx c/300$

beta function for the electron velocity

$$p\frac{\partial v(p)}{\partial p} = -\frac{4}{\pi^2 N}v(p)$$

v gets larger at low energies.
Moral sense in which our mixed QED is the IR fixed point of real world graphene.

Mixed QED is a bCFT

$$g_0 Z_{A_\mu}^{1/2} Z_\psi = g Z_g$$

The usual Ward identity for QED relates $Z_\psi = Z_g$

The superficial degree of divergence of the photon self energy is one (compared with two in four dimensional QED).

The gauge invariant prefactor $p_\mu p_\nu - \delta_{\mu\nu} p^2$ of $\Pi^{\mu\nu}(p)$ cuts down the degree of divergence to -1.

In other words, Z_{A_μ} is finite.

$\Longrightarrow$ coupling is not perturbatively renormalized.

Putting Graphene Like Theories to Work Computing Conductivity

The Plan:

We can compute the hemisphere partition function (or path integral) for super graphene exactly, as a function of the coupling, through a technique called localization. The partition function in turn will allow us to compute a variety of current-current correlation functions. The current correlation functions give us conductivities.

$\mathcal{N} = 2$ Super Graphene

$(A_\mu, \lambda, \lambda', X, Y)$

one photon,
two photini,
and two real scalars

4=4

(there is a simpler supersymmetric model,
but it won't allow us to use localization)

(ψ, ϕ)

electron and
complex scalar super partner

2=2

$\mathcal{N}=2$ Super Graphene

$$S_{\rm bulk} = \int_{\mathcal{M}} \mathrm{d}^4x \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\lambda}_i \gamma^\mu \partial_\mu \lambda^i - \frac{1}{2}(\partial_\mu X)^2 - \frac{1}{2}(\partial_\mu Y)^2 + \frac{1}{2}\vec{D}^2 \right)$$

$$\begin{aligned} S_{\rm bry} = \int_{\partial\mathcal{M}} \mathrm{d}^3x \Bigg(& -\frac{1}{4}\bar{\lambda}_i\, \vec{v}\cdot\vec{\tau}^{\,i}{}_j \gamma^5 e^{\eta\gamma^5}\lambda^j - X(\vec{v}\cdot\vec{D} + \partial_n X) \\ & + i\widetilde{\psi}\Gamma^A D_A \psi - |D_A\phi|^2 + |F|^2 + \sqrt{2} i g \big(\phi^* \,\widetilde{\lambda}_+\psi - \phi\,\widetilde{\psi}\lambda_+\big) \\ & + g\widetilde{\psi}Y\psi - g^2|\phi|^2 Y^2 - g(\vec{v}\cdot\vec{D} + \partial_n X)|\phi|^2 \Bigg) \end{aligned}$$

Claim: the coupling g is not renormalized.

We can add a theta angle too, suppressed here.

$$\boxed{D_\mu = \nabla_\mu - igA_\mu}$$

Where does our localization result come from?

- We put together a bunch of earlier results
- Localization of SUSY gauge theories on S^4 (Pestun ’07)
- Localization of Chern-Simons theories on S^3 (Kapustin, Willett, Yaakov ’09)
- Localization on a 4d hemisphere but without boundary degrees of freedom (Gava, Narain, Muteeb, Giraldo-Rivera ’16)

Localization

The partition function $Z = \int_M e^{-S(x)}$

Suppose we have a symmetry of the action $\delta S = 0$

(for us, it will be SUSY)

Construct a $W(x)$ such that $\delta^2 W(x) = 0$

Stokes Theorem

Modify the partition function: $Z(t) = \int_M e^{-S(x) - t\delta W(x)}$

Turns out it doesn't depend on t!

$$\frac{dZ}{dt} = -\int_M \delta W e^{-S(x) - t\delta W(x)} = -\int_M \delta(W e^{-S(x) - t\delta W(x)}) = 0$$

Localization Continued

As $Z(t)$ does not depend on t, we can make t as large as we want and evaluate by saddle point

$$Z(t) = \int_M e^{-S(x) - t\delta W(x)} dx$$

For cleverly chosen $W(x)$, the result can be much, much simpler than the original problem

In our case, a path integral over the function space of the 7 fields reduces to an ordinary integral

But first a trivial example…

A Trivial Example: Gaussian on the Plane

$$Z(t) = \int_M e^{-S(x) - t\delta W(x)}$$

$\delta = d + i_\theta$ rotational symmetry

$$e^{-S} = e^{-r^2}\left(\frac{1}{2} - r\,dr \wedge d\theta\right)$$

$$W = \frac{1}{2} r^2\, d\theta$$

$$\delta W = r\,dr \wedge d\theta + \frac{1}{2} r^2$$

$$\Rightarrow \quad -\int e^{-\left(\frac{t}{2}+1\right)r^2}\left(1 + \frac{t}{2}\right) r\,dr\,d\theta = -\int e^{-r^2} r\,dr\,d\theta$$

From which we learn we can do a change of variables: $r \to r\sqrt{\frac{t}{2} + 1}$

A Trivial Example: Gaussian on the Plane

$\delta = d + i_\theta$ rotational symmetry

$$e^{-S} = e^{-r^2}\left(\frac{1}{2} - r\,dr \wedge d\theta\right)$$

$$W = \frac{1}{2} r^2\, d\theta$$

$$\delta W = r\,dr \wedge d\theta + \frac{1}{2} r^2$$

Perhaps not a very good illustration of the power of the method....

$$\Rightarrow \quad -\int e^{-\left(\frac{t}{2}+1\right) r^2}\left(1 + \frac{t}{2}\right) r\,dr\,d\theta = -\int e^{-r^2} r\,dr\,d\theta$$

From which we learn we can do a change of variables: $r \to r\sqrt{\frac{t}{2} + 1}$

A more impressive use of the technique: super graphene on the hemisphere

classical terms

$$Z = \int d\sigma\, e^{i\pi\tau\sigma^2} \exp\left[n_+ \ell(1 - q_+ + i\sigma) + n_- \ell(1 - q_- - i\sigma) + 2\pi q_t \sigma\right]$$

one loop matter contributions

$$\tau = \frac{2\pi i}{g^2} + \frac{\theta}{2\pi}$$ complexified coupling

$n_\pm$ number of (ψ, ϕ) pairs on the boundary with scaling dimension $q_\pm$

q_t scaling dimension of a monopole operator

σ localization sets the value of one of the bulk scalars to this constant

$$\ell(z) = -z \log\left(1 - e^{2\pi i z}\right) + \frac{i}{2}\left(\pi z^2 + \frac{1}{\pi}\mathrm{Li}_2\left(e^{2\pi i z}\right)\right) - \frac{i\pi}{12}$$

Relation to the S^3 partition function

$$Z = \int d\sigma\, e^{i\pi\tau\sigma^2} \exp\left[n_+\ell(1-q_+ + i\sigma) + n_-\ell(1-q_- - i\sigma) + 2\pi q_t \sigma\right]$$

Kapustin, Willett, Yaakov (2009) found the same result for SUSY Chern-Simons theory on an S^3, but with τ equal to the integer Chern-Simons level

Since the bulk is free, we believe we can use many of the results that were derived in this case, e.g. relation to current-current correlation functions and the concept of extremizing $|Z|$ with respect to the q.

Symmetries

- The main reason we needed the more complicated SUSY model was to preserve a U(1) R-symmetry.
- The q parametrize how other abelian symmetries can mix with the U(1) R-symmetry. There is a particular mixture required to preserve conformal symmetry, minimizes $|Z|$.
- q_f flavor symmetry; q_t topological symmetry; q_g gauge symmetry
- The magic of localization will let us compute two point functions of currents associated with these symmetries.

Current-Current Correlation Functions

By symmetry, a current-current correlation function in a conformal field theory is fixed up to one complex number

$$\langle j_\mu(x) j_\nu(0) \rangle = \frac{\Pi}{16\pi^2}(\delta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\frac{1}{x^2} + \frac{i\kappa}{2\pi}\epsilon_{\mu\nu\rho}\partial^\rho \delta(x)$$

$$\Sigma \equiv \kappa + \frac{i\pi}{4}\Pi$$

This number is also the regular and Hall conductivity: $\Sigma = 2\pi(\sigma_H + i\sigma)$

The hemisphere and flat space are related by a conformal transformation. Thus Σ in flat space can be determined through a computation on the hemisphere.

Extremization

The dependence of Z on the q tells us these conductivities.

$$\Sigma = \frac{i}{2\pi}\partial_q^2 \log Z\Big|_{q=q_*}$$

(Closset, Dumitrescu, Festuccia, Komargodski, Seiberg 2012)

where q_* is determined by $\partial_q (Z\bar{Z})\big|_{q=q_*} = 0$

Fancier version of the R-extremization procedure of Jafferis (2010).

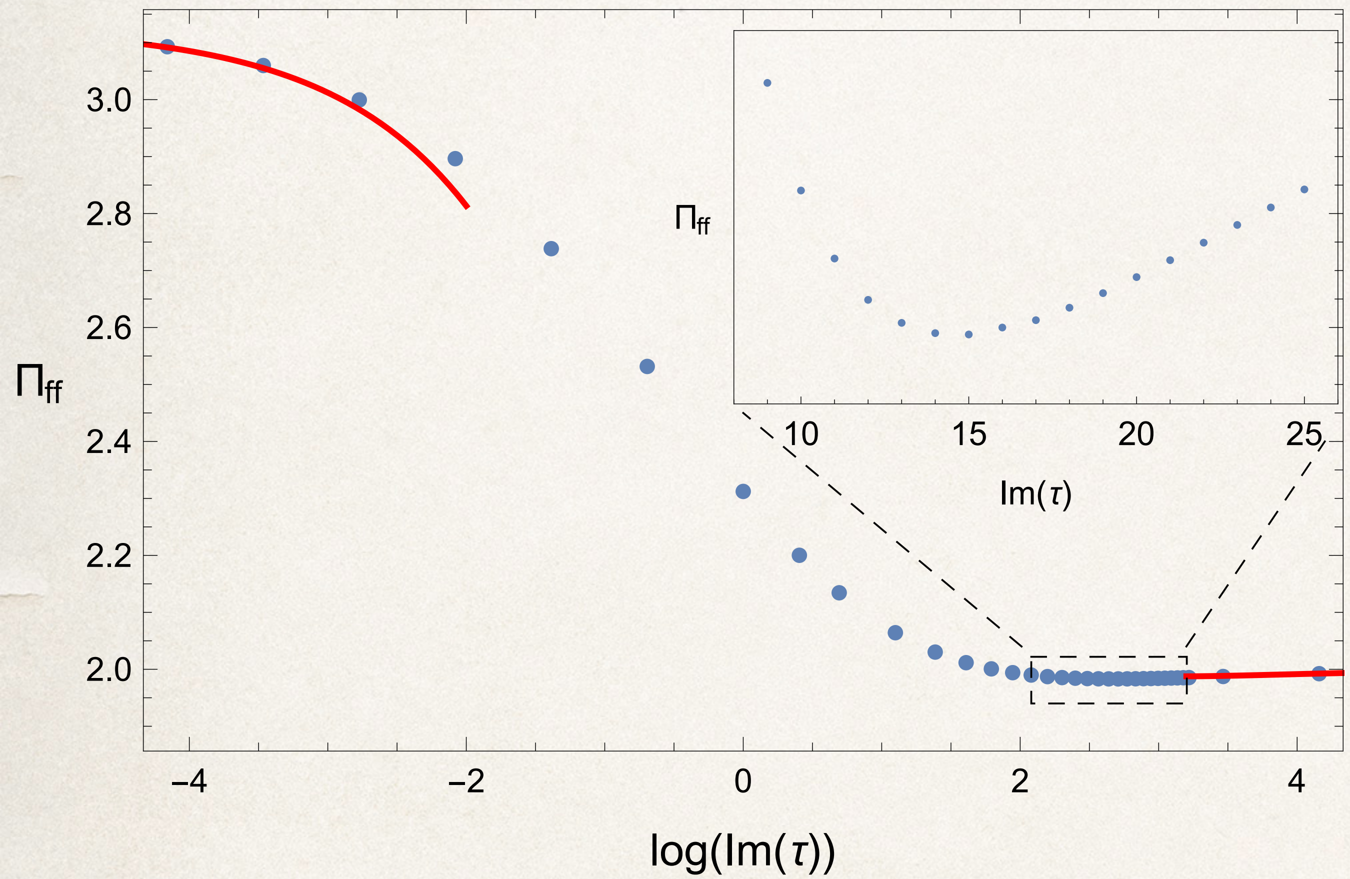
Relies on SUSY and the fact that the q are scalar components of vector super fields that couple to the respective symmetry currents.

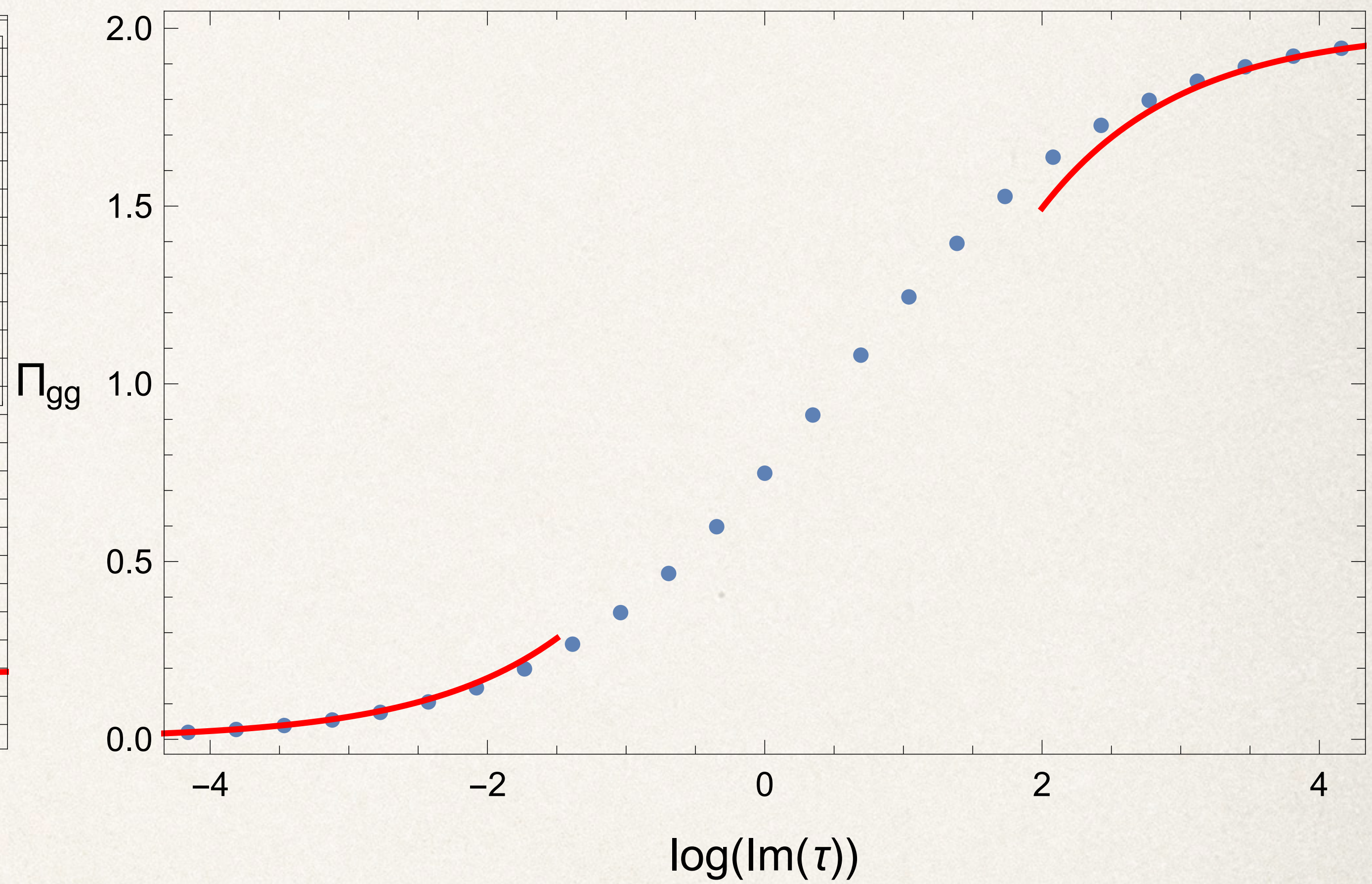
$$\tau = \frac{2\pi i}{g^2} + \frac{\theta}{2\pi}$$

An Exact Conductivity

$n_+ = 1\ ,\ \ n_- = 1$ theory at $\theta = 0$

$$q_\pm = q_f \pm q_g$$





points numerical

Very special that we can do this!

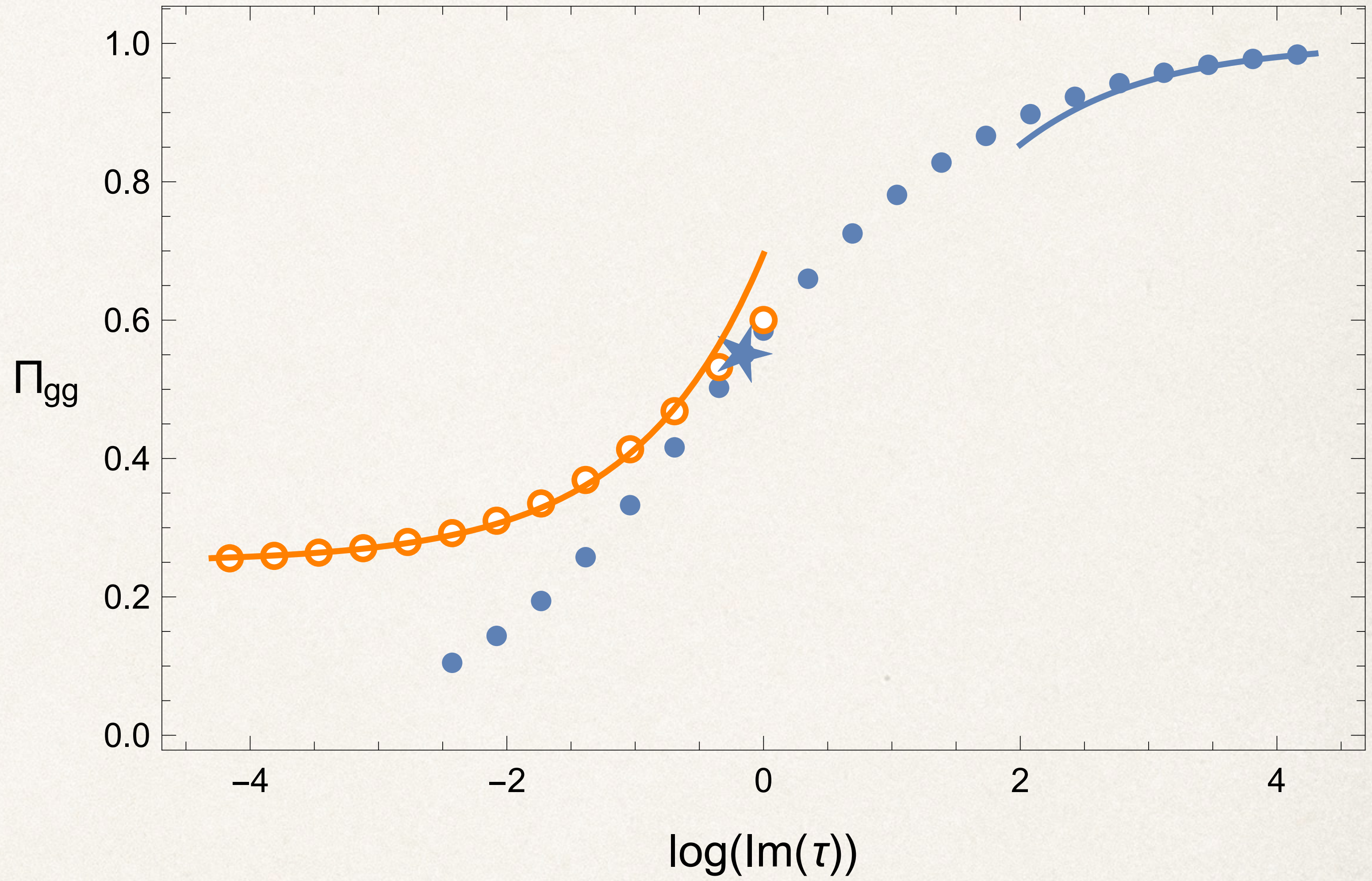
curves are saddle point approximations

Another Exact Conductivity

$n_+ = 1\,,\ \ n_- = 0$ theory

orange $\mathrm{Re}(\tau) = \dfrac{1}{2}$

blue $\mathrm{Re}(\tau) = 0$



Before addressing where the small τ curves come from and the meaning of the starred point, I want to make a few remarks.

Remarks

- We are usually limited to computations at weak coupling, and, when there is a known weak-strong coupling duality, strong coupling. Here we can do the computation at any value of the coupling.

- It may be possible to extend these results to other transport coefficients. Given the Lorentz and gauge symmetry, Ward identities can relate the charge conductivity to the thermoelectric coefficient and the heat conductivity (Herzog, 2009).

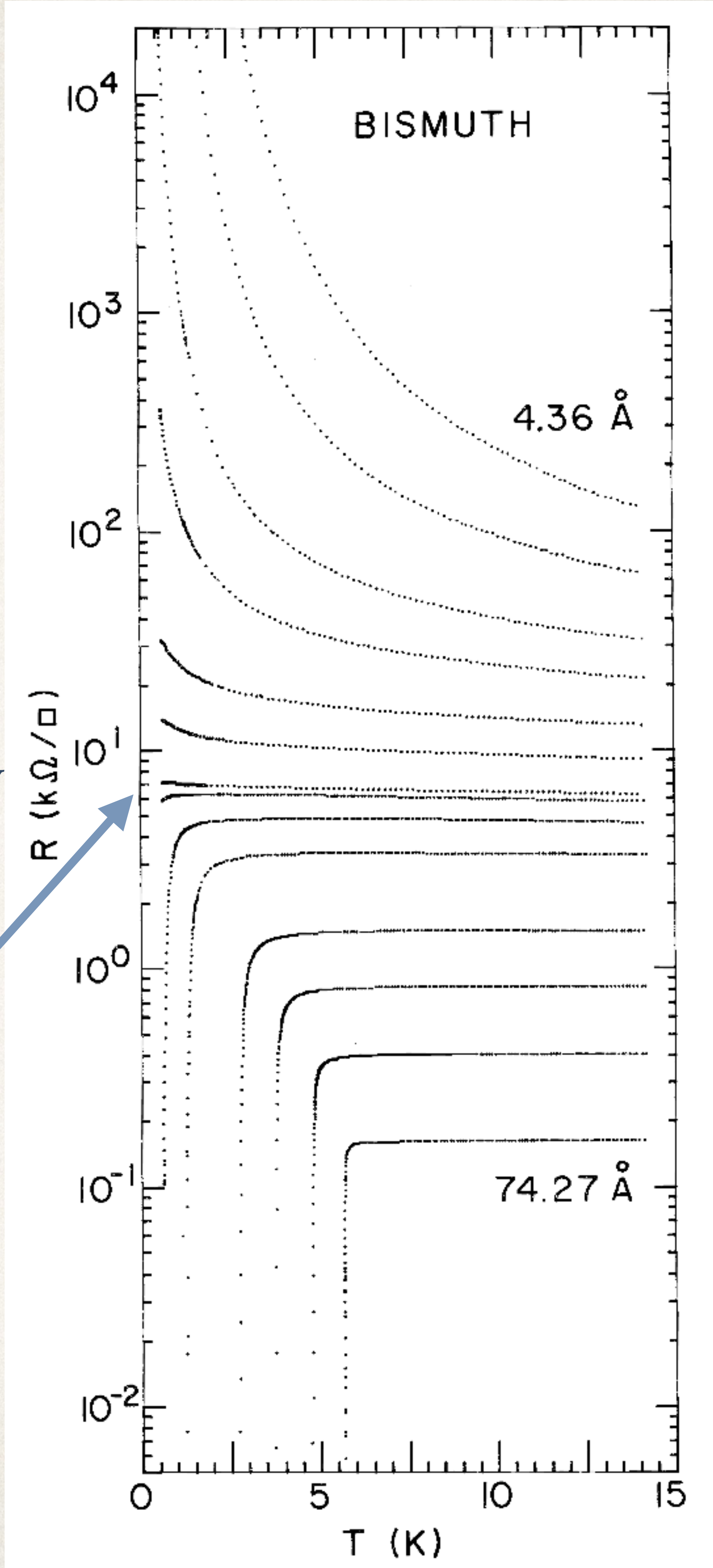
- This is a zero temperature result with no background charge density or magnetic field — may be a bit limiting in a condensed matter context however….

Thin Films and the Notion of a Quantum Phase Transition

thin films of bismuth

Phase transition at T=0 as a function of thickness of the sample

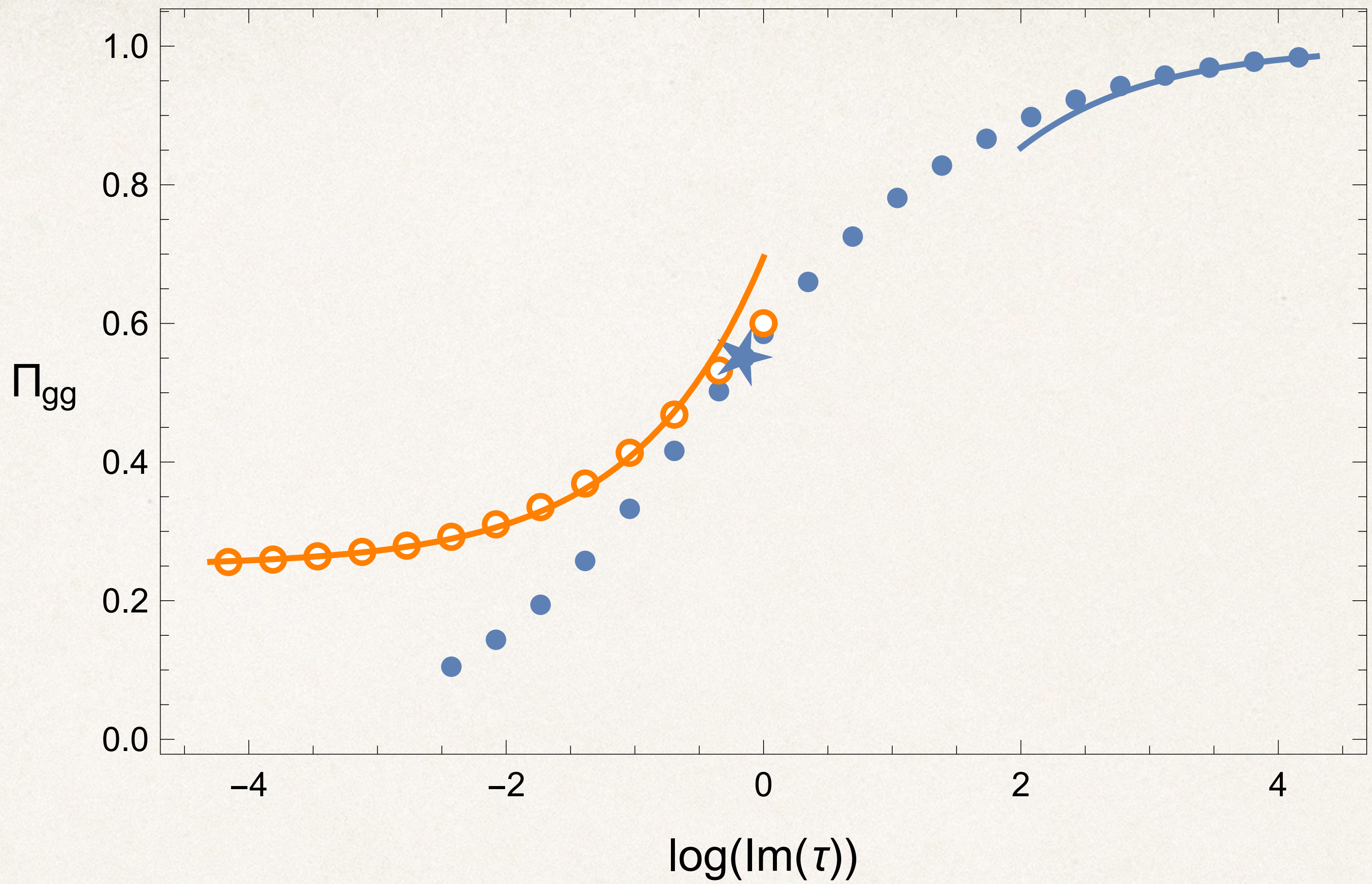
Resistivity



sense in which we are computing resistivity at the critical point in a similar system

Haviland, Liu, Goldman, PRL, 1989

Back to Duality



$n_+ = 1\,,\quad n_- = 0$ theory

How did we get the orange curve, and how do we explain that at the starred point $\Sigma_{gg} = \frac{\tau}{2}$?

There is a duality that maps this theory to a dual weakly coupled description when the original theory is strongly coupled.

Duality

$n_+ = 1\,,\quad n_- = 0$ theory

$$\tau' = -\frac{1}{\tau - \frac{1}{2}} - \frac{1}{2}$$

A TST element of SL(2, R)

$$\int d\sigma\, e^{i\pi\tau\sigma^2 + \ell(1-q_g+i\sigma) + 2\pi q_t \sigma} =$$

$$= \frac{e^{-\frac{3i\pi}{8}\left(q_g - \frac{1}{3}\right)^2 + \frac{i\pi}{2}q_t^2 - \frac{3i\pi}{2}\left(q_g-\frac{1}{3}\right)q_t + \frac{i\pi}{12}}}{\sqrt{-i(\tau - 1/2)}} \int dk\, e^{i\pi\tau' k^2 + \ell\left(\frac{q_g+1}{2} + q_t + ik\right) + 2\pi\left(\frac{3q_g - 1}{4} - \frac{q_t}{2}\right)k}$$

Note there is a self-dual point where $\boxed{\tau = \frac{i\sqrt{3}}{2} \qquad q_g = 1/3 \qquad q_t = 0}$

Duality lets us do a saddle point approximation close to $\tau = 1/2$.

Self-duality lets us calculate the conductivity -- the starred point.

Where did it come from?

There is a method for doing a Gaussian integral by contour integration that generalizes to our case

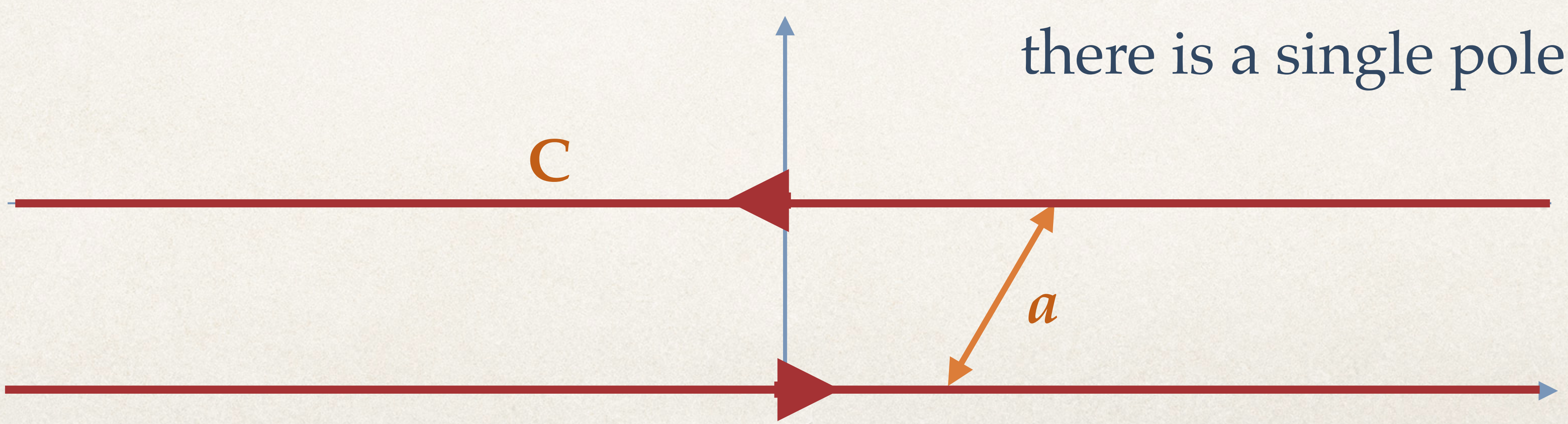
Given a "quasi-periodic" function $f(z+a)f(z-a) = f(z)^2$

$$\int f(\sigma)d\sigma = \oint_C \frac{f(z)}{1-\frac{f(z)}{f(z-a)}}dz$$

e.g. if $f(z) = e^{-\pi z^2}$

we can take $a^2 = i$

there is a single pole at $z = \frac{a}{2}$



Using this Contour Method

This contour method will only allow us to perform the partition function integral for rational τ, when the theory decouples and looks like 3d Chern-Simons.

Even though we can't do the integral in general, we can in general relate it to another through Fourier transform.

$$\int d\sigma e^{\frac{i}{2}\pi\sigma^2+2i\pi\sigma k+\ell(1-q_g+i\sigma)} = e^{-2i\pi k^2} e^{\ell\left(\frac{q_g+1}{2}+ik\right)-\frac{3i\pi}{8}\left(q_g-2ik-\frac{1}{3}\right)^2+\frac{i\pi}{12}}$$

This is the seed relation for producing the duality.

Conductivity at Self-Duality $\boxed{\Sigma_{gg} = \frac{\tau}{2}}$

$$\int d\sigma\, e^{i\pi\tau\sigma^2+\ell(1-q_g+i\sigma)+2\pi q_t\sigma} =$$

$$= \frac{e^{-\frac{3i\pi}{8}\left(q_g-\frac{1}{3}\right)^2+\frac{i\pi}{2}q_t^2-\frac{3i\pi}{2}\left(q_g-\frac{1}{3}\right)q_t+\frac{i\pi}{12}}}{\sqrt{-i(\tau-1/2)}} \int dk\, e^{i\pi\tau' k^2+\ell\left(1-q_g'+ik\right)+2\pi q_t' k}$$

where $\quad \tau' = TST(\tau)$

$$\begin{pmatrix} q_g' \\ q_t' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{4} \end{pmatrix} + (TST)^{-T} \begin{pmatrix} q_g \\ q_t \end{pmatrix}$$

$$\begin{pmatrix} J_g' \\ J_t' \end{pmatrix} = (TST) \begin{pmatrix} J_g \\ J_t \end{pmatrix} \quad \Rightarrow \quad \Sigma_{gg}' = \frac{1}{4}\Sigma_{gg} + \frac{3}{4}\Sigma_{gt} + \frac{9}{16}\Sigma_{tt}$$

$$\mathrm{TST} = \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\frac{3}{4} \\ 1 & -\frac{1}{2} \end{pmatrix}$$

Conductivity at Self-Duality Continued

$$Z = \int d\sigma \, e^{i\pi\tau\sigma^2} \exp\left[n_+\ell(1-q_f-q_g+i\sigma)+n_-\ell(1-q_f+q_g-i\sigma)+2\pi q_t\sigma\right]$$

Σ_{gg} , Σ_{gt} , and Σ_{tt} are not independent

performing a change of variables $\sigma \to \sigma - iq_g$

moves q_g out of the ell functions

$$\Sigma_{gg} = \tau + \tau^2\Sigma_{tt} \quad \text{and} \quad \Sigma_{gg} = \tau\Sigma_{gt}$$

$$\Sigma'_{gg} = \frac{1}{4}\Sigma_{gg} + \frac{3}{4}\Sigma_{gt} + \frac{9}{16}\Sigma_{tt} \quad \Rightarrow \quad \Sigma'_{gg} = \left(\frac{1}{4} + \frac{3}{4\tau} + \frac{9}{16\tau^2}\right)\Sigma_{gg} - \frac{9}{16\tau}$$

Last Steps

At self duality, we expect $\Sigma_{gg} = \Sigma'_{gg}$

However, that's not quite true because of the quadratic terms in q in our duality relation.

$$\int d\sigma\, e^{i\pi\tau\sigma^2+\ell(1-q_g+i\sigma)+2\pi q_t\sigma} =$$

$$= \frac{e^{-\frac{3i\pi}{8}\left(q_g-\frac{1}{3}\right)^2+\frac{i\pi}{2}q_t^2-\frac{3i\pi}{2}\left(q_g-\frac{1}{3}\right)q_t+\frac{i\pi}{12}}}{\sqrt{-i(\tau-1/2)}}\int dk\, e^{i\pi\tau' k^2+\ell\left(1-q'_g+ik\right)+2\pi q'_t k}$$

$$\Rightarrow \quad \Sigma_{gg}-\frac{3}{8} = \left(\frac{1}{4}+\frac{3}{4\tau}+\frac{9}{16\tau^2}\right)\Sigma_{gg}-\frac{9}{16\tau} \qquad \text{or} \qquad \Sigma_{gg} = \frac{\tau}{2}$$

Another self-dual example

We can play a similar game with the $n_+ = n_- = 1$ theory.

However, it turns out not to be self-dual. The dual theory has extra fields neutral under the gauged U(1) symmetry.

Playing with dilogarithm identities, there is a modification of the $n_+ = n_- = 1$ theory which is self-dual and in fact which only exists at the self-dual point $\tau = i$.

One finds again $\Sigma_{gg} = \dfrac{\tau}{2}$

Discussion of Self-Duality and Conductivity

- The idea that a complexified conductivity should transform under an element of SL(2,Z) is an old one in the condensed matter literature (e.g. Lutken and Ross '92).

- Usually Σ is taken to transform the same way as τ. (e.g. Witten '03, in a large n limit) $$\Sigma \to \frac{a\Sigma + b}{c\Sigma + d}$$

- Such a rule would imply $\Sigma = \tau$ at self-duality. However, we saw something a bit more intricate and $\Sigma = \tau/2$.

Summary

- Calculated the hemisphere partition function of super-graphene.
- Determined the conductivity exactly at arbitrary values of the coupling.
- Found a pair of theories that exhibit self-duality and determined the conductivity analytically at the point of self-duality.

Larger Vision: Structure of QFT

- Looking for new insights into QFT by studying defect and boundary CFT.
- Provide a more local view of QFT by figuring out how to deal with boundaries.

Thanks to my collaborators

- Rajesh Gupta (faculty, IIT Ropar)
- Kuo-Wei Huang (postdoc, Boston U)
- Kristan Jensen (faculty, San Francisco State)
- Imtak Jeon (postdoc, KCL)
- Itamar Shamir (postdoc, SISSA)
- Julio Virrueta (PhD student, Stony Brook)

Extra Slides

Mixed QED is a boundary conformal field theory

- Conformal field theories play a special role in the landscape of quantum field theories more generally — fixed points of the renormalization group flow.
- Mixed QED has vanishing beta function for g and no dependence on energy or (tangential) length scales.
- Slick argument — we can write the action so that the coupling appears multiplying the kinetic terms of the photons in the bulk. But the bulk is free.

$n_+ = n_- = 1$ theory

$$\int d\sigma e^{i\pi\tau\sigma^2+\ell(1-q_f-q_g+i\sigma)+\ell(1-q_f+q_g-i\sigma)+2\pi q_t\sigma} =$$

$$= \frac{e^{\ell(1-2q_f)}}{\sqrt{-i\tau}} \int dk\, e^{-i\pi k^2/\tau+\ell(q_f+q_t+ik)+\ell(q_f-q_t-ik)+2\pi(k-iq_t)q_g}$$

modified $n_+ = n_- = 1$ theory.

$$e^{\ell(q_f-1/2)+\ell(q_f)} \int d\sigma\, e^{i\pi\tau\sigma^2+\ell(1-q_f-q_g+i\sigma)+\ell(1-q_f+q_g-i\sigma)+2\pi q_t\sigma} =$$

$$= \frac{e^{\ell(1/2-q_f)+\ell(1-q_f)}}{\sqrt{-i\tau}} \int dk\, e^{-i\pi k^2/\tau+\ell(q_f+q_t+ik)+\ell(q_f-q_t-\imath k)+2\pi q_g k-2\pi i q_t q_g}$$

$\mathcal{N} = 1$ Super Graphene

$$S_{\text{bulk}} = \int_{\mathcal{M}} \mathrm{d}^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g^2\theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu} + \frac{i}{2} \bar{\lambda} \gamma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 \right)$$

$$S_{\text{bry}} = \int_{\partial\mathcal{M}} \mathrm{d}^3x \Big(i\widetilde{\psi} \Gamma^A D_A \psi - |\mathrm{D}_A \phi|^2 + |F|^2 + ig\big(\widetilde{\lambda}_+ \psi \phi^* - \widetilde{\psi} \lambda_+ \phi\big) \\ - \frac{1}{4} \bar{\lambda} \gamma^5 e^{\eta\gamma^5} \lambda - \frac{g^2\theta}{8\pi^2} \widetilde{\lambda}_+ \lambda_+ \Big)$$

Relation to large N_f QED$_3$

(Kotikov-Teber '13)

photon propagator for mixed dimensional QED
(don't FT the normal direction y)

$$-i\frac{e^{-py}}{p}\eta^{AB}$$ (Feynman gauge)

propagator for large N_f QED$_3$, resummed

$$-i\frac{\eta^{AB}}{p^2(1+\Pi(p))} \qquad \text{where} \qquad \Pi(p)=\frac{N_f e^2}{8|p|}+O(N_f^0)$$

Compensated by vertices, 3d e drops out of the amplitudes.
For scattering processes on the boundary (y=0),
the Feynman rules are the same in the IR with the identification $\frac{1}{N_f}\sim g^2$

Instability at small N_f / large g

- With N_f four-component massless fermions, there is a $U(2N_f)$ flavor symmetry.

- Arguments going back to Pisarski (1984) that spontaneous generation of a mass gap below a critical N_f can break the symmetry to $U(N_f) \times U(N_f)$ in QED3. Toy model for chiral symmetry breaking in QCD.

- Mixed QED gives physical meaning to a fractional value for the critical N_f.

Behavior under EM Duality

(Hsiao-Son '17)

Using recent progress in 2+1 dimensional non-SUSY dualities

$$\int d^3x \left[i\bar{\Psi}\gamma^A(\partial_A - ia_A)\Psi - \frac{1}{4\pi}\epsilon^{ABC}A_A\partial_B a_C \right] - \frac{1}{4g^2}\int d^4x F_{\mu\nu}^2$$

Integrating out a_B and A_μ yields same mixed QED theory but with a new

$\tilde{g} = 8\pi/g$ (one 2 component fermion)

Can use the duality to calculate the current-current and stress tensor correlation function at the self-dual point and at infinite coupling — calculate transport coefficients.

(similar in spirit to H, Kovtun, Sachdev, Son '07)

Transport Results

Ohm's Law: $J = \sigma E$

Boundary condition: $F^{nA}|_{\text{bry}} = \frac{g}{2}\bar{\psi}\gamma^A\psi$

$$\sigma_{xx} = \frac{J_x}{E_x} = \frac{2}{g}\left.\frac{F^{nx}}{F^{tx}}\right|_{\text{bry}} = \frac{2}{g}\left.\frac{\tilde{F}^{ty}}{\tilde{F}^{ny}}\right|_{\text{bry}} = \frac{2}{g\tilde{g}}\frac{\tilde{E}_y}{\tilde{J}_y} = \frac{2}{g\tilde{g}}\frac{1}{\tilde{\sigma}_{yy}}$$

At the self-dual point, $\sigma_{xx} = \tilde{\sigma}_{yy}$

$$\implies (\sigma_{xx})^2 = \frac{1}{4\pi}$$

(we used a similar argument back in 2007)

Relativistic Ward identities then allow one to compute heat conductivity and the thermoelectric effect (see my 2009 review).